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An Analysis of the Mathematics of Relativity

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Abstract

This article analyzes the mathematics of the Theory of Relativity.

Part 1: Dimensional Alchemy

Consider two towns 50 miles apart. If the distance between the towns (50 miles) is divided by the velocity of a train moving 50 miles per hour, it can be determined that it would require 1 hour for the train to travel the 50 miles:
i.e.,

50 miles / (50 miles per hour) = 1 hour,

or,

[Length]/ [Length/Time] = [Time].

It could be said, under these conditions, the two towns are 1 hour apart. In other words, it is common practice to divide a distance (from point A to point B) by a velocity, to determine the time interval necessary to travel from point A to point B.

Likewise, if the distance between the towns is unknown, it is common practice to multiply a velocity of a train by the time interval required for the train to travel from one town to the other (from point A to point B), to determine the distance between the towns (the distance from point A to point B). For example,

(50 miles per hour) (1 hour) = 50 miles,

or,

[Length/Time] [Time] = [Length].

These interpretations of defining (and measuring) distance and time, are applications of an equation for velocity, where average velocity is defined as the change in position of an object (displacement) divided by the time interval in which the change in position took place; or

velocity = displacement/time interval.

Using this equation as a basis, there are many instances in mathematics where time is defined as

time interval = displacement/velocity;

and distance is defined as

displacement = (velocity)(time interval).

For example, the term “light year” is used to describe the distance light travels in a year.

To make a another example, in Relativity, this interpretation of distance and time is applied in a thought experiment. This thought experiment sets out the conditions of a person traveling in a moving train, sitting on one side of the train looking out the window. Across from the person, on the opposite wall of the train (above the seats across the aisle), there is a mirror. The person (becoming bored) takes a small flashlight out of his pocket, and starts to sporadically send light pulses from his flashlight to the mirror to observe the reflected light (conceivably practicing morse code).

Under these conditions, it would appear to the person on the train that the light travels to the mirror and back in a straight line. However, from the vantage point of a stationary observer outside the train (perhaps a woman standing on the station platform as the train goes by), the displacement of the person traveling on the train, and the displacement of the light traveling from the flashlight to the mirror and back, would appear to form a triangle.

Consider Relativity’s interpretation of these conditions.

As the light appears to travel in a straight line (from the flashlight to the mirror) for the person on the train, and, in comparison, as the light appears to travel at an angle (from the flashlight to the mirror) for the woman standing on the platform, Relativity theorizes the light takes two paths - one path for the person on the train, and a different path for the woman standing on the station platform! These two hypothetical paths are then theorized to be of different lengths (i.e., the light is theorized to travel farther, from the flashlight to the mirror, for the woman on the platform than it does for the person on the train); and, as a consequence, the time interval for the travel of the light from the flashlight to the mirror and back is theorized to be different for the person on the train as compared to the woman standing on the platform. Relativity then draws the conclusion that (if the speed of the light is measured to be the same for both observers) the “passage” of time would be different for each observer. (Meaning, the wrist watch of the person on the train would run at a different rate than the wristwatch of the woman on the platform).

The most obvious problem with this Relativistic interpretation is - there are not two paths - there is only one path (one distance) the light pulse travels from the flashlight to the mirror and back. Though the path of the light might appear to be different for the two observers, the motion of the train is not splitting the light into two paths (causing the light to travel two separate distances). Therefore, as there is only one actual path (one distance) the light travels from the flashlight to the mirror, there is only one time interval for the light’s travel, and thus, the passage of time is really the same for both observers.

But in spite of this...
Relativity goes on to define the hypothesized two "different" paths (distances) of the light's travel, in terms of two hypothetically "different" time intervals.

For the person traveling on the train:
Relativity defines the distance between the flashlight and the mirror as being equal to one-half the time interval (that would be required for the light to travel from the flashlight to the mirror and back in a straight line) multiplied by the velocity of light, or

$$d = t_p c / 2.$$

In turn, Relativity defines the time interval (that would be required for the light to travel from the flashlight to the mirror and back in a straight line) as

$$t_p = 2d / c.$$

(Here, the velocity of light is represented by c, the distance from the flashlight to the mirror in a straight line is represented by d, and the Relativistic time interval for the person on the train is represented by t_p).

The question to ask concerning these two Relativistic equations is, "Does this circular method, of defining the distance by the time interval and then the time interval by the distance, provide a rigorous (and accurate) measurement for either distance or time?"

Now, for the woman standing on the station platform:
According to Relativistic theory, in the time interval required for the light to travel the distance from the flashlight to the mirror, the distance the mirror has moved with the train is defined as being equal to the velocity of the train multiplied by one-half the time interval required for the light to travel from the flashlight to the mirror and back, or

$$d_m = (v)(t_w) / 2.$$

Next, the time interval for the light to travel from the flashlight to the mirror and back (for the woman standing on the platform) is defined as

$$t_w = 2 d_m / v.$$

(Here, the term, d_m, represents the distance the mirror has moved with the train, the term, v, represents the velocity of the train, and the term, t_w, represents the Relativistic time interval for the woman standing on the platform).

As this circuitous method, of defining the distance by the time interval and then the time interval by the distance, does not allow for a direct calculation and comparison of the values of the two hypothetically "different" time intervals and distances (for the person on the train and the woman standing on the platform), Relativity then puts the hypothesized different distances involved in the light's movement into the form of the equation

$$(c\, t_w /2)^2 = (v\, t_w /2)^2 + d^{\,2}.$$

Dimensionally speaking this equation would be analyzed as,

[(velocity of light)(time interval)/ 2]^2 = [(velocity of train)(time interval)/ 2]^2 + [distance]^2,

or,

[(Length/Time) (Time)]^2 = [(Length/Time) (Time)]^2 + [Length]^2,

i.e.,

[Length]^2 = [Length]^2 + [Length]^2.

This equation is a form of the Pythagorean theorem, $z^2 = x^2 + y^2$ (an equation which describes the relationship between the lengths of the sides of a right triangle as the sides of three squares). However, in Relativity's interpretation, the lengths of two sides of the right triangle have been expressed in terms of velocity multiplied by time interval. If drawing the theoretical right triangle relationship expressed in this Relativistic equation, the distance the light has traveled (from the flashlight to the mirror) for the woman on the platform, $c\, t_w /2$, would be considered as the hypotenuse, the distance the mirror has moved with the train, $v\, t_w /2$, would be the base, and the distance the light has traveled (from the flashlight to the mirror) in a straight line for the person on the train, d, would be the altitude.

This equation, $(c\, t_w /2)^2 = (v\, t_w /2)^2 + d^{\,2}$, describing distances along the sides of a right triangle, is then developed in the manner of,

$$d^{\,2} = (c\, t_w /2)^2 - (v\, t_w /2)^2,$$

$$t_w = 2d/ (c^2 - v^2)^{½},$$

$$t_w = 2d/ [c\, (1- v^2 /c^2)^{½}].$$

You will note, here the Pythagorean equation solving for distance squared, $d^{\,2}$, has been developed into an equation solving for time, t_w. Can this be correct?

Dimensionally analyze the steps:
The first step, $d^{\,2} = (c\, t_w /2)^2 - (v\, t_w /2)^2$, has

[Length]^2 = {[Length/Time][Time]}^{2} - {[Length/Time][Time]}^{2}.

In this step, the dimensions of the equation still represent

$$[\text{Length}]^2 = [\text{Length}]^2 - [\text{Length}]^2.$$

However, the second step, $t_w = 2d/ (c^2 - v^2)^{½}$, has the dimensions of

$$[\text{Time}] = [\text{Length}]/\{[\text{Length/Time}]^2 - [\text{Length/Time}]^2\}^{½},$$

$$[\text{Time}] = [\text{Length}]/ [\text{Length}^2 /\text{Time}^2]^{½},$$

or,

$$[\text{Time}] = [\text{Time}].$$

Though the dimensions appear to be consistent throughout, has the Pythagorean right triangle (solving for length squared, i.e., area) just been transformed into a triangle of time? Have the dimensions of area been transformed into a dimension of time?

But, to step away from the dimensional analysis and continue with Relativity's development of the equation...

As the time interval for the person on the train was previously defined as

$$t_p = 2d/c,$$

Relativity defines the velocity of light as,

$$c = 2d/ t_p;$$

and, then, by selectively substituting the term $2d/t_p$ for c (note, $2d/t_p$ is substituted only for the c preceding the radical) in the equation,

$$t_w = 2d/ [c\ (1- v^2 /c^2)^{½}],$$

arrives at,

$$t_w = 2d/ [(2d/ t_p)\ (1- v^2 /c^2)^{½}],$$

$$t_w = t_p / (1- v^2 /c^2)^{½};$$

or,

$$t_w = t_p\ [1/ (1- v^2 /c^2)^{½}].$$

Then, stipulating the quantity,

$$1/ (1- v^2 /c^2)^{½},$$

is always greater than unity (one),

Relativity concludes the time interval for the person on the moving train, t_p , will be shorter than the time interval for the woman standing on the platform, t_w. In other words, Relativity theorizes the wristwatch of the person on the moving train will run slower than the wristwatch of the woman standing on the platform by a factor of $1/ (1- v^2 /c^2)^{½}$. This theory that the passage of time is affected by relative velocity is called time dilation.

There is, however, a problem with these equations (and theory of time dilation), specifically the quantity,

$$1/ (1- v^2 /c^2)^{½}.$$

This quantity is Relativity's Achilles Heel.

Remember, Relativity states the quantity, $1/ (1 - v^2 /c^2)^{½}$, is always greater than one. The obvious reason for this stipulation is, for the quantity, $1/ (1- v^2 /c^2)^{½}$, to equal one, the equation for time dilation,

$$t_w = t_p [1/ (1- v^2 /c^2)^{½}],$$

would have to be

$$t_w = t_p [1/ (1- 0)^{½}],$$

or,

$$t_w = t_p (1/1),$$

i.e.,

$$t_w = t_p.$$

In other words, the ratio of the velocity of the train to the velocity of light, v^2 /c^2, would have to equal zero: Meaning the velocity of the train, v, would have to be zero (the train would not be moving). So, this would not fulfill the conditions of relative movement for the thought experiment.

There is more to this. In analyzing the time dilation equation, $t_w = t_p [1/ (1- v^2 /c^2)^{½}]$, besides the obvious, that the ratio of the velocity of the train to the velocity of light, v^2 /c^2, cannot equal zero, there are two other possibilities to consider.

On one hand, if the ratio, v^2 /c^2, equaled one, the quantity, $(1- v^2 /c^2)$, would be equal to zero; and consequently, $1 /(1- v^2 /c^2)$, would be dividing by zero; and since division by zero is not allowed, this would render the equation for time dilation, $t_w = t_p [1/ (1- v^2 /c^2)^{½}]$, invalid.

On the other hand, if the ratio, v^2 /c^2, were greater than one, the quantity, $(1- v^2 /c^2)$, would be a negative number; thus, $1/ (1- v^2 /c^2)^{½}$, would be taking the square root of a negative number, and, as such, the quantity, $1/ (1- v^2 /c^2)^{½}$, in the time dilation equation, $t_w = t_p [1/ (1- v^2 /c^2)^{½}]$, would be an imaginary number.

So, it would appear, in the time dilation equation,

$$t_w = t_p [1/ (1- v^2 /c^2)^{½}],$$

the ratio, v^2 /c^2, must be greater than zero but less than one.

These limits of the ratio, v^2 /c^2, can be interpreted in terms of velocity as:

1. The limitation that, v^2/c^2, cannot equal zero - indicates the train must be moving.
2. The limitation that, v^2/c^2, cannot equal one - indicates the velocity of the train, v, cannot equal the velocity of light, c.
3. The limitation that, v^2/c^2, cannot be greater than one - indicates the velocity of the train, v, cannot exceed the velocity of light, c.

In other words, the limits for the ratio, v^2/c^2, in the Relativistic equation, $t_w = t_p\,[1/(1- v^2/c^2)^{½}]$, make it theoretically impossible for the velocity of a moving object, v, to equal or exceed the velocity of light, c.

The question is, though, "Is it valid for Relativity to develop an equation that sets the velocity of light as a theoretical, universal speed limit (or barrier) - where no moving object, v, can equal or exceed the velocity of light, c?"

Part 2: Tennis Ball Relativity

To address this question, return to the initial thought experiment of the person on a train. But this time (rather than having a light signal traveling from a flashlight to a mirror and back), imagine the person on the train throwing a tennis ball across the aisle, against the opposite wall of the train (playing a game of solitary catch).

Under these conditions, it would appear to the person on the train that the tennis ball travels to the wall and back in a straight line. However, from the vantage point of a stationary observer outside the train (the woman standing on the station platform), the displacement of the person traveling on the train, and the displacement of the tennis ball traveling to the opposite wall of the train and back, would appear to form a triangle. So, because of the perceived difference in the path of the tennis ball, Relativity could (in keeping with its previous logic) theorize the tennis ball takes two paths - one for the person on the train and a different path for the woman on the platform! Relativity could then theorize the two hypothetical paths are of different lengths (i.e., the ball will travel farther, to the wall and back, for the woman on the platform than it does for the person on the train); and, as a consequence of this theoretical difference in length, draw the conclusion that the time interval for the ball to travel to the wall and back will be different for the person on the train as compared to the woman on the platform.

Relativity could, next (after defining the distance by the time interval and the time interval by the distance), put the hypothesized different "distances" involved in the tennis ball's movement into the form of the Pythagorean equation for a right triangle (noting the term, b, represents the velocity of the tennis ball),

$$(b\,t_w/2)^2 = (v\,t_w/2)^2 + d^2,$$

i.e.,

[(velocity of the ball)(time interval)/ 2]^2 = [(velocity of train)(time interval)/ 2]^2 + [distance]^2 ;

and, proceeding as before, generate an equation for time dilation of,

$$t_w = t_p\,[1/\,(1- v^2/b^2)^{1/2}].$$

So, Relativity (applying its own logic and mathematical structure as a model) could create an equation to indicate the passage of time would be slower (would dilate) for the person on the train, in relation to the passage of time for the woman standing on the platform by a factor of

$$1/\,(1- v^2/b^2)^{1/2},$$

i.e.,

$$1/\,\{1- (\text{velocity of the train})^2/\,(\text{velocity of the tennis ball})^2\}^{1/2}.$$

Also, as this equation would require (to maintain its mathematical "integrity") that the ratio of the velocity of the train to the velocity of the tennis ball, v^2/b^2, must be greater than zero and less than one, Relativity, with this equation, $t_w = t_p\,[1/\,(1- v^2/b^2)^{1/2}]$, would have just set the velocity of the tennis ball as a universal speed limit - where the velocity of the train could approach, but could never equal or exceed the velocity of the tennis ball!

So, does time dilate according to the ratio of the velocity of train to the velocity of the light, as in the equation,

$$t_w = t_p\,[1/\,(1- v^2/c^2)^{1/2}];$$

or, does time dilate according to the ratio of the velocity of the train to the velocity of the tennis ball, as in the equation,

$$t_w = t_p\,[1/\,(1- v^2/b^2)^{1/2}]?$$

The question is, "Which should be considered as the universal speed limit (or barrier); the velocity of light or the velocity of the tennis ball?"

The point being, as both of these time dilation equations are developed with the same Relativistic logic, both equations should be equally true, or equally false. Obviously, as Relativity can only theorize one universal speed limit, both equations cannot be true. Therefore, as both cannot be equally true, it follows that both time dilation equations are equally false; and, as such, the logic structure of Relativity is invalid.

To look at this in another way, as Relativity postulates, with the equation

$$t_w = t_p\,[1/\,(1- v^2/c^2)^{1/2}],$$

that the speed of light is measured the same for all observers regardless of their motion; should it then be postulated, with the equation

$$t_w = t_p\,[1/\,(1- v^2/b^2)^{1/2}],$$

the velocity of the tennis ball is measured the same for all observers regardless of their motion?

To consider this question: Even with it established that the velocity of the tennis ball would be different relative to each of the observers (and, the velocity of the train could equal or exceed the velocity of the tennis ball), if Relativistic logic theorized the perceived difference in the path of the tennis ball caused the tennis ball to travel two separate paths (i.e., a different distance and a different time interval for the person on the train as compared to the woman on the platform), Relativity, could still produce an equation for time dilation of,

$$t_w = t_p [1/ (1- v^2 /b^2)^{½}];$$

which (in requiring the ratio, v^2 /b^2, to be less than one) would fictitiously set the velocity of the tennis ball as a universal speed barrier the velocity of the train could not equal or exceed, and imply the velocity of the tennis ball is measured the same for both observers.

Thus, as it is possible (with Relativistic logic) to develop an equation that contradicts reality in the thought experiment concerning the tennis ball, the same holds true for the thought experiment concerning the velocity of light: That is to say, regardless of whether the velocity of light is defined as having the same value, or a different value, for each of the observers, by hypothesizing two different distances for the light to travel, Relativity creates a bias that allows for the development of an equation for two different time intervals, i.e., an equation for time dilation of

$$t_w = t_p [1/ (1- v^2 /c^2)^{½}].$$

To emphasize this, even if an experiment were set up with conditions where a light pulse traveled one set distance (from a light source to a mirror and back); if Relativity chose to theorize the light pulse traveled two separate distances and two time intervals, Relativity could, regardless of the realities of the experiment, develop an equation for time dilation. In other words, all that would be required for Relativity to create a time dilation equation is for something traveling one distance to be said to be traveling two distances.

The point being, the Relativistic time dilation equation does not indicate anything about velocity relative to light, or about the velocity of light itself. In examining the equation,

$$t_w = t_p [1/ (1- v^2 /c^2)^{½}],$$

with the ratio, v^2 /c^2, having to be greater than zero and less than one, regardless of what velocity is substituted for the symbol, c, the time dilation equation will automatically set c as a universal speed limit. It is fair to say, with its equation, Relativity could theoretically set the velocity of any moving object (a tennis ball, a snail, even the train itself, etc.) as a universal speed barrier the velocity of a train, v, could not equal or exceed, and imply that velocity to be measured the same value for all observers.

So, the engine that drives the time dilation equation, the ratio of velocities, v^2 /c^2, has no significance in terms of relative velocity: The ratio is only being employed as a deus ex machina (an artificial mathematical device) to allow Relativity to appear to have solved the difficulties concerning relative motion and the velocity of light.

This brings up the last point: Despite its mathematical ingenuity (and countless experiments transmitting light signals back and forth between mirrors), Relativity has not been able to produce the results projected by its equations and hypotheses concerning relative motion and the velocity of light. However, instead of coming to the conclusion that its theory (and equations) could be incorrect, Relativity asserts "this very lack of evidence" is proof of the theory itself!

To explain this logic (as if this logic could be explained), Relativity theorizes "this lack of evidence" demonstrates another phenomena - that of length contraction. In other words, not being able to produce results to support its time dilation equation, Relativity has developed another equation, namely,

$$L = L_1 \, (1- v^2 /c^2)^{½},$$

to theorize that length contracts along the line of direction of travel by a factor of $(1- v^2 /c^2)^{½}$, thus making it impossible to detect or measure the differences Relativity hypothesizes would occur in the distance and time of light's travel due to relative motion.

There are three reasons this Relativistic logic should be brought into question:

First, to state the obvious, with the time dilation equation being "supported" by the length contraction equation, what is supporting the length contraction equation? Relativistic logic sets up the potential for the development of an infinite series of unsupportable equations.

Second, to say that "the lack of proof " of one equation is supported by "the lack of proof " of another equation seems ludicrous. In lieu of rigorous proof, Relativity seems to be offering the equations themselves as proof of the theory: The mathematical logic is not supported, just perpetuated.

To make a comparison, as Relativity "supports" its time dilation equation with a length contraction equation, would it be valid, then, to support the "Tennis Ball Relativity" time dilation equation, $t_w = t_p \, [1/ (1- v^2 /b^2)^{½}]$, with a "Tennis Ball Relativity" length contraction equation, such as,

$$L = L_1 \, (1- v^2 /b^2)^{½};$$

to theorize that length contracts along the line of direction of travel by a factor of $(1- v^2 /b^2)^{½}$, i.e.,

$$\{1 - (\text{velocity of the train})^2 / (\text{velocity of the tennis ball})^2\}^{½} \; ?$$

Again, supporting one unsupportable equation with another seems mathematically unrealistic.

Third, there is a mathematical inconsistency between the Relativistic equation for time dilation and the equation for length contraction.

In comparing the time dilation equation,

$$t_w = t_p [1/ (1- v^2 /c^2)^{½}],$$

in the form of,

$$t_w = t_p / (1- v^2 /c^2)^{½},$$

to the length contraction equation,

$$L = L_1 (1- v^2 /c^2)^{½},$$

note, where the time dilation equation divides dimension by the quantity, $(1- v^2 /c^2)^{½}$, the length contraction equation multiplies dimension by the quantity, $(1- v^2 /c^2)^{½}$. That is, from one equation to the other, the quantity, $(1- v^2 /c^2)^{½}$, changes from being a divisor to a multiplicand: Meaning, the limits for the ratio, v^2 /c^2, are not the same in the two equations.

To explain, in analyzing the time dilation equation,

$$t_w = t_p / (1- v^2 /c^2)^{½},$$

as division by zero is not allowed in mathematics, the quantity, $(1- v^2 /c^2)^{½}$, cannot equal zero; which means the ratio, v^2 /c^2, cannot equal one, and thus, the velocity of v cannot equal the velocity of c.

However, in analyzing the length contraction equation,

$$L = L_1 (1- v^2 /c^2)^{½},$$

as multiplication by zero is allowed in mathematics, here, the quantity, $(1- v^2 /c^2)^{½}$, can equal zero; meaning the ratio, v^2 /c^2, can equal one: Therefore, in this equation, the velocity of v can equal the velocity of c.

So, with the limits of the ratio, v^2 /c^2, in the Relativistic time dilation equation indicating no moving object, v, can equal the speed of light, and the limits of same ratio, v^2 /c^2, in the length contraction equation indicating the velocity of a moving object, v, can equal the speed of light, c, the two Relativistic equations present a contradiction - each express a completely different theory concerning the velocity of light and velocity relative to light.

In other words, because there is a difference in the limit for the ratio, v^2 /c^2, and, thus, a difference in what this ratio represents in these two equations, the length contraction equation does not mathematically, nor theoretically, support the time dilation equation.

Therefore, the basic postulate of Relativistic theory, that the speed of light is measured to have the same value for all observers and no moving object can equal or exceed the velocity of light, does not even hold within the mathematics of its own equations.

Conclusion

Consider the Relativistic equation

$$t = t_1 / (1- v^2 /c^2)^{½},$$

here, the limits of the quantity, $(1- v^2 /c^2)^{½}$, as a divisor, indicate the velocity of any moving object, v, cannot equal the velocity of c.

However, if both sides of the equation are multiplied by the quantity, $(1- v^2 /c^2)^{½}$, to produce the equation

$$t\,(1- v^2 /c^2)^{½} = t_1\,;$$

the limits of the quantity, $(1- v^2 /c^2)^{½}$, as a multiplicand, now allow for the velocity of a moving object, v, to equal the velocity of c.

So, in analyzing the series of Relativistic equations;

$$t = t_1 / (1- v^2 /c^2)^{½},$$

$$L = L_1 \;(1- v^2 /c^2)^{½},$$

$$m \equiv m_o / (1- v^2 /c^2)^{½},$$

$$p \equiv m_o\, v / (1- v^2 /c^2)^{½},$$

and

$$E = m_o\, c^2 / (1- v^2 /c^2)^{½},$$

by simply changing the quantity, $(1- v^2 /c^2)^{½}$, in any of these equations, from one side of the equation to the other (i.e., from a divisor to a multiplicand, or from a multiplicand to a divisor) the limits of the quantity are changed. So, as these quantities, $(1- v^2 /c^2)^{½}$ and $1/ (1- v^2 /c^2)^{½}$, do not express equivalent limits, though they might appear to be reciprocals in the sense that one can be multiplied by the other to produce a result of one, they are not reciprocals in terms of the limits they represent. Thus, as these quantities do not maintain the Fundamental Property of Equality when used as reciprocals, this renders these quantities, and any equation employing them or based upon them, invalid.

In closing, Relativity's use of velocity squared raises questions about any equation that creates a limit with a rate of change divided by a rate of change, such as, $1/ (1- v^2 /c^2)^{½}$.

Any equation employing limits such as these should be analyzed very carefully to see if dimensional conditions are being represented truthfully.

References

1. Raymond A. Serway and Jerry S. Faughn, *College Physics* (Saunders College Publishing, New York, 1985).

2. H.A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity* (Dover Publications, New York, 1952).