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A Mathematical Disproof of The Equation E = mc^2

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Introduction

Einstein's equation, E = mc^2, and Lorentz' ratio, 1 : $(1- v^2/ c^2)^{1/2}$, bring to light an issue concerning the use of the exponent that has been studiously avoided since the 1600's- an issue which actually invalidates these two equations and, consequently, invalidates Relativity.

Cartesian

In the 1600's Rene Descartes wrote (founding modern analytical geometry),

"Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction. Just as arithmetic consists of only four or five operations, namely, addition, subtraction, multiplication, division and the extraction of roots, which may be considered a kind of division, so in geometry, to find required lines it is merely necessary to add or subtract other lines .... Often it is not necessary thus to draw the lines on paper, but it is sufficient to designate each by a single letter. Thus, to add the lines ... I call one a and the other b, and write a + b. Then a - b will indicate that b is subtracted from a; ab that a is multiplied by b; a/b that a is divided by b; aa or a^2 that a is multiplied by itself; a^3 that this result is multiplied by a, and so on, indefinitely .... Here it must be observed that by a^2, b^3, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes, etc., so that I may make use of the terms employed in algebra." (1)

Here, Descartes introduced the concept that the exponent, in a^2, b^3 ... b, could be used to designate proportion of *line* (rather than a^2 being restricted to representing the surface of a square whose side is a, and b^3 representing the volume of a cube whose side is b). Descartes defined his use of the exponent as designating lengths of ***lines*** and component parts of ***lines***; with the emphasis on facilitating expression: of not having to draw the lines on paper, but to abstractly represent the *lines* and their proportionals by symbol (for ease of abstract representation of squaring, cubing, extracting the square root, cube root, etc., of a given *line*). Descartes' transliteration of geometric ***line*** and proportionals of ***line*** into algebraic form is similar to not having to draw an apple, but rather, expressing the apple with the word ***apple***; in Descartes' own words, ***"And I shall not hesitate to introduce these arithmetical terms into geometry, for the sake of greater clearness."*** (2)

Descartes illustrated a specific example of his use of the exponent (the extraction of root) as it applies to ***line***, in his ***Geometry***.

To quote Descartes again, ***"... taking one line which I shall call unity in order to relate it as closely as possible to numbers, and can in general be chosen arbitrarily .... If the square root of* (a line) *GH is desired, I add, along the same straight line,* (a line) *FG equal to unity; then, bisecting FH at K, I describe the circle FIH about K as a center, and draw from G a perpendicular and extend it to I, and GI is the required root."*** **(3)**

Please, take a moment to either draw or visualize this relationship that Descartes has described (or look at Descartes' own diagram on page 4 of his ***Geometry***). By analyzing this explanation (and diagram) of Descartes' concerning the extraction of the square root, the potential problem that can arise when applying the concept of squares, square roots, cubes, and cube roots, i.e., exponents in general, to ***lines*** in geometry becomes apparent. The problem is how the arbitrary choice of unity, combined with the use of the square, square root, etc., can, in certain instances, affect the outcome of the result. Looking again to Descartes' explanation (and diagram), you can see that the shorter the arbitrary length of unity chosen, the shorter the length of the resultant root; and the greater the length of unity chosen, the greater the length of the resultant root. The length and value of the resultant root (in Descartes' application) is not governed by the ***line*** it is supposed to be the root of, but is, surprisingly enough, governed by the arbitrary chosen length of unity - the choice of unity dictates the result!

To put this another way, draw three lines of equal length parallel to each other. Now divide one line into four units, one into nine units, and the other into twenty-five units. In each line of equal length, there has been chosen, arbitrarily, a different length for unity. Although each line is of the same length, if you extract the square root of each of the ***lines***, each will have a different end result (both numerically and in actual length). For the line divided into four units: the square root of the ***line*** will be two numerical units, or one-half the length of the line. For the line divided into nine units: the square root of the ***line*** will be three numerical units, or one-third the length of the line; and, for the line divided into twenty-five units: the square root of the ***line*** will be five numerical units, or one-fifth the length of the line.

To give this concept more emphasis, set a meter rod next to a yardstick. Here you can see, in actual application, the implication of how greatly the arbitrary assignment of unit, combined with the use of the square root, can color (skew) the perception of mathematics with respect to actual distance. The length of 1 meter is equal to the length of approximately 39.37 inches. If you take the square root of the ***length of line*** represented in inches (39.37 inches), the resultant root will be equal to 6.27 inches; but if you take the square root of the same ***length of line***, represented in centimeters (100 centimeters), the resultant square root will be 10 centimeters, or 3.93 inches; and if you take the square root of the same ***length of line*** represented in meters (1 meter), the resultant root will be 1 meter, or 39.37 inches. Although, beginning with and representing the same ***length of line*** geometrically, the resultant ***length*** of the extraction of the root is entirely different for each arbitrary assignment of unit. Comparing the results of 6.27 inches, 3.93 inches, and 39.37 inches, the disparity is remarkable in its demonstration of the inconsistency that can arise in the application of roots (extraction of roots), as applied to ***line*** (distance) and proportion of ***line***; as the entire perception of reality can be biased by the arbitrary choice and use of unity, when combined with the extraction of root.

Dimensional Analysis

A similar type of distortion can occur when multiplying a ***length of line*** by itself. Again, to quote Descartes, **"*Here it must be observed that by a^2, b^3, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes, etc., so that I may make use of the terms employed in algebra.*"** (4) **Based upon this Cartesian definition, [L] (where [L] represents *length*, and n is equal to 2, 3, 4, 5, 6, and so on, indefinitely), as an expression of *simple line* (a simple multiplication of *length of line*), will naturally be expressed in *linear* units. Therefore, in this article, to make a dimensional distinction and a break from current usage, [L] , that is L in brackets, will denote only *linear* dimension, and L without brackets will denote dimension of area or volume (where $A = L^2$ and $V = L^3$); a distinction necessary for correct calculation and conversion.**

The difference in the use of the exponent in calculation and conversion concerning volume and area in comparison to ***linear*** dimension is illustrated by the following examples.

<u>Example 1.</u>

The volume of a cube with side length of 1 yd or 3 ft is equal to $(3\ ft)^3$, or 3 ft 3ft 3 ft, or 27 cubic ft, which if converted is equal to 1 cubic yard. There is consistency of conversion, as 27 cubic ft and 1 cubic yd equal the same volume. However, when "cubing" ***linear*** units of [1] yd or [3] ft, the resultant length differs: [3^3] ***linear*** ft is equal to [3] [3] [3], or [27] ***linear*** ft, which if converted, is equal to [9] ***linear*** yds; and, in contrast, the resultant length of [1^3] ***linear*** yd is equal to [1] [1] [1], or [1] ***linear*** yd, which if converted will equal [3] ***linear*** ft. **Again, in the Cartesian use of the exponent to denote multiplying *length of line* by itself (to determine a proportional extension of that *length of line*) the result is *linear*, conversion must remain *linear*, and, as illustrated, the resultant *length* will vary with unit chosen.**

<u>Example 2.</u>

If the area of a square with side length of 1 meter is expressed in meters, the resultant area of the square: 1 meter multiplied by 1 meter will result in one square meter; and similarly, if the area of the same square is expressed in inches, i.e., 39.37 inches multiplied by 39.37 inches, the resultant area 1,549.99 square inches, if converted to square meters, will again equal one square meter. In contrast, when "squaring" ***linear*** units, the result differs with unit assigned: [1^2] ***linear*** meter is equal to [1] [1] or [1] ***linear*** meter; which, if converted, is equal to [39.37] ***linear*** inches; and "squaring" the same length of line, with a unit assignment of inches, [39.37^2] ***linear*** inches, the resultant length is equal to [39.37] [39.37] or [1,549.99] ***linear*** inches. Although beginning with the same initial length of line, [1] meter, the result of the "squared" ***length of line*** in question differs for each different assigned unit; compare the results of [39.37] inches to [1549.99] inches. Imagine the disparity of result that occurs if a coefficient is introduced along with the exponent, e.g., 25 [L^2], that is, if applied and repeated over distance.

To make a dimensional distinction:

1. Multiplying a side of a square or cube by itself, L^2 or L^3, to find the area of the square or volume of a cube, is a calculation to determine the number of square or cube units comprising a specific, delineated, geometric area. The result is expressed in number of square or cube units, the combined area or volume of which will always equal the initial area of the square or volume of the cube. In the calculation of area or volume, regardless of the arbitrary unity assigned, as the actual geometric area described is consistent, there is always consistency of conversion between the results received from different assigned units.
2. In comparison, multiplying a ***length*** of line by itself, to determine a proportional extension of that ***length*** of line (e.g., [L] [L], or [L^2], or [L] [L] [L] or [L^3]), is a calculation to determine an unknown ***length***. In this Cartesian application, [L^2] and [L^3] indicate ***simple line*** and are expressed as simple ***length*** of line, i.e., in ***linear*** units (in inches, meters, etc.), not in square or cube units; with conversion remaining ***linear***, and resultant length varying with assigned unit.

As illustrated by these examples, applying the exponent to ***line*** and ***distance*** to indicate proportional ***length of simple line*** is dimensionally and mathematically distinct from the use of the exponent to describe area and volume. As there is a difference in respective application and expected result, a dimensional distinction has to be made with respect to the use of the exponent in the language of mathematics. Is the exponent denoting area or volume of a particular delineated, geometric figure - with the result expressed in either square or cube units? Or, is the exponent representing multiplication of ***simple length of line*** - a mathematical application of scale, reminiscent of Democritus' Necklace, where the smaller the unit chosen (the greater number of unit divisions for a given length), the greater the resultant "exponential" length; and, theoretically, with the potential of infinite divisibility of unit (i.e., with the potential of infinite divisibility numerically), there is the potential for infinite length of result?

$E = mc^2$?

This question and concept, concerning the use of the exponent in the language of mathematics, has great significance in modern mathematics - as demonstrated by the familiar equation:

$$E = mc^2$$

In the equation $E = mc^2$ (with *c* denoting velocity, or ***linear*** displacement over time) you can see that each different choice of unity (e.g., centimeters, meters, feet, yards, etc.) will present the mathematician with a different end result (whether squaring, $E = mc^2$, or extracting the square root, [E/m] = *c*); an apparent distortion of reality: a mathematically generated contraction or expansion of length, increase or decrease in mass, temporal distortion, etc., created and based entirely upon the arbitrarily chosen length of unity combined with the application of the exponent.

Unlike equations describing specific proportional truths that exist within a geometric figure, proportional truths that hold true and consistent ***regardless of arbitrary choice of unity*** or assignment of numerical value, such as:

1. the Pythagorean Theorem, where the exponent in the equation, $x^2 + y^2 = z^2$, is used to describe an actual proportional relationship that exists between (squares of the) sides of a right triangle; or
2. the proportional relationship that exists between the circumference and the diameter of a circle, attempted to be expressed by c/d or ; or even,
3. the proportional relationship that exists between the sides of a square and the diagonal of a square, the diagonal being equal to the length of the side of a square multiplied by the square root of two;

equations, such as $E = mc^2$, that attempt to apply the exponent to *line and distance (where the end proportional result is dictated by the arbitrary choice of unity)*, simply result in a mathematical manipulation of scale, similar to what can be found applied in a map, a globe or in cartography in general.

Pythagorean

This specific problem that arises with the use of the exponent (when attempting, in the abstract language of mathematics, to combine geometry and algebra) can be further understood and demonstrated by analyzing the Pythagorean Theorem. The Pythagorean Theorem addresses the proportional relationship existing between the sides of a right triangle, by treating the sides of a right triangle as sides of abstract squares. With the area of a square being a side multiplied by itself, or x^2, the Pythagorean Theorem states that in a right triangle: the sum of the (area of the) squares of the two sides of the triangle adjacent to the right angle are equal to the (area of the) square of the side of the triangle opposite the right angle. This theorem is expressed in equation form as:

$$x^2 + y^2 = z^2$$

In this equation, x and y represent the two sides of the triangle adjacent to the right angle, and z represents the side opposite (the hypotenuse). This Pythagorean Theorem and equation, applying to the proportional relationship of squares of the sides of a right triangle, holds true as long as it applies to a right triangle and the squares of the sides of the right triangle. This theorem can be applied correctly, two-dimensionally, to a plane geometric figure; or, it can be applied correctly to a three-dimensional geometric figure, as a two-dimensional figure stacked or layered to a desired height. This relationship is expressed as:

$$h\ (x^2 + y^2) = h\ (z^2)$$

where h represents the desired, stacked height. However, if this proportional relationship is attempted to be applied to the cube, as described by the equation:

$$x^3 + y^3 = z^3$$

the proportional relationship no longer holds true. The cubes of the sides of the right triangle do not relate proportionally in the same manner as squares of the sides of the right triangle. The sum of the (volumes of the) cubes of the two sides of the triangle adjacent to the right angle do not equal the (volume of the) cube of the side opposite, as described in the equation $x^3 + y^3 = z^3$. The equation, $x^3 + y^3 = z^3$, an equation dealing with the volume of cubes, and, $x^2 + y^2 = z^2$, an equation dealing with areas of squares, are entirely different in what they represent. Again, the cube, different from the square, cannot be expressed in terms of the square: quadrature and cubature are different, serve different purposes, and cannot be used interchangeably. The proportional relationship that is true for the plane geometric figure of squares of the sides of a right triangle, expressed in the equation $x^2 + y^2 = z^2$, is not true of cubes of the sides of a right triangle $x^3 + y^3 = z^3$, nor for any equation $x^n + y^n = z^n$, where n represents anything other than the square; for example, where n represents any number greater than 3 (such as 4, 5, 6, etc.), unintelligible as geometric form. These equations, though similar appearing algebraically, are different in what they represent geometrically.

Fermat?

Taken by itself, this attempt to resolve Fermat's "Last Theorem"(5) by relating the exponent value directly to the actual geometric figure (this literal association of the x^2 to the square and the x^3 to the cube, in the Pythagorean Theorem), may appear superficial, even naive and primitive in its simplistic approach; however, it serves to illustrate and emphasize the necessary mathematical distinction that must be made between Descartes' application of the exponent to denote proportional ***length of line*** and the Pythagorean use of the exponent to denote proportional relationships (area or volume) of a given geometric figure.

Although the use of the exponent in the Pythagorean Theorem and by Descartes may appear similar, and, on the surface, seem to have a common point of agreement in the square, nothing could be farther from the truth; as they differ in intent, application, representation, and expected result. Descartes' use of the exponent is clear, direct, and consistent in its application as a simple device for ease of ***expressing the proportional components of a given line in relation to an arbitrarily chosen unity***: serving both as a shorthand for repeated multiplication of a given ***length of line*** by itself, and as a means to apply scale. In contrast, the use of the exponent in the Pythagorean Theorem ($x^2 + y^2 = z^2$) specifically defines the proportional relationship that exists between surface areas of squares of the sides of a right triangle: a transliteration of geometry into algebra that if attempted to be applied to address anything other than the actual proportional relationship it is intended to describe simply does not apply. Hence, regarding the use of the exponent:

1. in the Pythagorean Theorem, equations of $x^n + y^n = z^n$, where n represents anything other than the square (not describing any proportional actuality or truth), fall into the category of non-representative mathematics; and,
2. equations such as $E = mc^2$, that combine arbitrary assigned unit with the use of the exponent, to express proportionals of a given unity (with the end result dictated by the arbitrarily chosen unity), fall into the category of mathematical manipulations of scale and proportion.

Regarding the extraction of roots and the use of the exponent in the language of mathematics: again, the use

of the exponent to denote a given, delineated, geometric figure, as in the Pythagorean Theorem (where the arbitrary assignment of unity has no affect on the actual geometric figure), and Descartes' use of the exponent, concerning proportionals of line (where the choice of unity dictates the resultant length of line), are different. Although each have a distinct truth and purpose unto themselves, if attempted to be used interchangeably, indiscriminately without regard to what they represent in the language of mathematics, they (the use of the exponent and the extraction of root) lose their truth of application; and without truth of application, the language of mathematics loses its truth.

The Apple of Discord

As the equation $E = mc^2$ is shown to be flawed and invalid in its mathematical application (its use of the exponent), is it also flawed in its mathematical basis, and/or flawed in what it is intending to represent with its mathematical symbols? This equation brings up the question, "Are there misrepresentations or misunderstandings created when representing and combining such diverse actualities as space, matter, energy, time, and relative motion as abstract mathematical symbols within one equation?"

Ask yourself,

"What are the square/square root of inertia, time, and distance actually representing: length of line, geometric form, or simply arbitrary assignment of unity? Is there any mathematical truth in equations combining the use of seconds squared, meters squared, and/or grams squared - not describing geometric truths but only describing numerical proportions of arbitrary assigned unity? Is there any hope of dimensional consistency between such disparate actualities? What is the common denominator (dimensional conversion) between time, inertia, and distance...number alone?"

Conclusions

Mathematical Reformation (Post-Relativism)

The use of the exponent as applied in the mathematics of Relativity, specifically, Lorentz' mathematical ratio, $1:(1-v^2/c^2)^{1/2}$, and Einstein's equation $E = mc^2$, by demonstrating and expressing a fundamental error in the application and use of the exponent in the language of mathematics, has brought to light the need for a Mathematical Reformation, namely:

1. A dimensional distinction must be made in the use of the exponent in mathematical language. The Cartesian use of exponent applied to line and distance $[L^2]$, $[L^3]$ ... $[L]$, as a simple device for mathematical calculation and manipulation of scale and proportion (where the exponential result is ***linear***, and subsequent conversion must remain ***linear***), is dimensionally, and therefore algebraically, different than the use of the exponent as applied to describe delineated, geometric form (i.e., area or volume, number of square or cube units), as in the Pythagorean Theorem ($x^2 + y^2 = z^2$). This is a dimensional distinction that also "solves" Fermat's "Last Theorem"; as any Pythagorean equation, $x^n + y^n = z^n$, where n represents anything other than the square, does not describe any proportional, numerical, or geometric truth.
2. Most importantly, there is a need to re-evaluate the perception of the language of mathematics as an absolute truth unto itself; and to acknowledge that the language of mathematics, as a language (as an abstract representation of thought and understanding), can be limited, and/or biased, by individual thought and perspective; and as such, can be applied to express either truth or fiction in conformance to the user's perception.

Epilogue

Mathematical Fiction vs. Fact (Dragons and Unicorns)

There is one last issue (concerning the equation $E = mc^2$, "modern mathematics", and the concept of "pure" as opposed to "applied" mathematics) that I would like to address in closing.

In my mind's eye, I can envision chimera, griffins, dragons, and unicorns. I can look into the field (of imagination) and see, in place of horses, unicorns, pure and bright, argent, their narwhal-like horns glistening in the sun; an idyllic pastoral. Listening, I can almost hear the sound of nickering, whinnying, the rhythmic thudding of hooves as unicorns gambol and spar playfully in golden fields, frolicking across sylvan glades; a sound becoming fainter and fainter as they disappear into the verdant, forested hills. Anything imagined can be communicated through language. In language I can "create" a unicorn; giving form and expression to my imagination, I can endow my "creation" with dimension, shape, color, characteristics, and subtle demeanor. But, even with "correct" technique (no errors in spelling, punctuation, or sentence structure) and the language true to what I am describing, the expression of my imagination is not reality - still the unicorn does not exist. Similarly, though the relativistic equations such as $E = mc^2$ (theorizing c as a mathematical vanishing point) give full expression to the imagination through the language of mathematics; like mythical, mathematical "mythematical" dragons and unicorns, representing and oriented to the thoughts and dreams of their developers rather than to reality, these expressions of conceptual ideas (hypothesis) have to be recognized for what they are: theory, hypothetical constructs, fantasy, and imagination, expressed in the language of mathematics.

Though theoretical equations, such as $1:(1-v^2/c^2)^{1/2}$ and $E = mc^2$, provide a vehicle for the imagination, these expressions of hypothesis through mathematical language can be misleading. To treat these (or any other) ***theoretical*** equations as fact is to set hypothesis and imagination as a standard, i.e., to create a false set of mathematical expectations, which can establish its own wall to progress. Attempting to hypothetically combine and mesh such diverse actualities as space, energy, motion, matter, and time (sequence of occurrence) within a mathematical pattern/equation (to paint all reality with the same mathematical brush), relativistic hypothetical equations (such as $E = mc^2$) may articulate the desire to develop a unified form of mathematical expression (a unified field), but also clearly demonstrate this desire is as yet unrealized: that the relationship between space, energy, motion, matter, and time is still not resolved, or at least has not been put to a common scientific or mathematical language pattern. Displaying not only the flaw in relativity, but the flaw in "modern mathematics"; the equation $E = mc^2$ stands as a modern scientific mile marker: silently attesting to the fact that each step toward the development of a mathematical language of unified interpretation (whether, Euclidian, Ptolemaic, Cartesian, Newtonian, Riemannian, etc.) must be examined and considered objectively to determine if supported only by desire, imagination, and vision, or firmly grounded in truth and, therefore, suitable as a stepping stone on the path toward progress (and unified understanding and communication).

1. Rene Descartes, *The Geometry Of Rene Descartes*, (Dover Publications Inc., New York, 1954), pp. 2-5.

2. Descartes, *The Geometry Of Rene Descartes*, p. 5.

3. Descartes, *The Geometry Of Rene Descartes*, p. 5.

4. Descartes, *The Geometry Of Rene Descartes*, p. 5.

5. *Fermat, Pierre De*, The World Book Encyclopedia, (Field Enterprises Educational Corporation, Chicago, 1966)