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Solution of the Quantum Electrohydrodynamic Fundamental Equations for the inhomogeneously charged Proton-Antiproton Pair

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Abstract

In the present work, the quantum electrohydrodynamic fundamental equations are solved for the inhomogeneously charged proton-antiproton pair. Antiproton and proton are described as nested spherical shells. In addition, each spherical shell is preceded by a cavity (by a buffer zone) which exerts Casimir forces on the proton and antiproton. Quantum electrohydrodynamic calculations give for the antiproton on the inner spherical shell an cosmological initial expansion from $R_{01} = 0.0943263\ fm$ up to $R_1 = 0.0951547\ fm$ and for the proton on the outer spherical shell an cosmological initial expansion from $R_{02} = 0.105696\ fm$ up to $R_2 = 0.106562\ fm$.

Key words: Quantum electrohydrodynamics, Casimir energy pressure, inhomogeneously charged proton-antiproton pair.

1. Introduction

In a previous work [1] we solved the fundamental equations of quantum electrohydrodynamics (QEHD) [2] in the first approximation for a homogeneously charged proton-antiproton pair and concluded that for an adequate treatment of this problem is not required a homogeneous, but an inhomogeneous mass and charge distribution. For this reason, in the present work we start from the model concept of a proton-antiproton pair whose mass and charge density is inhomogeneously distributed. In this model concept, proton and antiproton are described as nested spherical shells. In addition, each spherical shell is preceded by a cavity (by a buffer zone) that exerts Casimir forces on the antiproton and proton. The antiproton, which forms the inner spherical shell, exerts on the proton – on the outer spherical shell – an attractive Coulomb force. Conversely, however, the proton as the particle on the outer spherical shell cannot exert electromagnetic forces on the antiproton on the inner spherical shell; instead, it pushes with its entire "weight" – with its total energy-mass – the antiproton and compresses it down to fractions of $1\,fm$.

In the present work, the fundamental equations of QEHD are solved for the inhomogeneously charged proton-antiproton pair. The paper is structured as follows: In Section 2 we introduce the fundamental equations of QEHD for a proton-antiproton pair. In section 3, the electromagnetic field variables occurring in the fundamental equations are calculated for an inhomogeneous charge distribution. In Section 4, the radial differential equation is solved for the inhomogeneously charged antiproton on the inner spherical shell. In Section 5, the same calculation is performed for the inhomogeneously charged proton on the outer spherical shell. Section 6 is provided for a final remark.

2. The fundamental equations of QEHD

The fundamental equations of QEHD are given by the hydrodynamic formulation of the Dirac equation [2]:

$$\frac{\partial}{\partial t}\left(\frac{\rho\,(E - e\Phi)}{c^2}\right) + \nabla\cdot\left(\rho\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)\right) = 0, \tag{2.1}$$

$$(E - e\Phi)^2 = m_0^2\, c^4 + \left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)^2 c^2 - e\,\hbar\, c\,\boldsymbol{\sigma}\boldsymbol{B} - \hbar^2 c^2\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\,, \tag{2.2}$$

$$\oint\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)\cdot d\boldsymbol{r} = 2\pi\hbar\, m, \qquad m = 0, \pm 1, \pm 2, \ldots \tag{2.3}$$

where are

c	the constant vacuum speed of light,
$\hbar$	the reduced Planck constant,
e	the elementary charge,
$\boldsymbol{\sigma}$	the 2x2 Pauli matrices,
m_0	the naked mass,
$\boldsymbol{p}$	the particle momentum,
ρ	the probability density (= mass and charge density),
E	the energy,
$\Phi, \boldsymbol{A}, \boldsymbol{B} = \nabla \times \boldsymbol{A}$	the electromagnetic field variables.

Eq. (2.1) is the equation of continuity of QEHD, Eq. (2.2) the equation of motion, and Eq. (2.3) a quantization rule for the angular momentum going back to Bohr-Sommerfeld-Wilson.

The electromagnetic potentials $\boldsymbol{\Phi}$ and $\boldsymbol{A}$ occurring in the fundamental equations (2.1)-(2.3) are defined by the equations:

$$\Phi(\boldsymbol{r}, t) = -\,4\pi Z\, e \int d^3 r' \int dt'\, \rho\,(\boldsymbol{r'}, t')\; G\,(\boldsymbol{r} - \boldsymbol{r'}, t - t')\,, \tag{2.4}$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = -\,4\pi Z\, e \int d^3 r' \int dt'\, \frac{\boldsymbol{p}}{m_0\, c}\, \rho\,(\boldsymbol{r'}, t')\; G\,(\boldsymbol{r} - \boldsymbol{r'}, t - t') \tag{2.5}$$

where

$$G(\boldsymbol{r} - \boldsymbol{r'}, t - t') = - \frac{\delta(\boldsymbol{t} - \boldsymbol{t'} - /\boldsymbol{r} - \boldsymbol{r'}//\boldsymbol{c})}{(4\pi/\boldsymbol{r} - \boldsymbol{r'}/)} \tag{2.6}$$

is the Green's function and $Z\,e$ the total charge of the considered quantum hydrodynamic system.

In the present work, we want to solve the equation system (2.1)-(2.6) for a proton-antiproton pair and thereby determine the mass or charge density ρ. Since in this study we are interested only in stationary solutions and the mass density ρ must satisfy the boundary condition $\rho\,(\boldsymbol{r_o}) = 0$, the equation of continuity (2.1) gives

$$\nabla \cdot (\rho\, \boldsymbol{p}) = 0, \ \boldsymbol{p}_r(\boldsymbol{r}) = 0\,. \tag{2.7}$$

Since, furthermore, the particle momentum can only flow in closed lines and because of the azimuthal symmetry of the system, the momentum field in spherical coordinates is independent of the angle φ:

$$\boldsymbol{p} - \frac{e}{c}\boldsymbol{A} = \frac{\hbar}{r\,sin(\vartheta)}\boldsymbol{e}_\varphi\, m \qquad\qquad m = 0, \pm 1, \pm 2, ... \tag{2.8}$$

Eq. (2.8) is consistent with the quantization rule (2.3):

$$\oint(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}) \cdot d\boldsymbol{r} = \oint \frac{\hbar\, m}{r\,sin(\vartheta)}\boldsymbol{e}_\varphi\, d\boldsymbol{r} = \hbar\, m \oint d\varphi = 2\pi\,\hbar\, m\,. \tag{2.9}$$

Substituting Eq. (2.8) into the equation of motion (2.2) we obtain

$$(E - e\Phi)^2 = m_0^2\, c^4 + (\frac{\hbar\, c\, m}{r\,sin(\vartheta)})^2 - e\,\hbar\, c\,\boldsymbol{\sigma B} - \hbar^2 c^2 \frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\,. \tag{2.10}$$

If we assume

$$\sqrt{\rho} = f(r)\, g(\varphi, \vartheta)\,, \tag{2.11}$$

Eq. (2.10) can be separated into two differential equations with respect to r and ϑ (λ = separation parameter) [3]:

$$\left(\frac{r^2}{f}\right)\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \left((E - e\Phi)^2 - m_0^2\, c^4 + e\,\hbar\, c\, \boldsymbol{\sigma B}\right)\frac{f}{\hbar^2 c^2}\right] = \lambda \tag{2.12}$$

and

$$\lambda = -\frac{1}{g}\left[\frac{1}{sin(\vartheta)}\frac{\partial}{\partial \vartheta}\left(sin(\vartheta)\frac{\partial g}{\partial \vartheta}\right) - \frac{m^2}{sin^2(\vartheta)}\, g\right]. \tag{2.13}$$

Solutions of the differential equation (2.13) are the spherical harmonics

$$g(\varphi, \vartheta) = (-1)^m \sqrt{\frac{(l-|m|)!\,(2\,l+1)}{4\,\pi\,(l+|m|)!}}\, e^{im\varphi}\, \frac{1}{2^l\, l!}\, \frac{d}{d(cos(\vartheta))^l}\,(cos^2(\vartheta) - 1)^l, \tag{2.14}$$

where the constants λ and m have the eigenvalues

$$\lambda = l\,(l+1), \qquad l = 0, \pm 1, \pm 2, \ldots \tag{2.15}$$

and

$$-l \leq m \leq +l\,. \tag{2.16}$$

3. Determination of the electromagnetic field variables

In order to solve the radial differential equation (2.12) we need the electromagnetic potentials Φ and $\boldsymbol{A}$ which are defined by the equations (2.4) and (2.5). The scalar potential Φ for a self-interacting proton or antiproton ($Z = 1$) is obtained according to Eq. (2.4) by performing in polar coordinates the integral

$$\Phi(r) = -\, c_0^2\, \frac{e}{4\,\pi} \int_0^{2\pi}\int_0^{\pi}\int_{R_0}^{r} r'^2\, sin(\vartheta)\, \frac{\rho(r')}{\sqrt{r^2 + r'^2 - 2\, r\, r'\, cos(\vartheta)}}\, d\varphi\, d\vartheta\, d\, r'\,, \tag{3.1}$$

where c_0^2 is the normalization constant and R_0 the starting point of the charge distribution.

Integrating over the angle coordinates φ and ϑ, Eq. (3.1) gives:

$$\Phi(r) = -\, c_0^2 \, \frac{e}{r} \int_{R_0}^{r} r'^2 \rho\,(r')\, d\,r' \, . \tag{3.2}$$

In a previous work we assumed [1] that in the first approximation the proton-antiproton pair is homogeneously charged, i.e.

$$c_0^2 \; \rho\,(r') = a_0 = constant\,, \tag{3.3}$$

so that from Eq. (3.2) for the scalar potential of a self-interacting proton or antiproton follows:

$$e\,\Phi\,(r) = -\, e^2 a_0 \, \frac{r^3 - {R_0}^3}{r} \, . \tag{3.4}$$

Solving the radial differential equation (2.12) for the scalar potential (3.4), we get the density function [1]

$$z = z_0 \, \sin\!\left(\frac{e^2 a_0 \, r'^3}{3}\right) r'^n \, e^{-\,a\,r'} \, . \tag{3.5}$$

In order to calculate the scalar potential for the inhomogeneous mass and charge distribution, we must now perform the integral in Eq. (3.2) with the density function (3.5). It turns out, however, that the integration (3.2) with the new density function (3.5) cannot be performed, i.e. there is no analytical solution for this problem. Therefore, in the context of the present work, we make for the scalar potential the assumption:

$$e\,\Phi\,(r) = -\,\frac{e^2}{r} \, a_0 \,\left(a_1 \,(r - R_0) + a_2 \,(r - R_0)^2 + a_3 \,(r - R_0)^3 \right) . \tag{3.6}$$

This definition equation shows that at the point $r = R_0$, i.e. at the beginning of the charge distribution, the scalar potential Φ disappears as expected. Furthermore, there is an end point of the charge distribution $r = R_i$ where the scalar potential is equal to the Coulomb potential $e\,\Phi(R_i) = -\frac{e^2}{R_i}$; from this, it follows that the parameter

$$a_0 = \frac{1}{a_1\,(R_i - R_0) + a_2\,(R_i - R_0)^2 + a_3\,(R_i - R_0)^3} \tag{3.7}$$

is always positive.

In the case of the proton, there is, in addition to the self-interaction term (3.6), an attractive Coulomb potential as external interaction term, so that the scalar potential for the proton looks as follows:

$$e\,\Phi_{Pr}\,(r) = -\frac{e^2}{r}\,\left(-\,a_0\,\left(a_1\,(r - R_0) + a_2\,(r - R_0)^2 + a_3\,(r - R_0)^3\right) + 1\right). \tag{3.8}$$

* * *

Now, we determine the vector potential $\boldsymbol{A}$ for a self-interacting proton-antiproton pair. Just as we have done by the scalar potential, for the vector potential we must also perform according to Eq. (2.5) in polar coordinates the integral

$$\boldsymbol{A}(r) = -\,c_0^2\,\frac{e}{4\,\pi}\int_0^{2\pi}\int_0^{\pi}\int_{R_0}^{r} r'^2\,sin(\vartheta)\,\frac{p}{m_0\,c}\,\frac{\rho\,(r')}{\sqrt{r^2 + r'^2 - 2\,r\,r'\,cos(\vartheta)}}\,d\varphi\,d\vartheta\,d\,r'. \tag{3.9}$$

Taking into account the relation

$$\frac{p}{m_0\,c} = \frac{\omega\,r\,sin(\vartheta)}{c}\,\boldsymbol{e_\varphi} = \frac{\hbar\,m}{m_0\,r^2\,sin(\vartheta)}\,\frac{r\,sin(\vartheta)}{c}\,\boldsymbol{e_\varphi} = \frac{\hbar\,m}{m_0\,c\,r}\,\boldsymbol{e_\varphi}, \tag{3.10}$$

and integrating in Eq. (3.9) over the angle coordinates φ and ϑ , we obtain for the vector potential of a self-interacting proton and antiproton:

$$\boldsymbol{A}\,(r) = -\,c_0^2\,e\,\frac{\hbar\,m}{m_0\,c\,r^2}\,\boldsymbol{e_\varphi}\int_{R_0}^{r} r'^2\rho\,(r')\,d\,r'. \tag{3.11}$$

In this case too, we make the assumption (3.6) for the inhomogeneous charge distribution and obtain for the vector potential of a self-interacting proton or antiproton:

$$\boldsymbol{A}\,(r) = -\,e\;a_0\,\frac{\hbar\,m}{m_0\,c}\,\boldsymbol{e_\varphi}\,\frac{a_1\,(r-R_0)+a_2\,(r-R_0)^2+a_3\,(r-R_0)^3}{r^2}\,. \tag{3.12}$$

From that, we calculate the magnetic flux density

$$\boldsymbol{B} = \boldsymbol{\nabla}\,x\,\boldsymbol{A} = -\,e\;a_0\,\frac{\hbar\,m}{m_0\,c}\,\boldsymbol{e_\vartheta}\,(\frac{2\,a_3\,R_0{}^3-2\,a_2\,R_0{}^2+2\,a_1\,R_0}{r^3}+\frac{-3\,a_3\,R_0{}^2+2\,a_2\,R_0-a_1}{r^2}+a_3), \tag{3.13}$$

so that for the term occurring in the differential equation (2.12)

$$e\,\hbar\,c\,\boldsymbol{\sigma B} = -\,e^2\;a_0\;m\,\frac{(\hbar\,c)^2}{m_0\,c^2}\,(\frac{2\,a_3\,R_0{}^3-2\,a_2\,R_0{}^2+2\,a_1\,R_0}{r^3}+\frac{-3\,a_3\,R_0{}^2+2\,a_2\,R_0-a_1}{r^2}+a_3) \tag{3.14}$$

results.

Considering the contribution of the Coulomb potential (3.8) to the magnetic flux density of the proton, we obtain, instead of (3.14), the term

$$e\,\hbar\,c\,\boldsymbol{\sigma B_{Pr}} = -\,e^2\;m\,\frac{(\hbar\,c)^2}{m_0\,c^2}\,[a_0\,(\frac{2\,a_3\,R_0{}^3-2\,a_2\,R_0{}^2+2\,a_1\,R_0}{r^3}+\frac{-3\,a_3\,R_0{}^2+2\,a_2\,R_0-a_1}{r^2}+a_3)-\frac{2}{r^3}]\,. \tag{3.15}$$

4. The inhomogeneously charged antiproton on the inner spherical shell

Substituting the electromagnetic field variables (3.6) and (3.14) for the inhomogeneously charged antiproton on the inner spherical shell into the radial differential equation (2.12), we obtain

$$\begin{aligned}&(\frac{r^2}{f})[\frac{1}{r^2}\,\frac{\partial}{\partial r}(r^2\,\frac{\partial f}{\partial r})+(E+\frac{e^2}{r}a_0\,(\,a_1\,(r-R_{01})+a_2\,(r-R_{01})^2+a_3\,(r-R_{01})^3\,))^2-m_0{}^2c^4\\&-e^2a_0m\,\frac{(\hbar\,c)^2}{m_0\,c^2}\,(\frac{2\,a_3\,R_{01}{}^3-2\,a_2\,R_{01}{}^2+2\,a_1\,R_{01}}{r^3}+\frac{-3\,a_3\,R_{01}{}^2+2\,a_2\,R_{01}-a_1}{r^2}+a_3\,)]\frac{f}{\hbar^2c^2} = l\,(l+1)\end{aligned} \tag{4.1}$$

where R_{01} is the starting point of the antiproton on the inner spherical shell.

For central symmetric potentials it is usual to make the assumption $f = \frac{z}{r}$. By this way, Eq. (4.1) can be reduced to:

$$z'' + \frac{1}{d^2}[(E + \frac{e^2}{r} a_0 (a_1 (r - R_{01}) + a_2 (r - R_{01})^2 + a_3 (r - R_{01})^3))^2 - b^2 \tag{4.2}$$
$$- e^2 a_0 \frac{d^2}{b} m (\frac{2 a_3 R_{01}{}^3 - 2 a_2 R_{01}{}^2 + 2 a_1 R_{01}}{r^3} + \frac{-3 a_3 R_{01}{}^2 + 2 a_2 R_{01} - a_1}{r^2} + a_3)] z = \frac{l(l+1)}{r^2} z$$

where $b = m_o c^2$ and $d = \hbar c$ are two very useful abbreviations.

Dividing the variables E, b, e^2, for simplicity, by $\frac{d}{fm}$, we obtain from (4.2) an equation with dimensionless quantities:

$$z'' + [(E + \frac{e^2}{r} a_0 (a_1 (r - R_{01}) + a_2 (r - R_{01})^2 + a_3 (r - R_{01})^3))^2 - b^2 \tag{4.3}$$
$$- \frac{e^2}{b} m a_0 (\frac{2 a_3 R_{01}{}^3 - 2 a_2 R_{01}{}^2 + 2 a_1 R_{01}}{r^3} + \frac{-3 a_3 R_{01}{}^2 + 2 a_2 R_{01} - a_1}{r^2} + a_3)] z = \frac{l(l+1)}{r^2} z$$

In order to solve the differential equation (4.3), we first consider the asymptotic behavior at $r = 0$. In this case, Eq. (4.3) reduces to:

$$z_0'' - e^2 \frac{m}{b} 2 a_0 \frac{a_3 R_{01}{}^3 - a_2 R_{01}{}^2 + a_1 R_{01}}{r^3} z_0 = 0 . \tag{4.4}$$

This differential equation can be solved by the modified Bessel functions of the second kind [4]:

$$z_0 = \sqrt{a\, r}\, K\_1(\frac{2}{\sqrt{a\, r}}) , \; a = \frac{e^2}{b} a_0 (a_3 R_{01}{}^3 - a_2 R_{01}{}^2 + a_1 R_{01}) \tag{4.5}$$

where for the antiproton $m = -½$ holds .

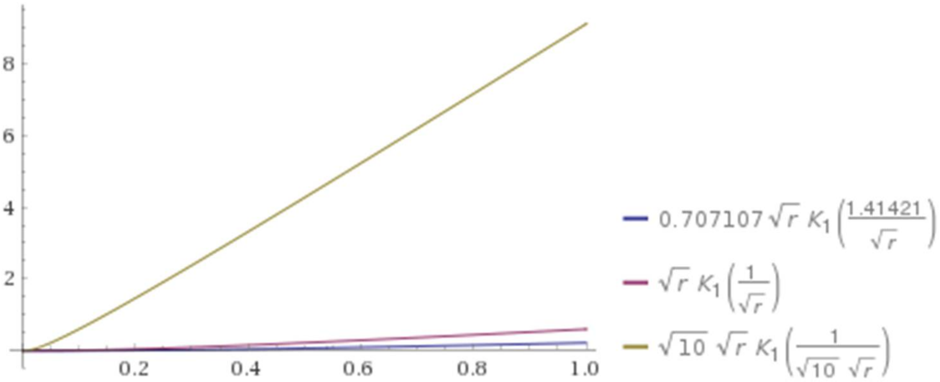

Figure 1: The parameter representation of the modified Bessel functions of the second kind: $z_0 = \sqrt{a\,r}\; K\_1(\frac{2}{\sqrt{a\,r}})$

Giving a graphic representation of the function z_0 for some selected parameters *a = 0.5, 1, 10* (Fig. 1), we recognize that this function can be approximated by a straight line equation through 0 and any points R_1 and R_2 :

$$z_0 = y_{12}\, r\,, \tag{4.6}$$

where

$$y_{12} = \frac{z_0\,(R_1) - z_0\,(R_2)}{R_1 - R_2} \tag{4.7}$$

holds.

Therefore, a general solution of the differential equation (4.3) is given by

$$z = z_0\,\sin(c_3 \frac{e^2\, r^3}{3})\; r^n\, e^{-\,a\,r}. \tag{4.8}$$

Substituting the solution (4.8) into the differential equation (4.3), a coefficient comparison for the powers of r yields the following seven equations:

$$c_3 = a_0\, a_3\,, \tag{4.9}$$

$$a_2 = 3\, a_0\, R_{01}\,, \tag{4.10}$$

$$(n+1)\,n + a_0{}^2\, e^4\, R_{01}{}^2\, (a_1 - 2\, a_3\, R_{01}{}^2)^2 + e^2\, \tfrac{m}{b}\, a_0\, (a_1 - 3\, a_3\, R_{01}{}^2) - l\,(l+1) = 0\,, \tag{4.11}$$

$$a\,(n+1) + a_0{}^2\, e^4\, R_{01}\, (a_1 - 3\, a_3\, R_{01}{}^2)(a_1 - 2\, a_3\, R_{01}{}^2) + a_0\, e^2 R_{01} (a_1 - 2\, a_3\, R_{01}{}^2)\, E = 0\,, \tag{4.12}$$

$$a^2 - b^2 + \left(a_0\, e^2\, (a_1 - 3\, a_3\, R_{01}{}^2) + E\right)^2 - a_0\, a_3\, e^2\, \tfrac{m}{b} = 0\,, \tag{4.13}$$

$$E = a_0\, e^2\, R_{01}\, (a_1 - 2\, a_3\, R_{01}{}^2) - a_0\, e^2\, (a_1 - 3\, a_3\, R_{01}{}^2)\,, \tag{4.14}$$

$$a = n + 2\,. \tag{4.15}$$

where according to Eq. (3.7) $a_0 = \frac{1}{a_1\,(R_1 - R_{01}) + a_3\,(R_1 - R_{01})^2\,(2\,R_{01} - R_1)}$ holds.

We obtain from these equations as a special case the solution to the homogeneous charge and mass distribution, if we set $a_1 = 3\, a_3 R_{01}{}^2$, $a_3 = 1$ [1]:

$$(n+1)\,n + a_0{}^2\, e^4\, R_{01}{}^6 - l\,(l+1) = 0\,, \tag{4.16}$$

$$a\,(n+1) + a_0\, e^2\, R_{01}{}^3\, E = 0\,, \tag{4.17}$$

$$a^2 - b^2 + E^2 - a_0\, e^2\, \tfrac{m}{b} = 0\,, \tag{4.18}$$

$$E = a_0\, e^2\, R_{01}{}^3\,, \tag{4.19}$$

$$a = n + 2\,, \tag{4.20}$$

$$a_0 = \frac{1}{R_1{}^3 - R_{01}{}^3}\,, \tag{4.21}$$

where R_1 is the end point of the antiproton on the inner spherical shell. The special case "homogeneous charge and mass distribution" must be included in the general case "inhomogeneous charge and mass distribution"; from this it follows that for the inhomogeneous charge and mass distribution we can set $a_1 = 3\, a_3 R_{01}{}^2 + x$, $a_3 = 1$:

$$(n+1)\,n + a_0{}^2\, e^4\, R_{01}{}^2\, (R_{01}{}^2 + x)^2 + e^2\, \tfrac{m}{b}\, a_0\, x - l\,(l+1) = 0\,, \tag{4.22}$$

$$a\,(n+1) + a_0{}^2\, e^4\, R_{01}\, x\, (R_{01}{}^2 + x) + a_0\, e^2 R_{01} (R_{01}{}^2 + x)\, E = 0\,, \tag{4.23}$$

$$a^2 - b^2 + (a_0\, e^2\, x + E)^2 - a_0\, e^2\, \tfrac{m}{b} = 0\,, \tag{4.24}$$

$$E = a_0\, e^2\, R_{01}\, (R_{01}{}^2 + x) - a_0\, e^2\, x\,, \tag{4.25}$$

$$a = n + 2\,, \tag{4.26}$$

$$a_0 = \frac{1}{R_1{}^3 - R_{01}{}^3 + (R_1 - R_{01})\, x}\,. \tag{4.27}$$

The equation system (4.22)-(4.27) consists of 6 equations for the determination of 8 unknowns (E, a, n, a_0, x, b, R_{01}, R_1); therefore, we need two more equations to solve (4.22) - (4.27). We get the first of these equations by using Eq. (4.3) in order to establish the energy balance of the inhomogeneously charged antiproton (m = - ½):

$$E_{11} = - \frac{e^2}{r} a_0 \,(a_1 (r - R_{01}) + a_2 (r - R_{01})^2 + a_3 (r - R_{01})^3) - \sqrt{b^2 + \frac{0.75}{r^2} - \frac{e^2}{2\,b} a_0 \,(\frac{2\,a_3\,R_{01}{}^3 - 2\,a_2\,R_{01}{}^2 + 2\,a_1\,R_{01}}{r^3} + \frac{-3\,a_3\,R_{01}{}^2 + 2\,a_2\,R_{01} - a_1}{r^2} + a_3)} \quad (4.28)$$

$$E_{12} = - \frac{e^2}{r} a_0 \,(a_1 (r - R_{01}) + a_2 (r - R_{01})^2 + a_3 (r - R_{01})^3) + \sqrt{b^2 + \frac{0.75}{r^2} - \frac{e^2}{2\,b} a_0 \,(\frac{2\,a_3\,R_{01}{}^3 - 2\,a_2\,R_{01}{}^2 + 2\,a_1\,R_{01}}{r^3} + \frac{-3\,a_3\,R_{01}{}^2 + 2\,a_2\,R_{01} - a_1}{r^2} + a_3)} \quad (4.29)$$

where E_{11} is the negative energy of the antiproton exerting pressure in the direction of the external vacuum and E_{12} is a positive energy exerting pressure in the direction of the center. E_{12} is not possible for the antiproton, because in this case it would fall into the center. In addition, E_{12} is not possible for the antiproton because it contains a root expression with a positive sign, even though the magnetic quantum number is m = - ½ . Solving the fundamental equations of the QEHD for the proton, we will see that E_{12} is the positive energy of the proton for m = ½ .

Considering the boundary condition $E_{11}(R_0) = 0$ at the position $r = R_0$, we get from (4.28):

$$b^2 + \frac{0.75}{R_{01}{}^2} - \frac{e^2}{2\,b} a_0 \,(\frac{x}{R_{01}{}^2} + 3) = 0 \,. \quad (4.30)$$

Reducing, for simplicity, $e^2 a_0$ to a_0, we obtain from equations (4.22)-(4.27) and (4.30):

$$(n + 1)\, n + a_0^{\,2} R_{01}^{\;2} (R_{01}^{\;2} + x)^2 + \frac{m}{b}\, a_0\, x - l\,(l+1) = 0\,, \tag{4.31}$$

$$a\,(n + 1) + a_0^{\,2} R_{01}\, x\left(R_{01}^{\;2} + x\right) + a_0\, R_{01}(R_{01}^{\;2} + x)\, E = 0\,, \tag{4.32}$$

$$a^2 - b^2 + (a_0\, x + E)^2 - a_0\, \frac{m}{b} = 0\,, \tag{4.33}$$

$$E = a_0\, R_{01}\,(R_{01}^{\;2} + x) - a_0\, x\,, \tag{4.34}$$

$$a = n + 2\,, \tag{4.35}$$

$$b^2 + \frac{0.75}{R_{01}^{\;2}} - \frac{1}{2\,b}\, a_0 \left(\frac{x}{R_{01}^{\;2}} + 3\right) = 0\,, \tag{4.36}$$

$$a_0 = \frac{e^2}{R_1^{\;3} - R_{01}^{\;3} + (R_1 - R_{01})\, x}\,. \tag{4.37}$$

Solving these 7 equations for 8 unknowns, we get the following solutions:

$$\frac{(4\, b\, R_{01}^{\;3}\,(-\, b^2 + n + 2) + b\,(8\, n + 11)\, R_{01})^2}{16\, m^2} + (n+1)(n+2) = 0\,, \tag{4.38}$$

$$-\,2\, b^2 + \frac{2\,(n + 1.75)}{R_{01}^{\;2}} + 3\, n + 6 = 0\,, \tag{4.39}$$

$$a = n + 2\,, \tag{4.40}$$

$$a_0 = \frac{b\,(-\, b^2 + n + 2)}{m}\,, \tag{4.41}$$

$$x = \frac{b\,(8\, n+11)}{4\, a_0\, m}\,, \tag{4.42}$$

$$E = a_0\,(R_{01}\,(R_{01}^{\;2} + \mathrm{x}) - \mathrm{x})\,, \tag{4.43}$$

$$a_0 = \frac{e^2}{R_1^{\;3} - R_{01}^{\;3} + (R_1 - R_{01})\, x} \tag{4.44}$$

with $e^2 = 0.007297412953$.

Since we have 7 equations for 8 unknowns, we must reduce the number of unknowns by 1 . From (4.38)-(4.44) we get the solution for the homogeneous charge and mass distribution if we set $x = 0$ [1] :

$$n = -1.375\,, \tag{4.45}$$
$$a = 0.625\,, \tag{4.46}$$
$$b = 3.92791\,, \tag{4.47}$$
$$R_{01} = 0.160867\,, \tag{4.48}$$
$$R_1 = 0.161671\,, \tag{4.49}$$
$$a_0 = 116.2935 \text{ für } m = -\tfrac{1}{2}\,, \tag{4.50}$$
$$E = 0.484124\,. \tag{4.51}$$

In order to obtain a positive value for a_0 in (4.50), we assumed $m = -½$ in Eq. (4.41). However, this assumption leads in (4.43) and (4.51) to a positive energy, which is not possible according to our energy balance considerations in Eq. (4.28) because in this case the antiproton would fall into the center. From this fact we conclude that the first approximation "homogeneous charge and mass distribution" does not lead to a satisfactory physical solution in the case of the antiproton.

In order to reduce the number of unknowns in equations (4.38)-(4.44) by 1 for the inhomogeneous charge and mass distribution, we need another equation, which we want to derive later from the normalization of the total energy. Therefore, we first arbitrarily set $b = 4.6494$ for the naked mass-energy and justify afterwards why we have just made this assumption. From (4.38) and (4.39) it follows:

$$\frac{21.6169\, {R_{01}}^2 \left(n\left({R_{01}}^2+2\right)-19.6169\, {R_{01}}^2+2.75\right)^2}{m^2}+(n+1)(n+2)=0\,, \qquad (4.52)$$

$$\frac{2\,(n+1.75)}{{R_{01}}^2}+3\,n-37.2338=0\,. \qquad (4.53)$$

For $m = -½$ we get from these two equations the following solutions with positive starting point R_{01}:

1. $n = -1.56349,\ R_{01} = 0.0943263\,,$ (4.54)
2. $n = -1.40122,\ R_{01} = 0.129746\,.$ (4.55)

The equations (4.40)-(4.44) then yield:

1. $n = -1.56349,\ R_{01} = 0.0943263\,,\ b = 4.6494\,,$ (4.56)

$a = 0.43651\,,$ (4.57)

$a_0 = 196.95239985\,,$ (4.58)

$x = 0.017798522\,,$ (4.59)

$E = -3.009509413\,,$ (4.60)

$R_1 = 0.0951547\,.$ (4.61)

2. $n = -1.40122,\ R_{01} = 0.129749,\ b = 4.6494\,,$ (4.62)

$a = 0.59878\,,$ (4.63)

$a_0 = 195.43063346\,,$ (4.64)

$x = 0.0024951\,,$ (4.65)

$E = 0.002528\,,$ (4.66)

$R_1 = 0.13045\,.$ (4.67)

Only the first solution (4.56)-(4.61) leads to a negative energy eigenvalue and thus, to a positive pressure in the direction of the external vacuum ("repulsion"). The second solution (4.62)-(4.67), on the other hand, is physically not correct because a positive energy eigenvalue leads to a negative pressure in the direction of the center (“attraction”).

For $m = ½$ we get from equations (4.52) and (4.53) the following solutions with positive starting point R_{01}:

1. $n = -1.56349,\ R_{01} = 0.094328\,,$ (4.68)
2. $n = -1.40122,\ R_{01} = 0.129749\,.$ (4.69)

These two solutions are physically not correct because they lead to a negative value for a_0 in (4.58) and (4.64). So, it turns out that only the solution (4.56)-(4.61) can be used for the inhomogeneously charged antiproton:

$$m=-0.5,\ n = -1.56349,\ a = 0.43651,\ E = -3.009509413,\ b =4.6494, \quad (4.70)$$
$$a_0 = 196.95239985,\ a_0 / e^2 = 26989.3455555,\ x = 0.017798522,$$
$$a_1 = 0.0444908746,\ a_2 = 0.2829789,\ a_3 = 1,\ R_{01} = 0.0943263,\ R_1 = 0.0951547\,.$$

Substituting these values into Eq. (4.8), we get the density function

$$\rho = (\sin(65.6508\,r^3)\,r^{-1.56349}\,e^{-0.43651})^2. \quad (4.71)$$

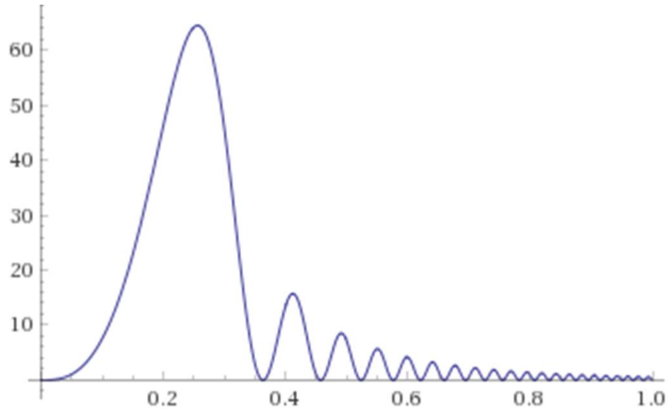

Figure 2: The mass or charge density of the self-interacting antiproton:

$$\rho \ = (\sin(65.6508\, r^3)\, r^{-1.56349}\, e^{-0.43651})^2$$

Fig. 2 shows graphically the mass or charge density of the self-interacting antiproton in the interval $0 \le r \le 1$. The first maximum of this curve lies at the point $r_{Max1} = 0.256017$. In order to place the first maximum in the starting point R_{01} , we shift in (4.71) r by $- R_{01} + r_{Max1} = - 0.0943263 + 0.256017 = 0.1616907$:

$$\rho \ = (0.931854\, sin(65.6465\, (r + 0.161691)^3)\, (r + 0.161691)^{-1.56349}\, e^{-0.43651})^2 . \qquad (4.72)$$

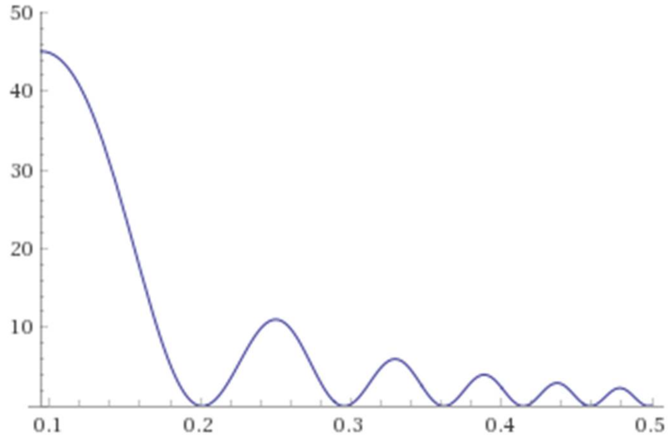

Figure 3: The mass or charge density curve of the self-interacting antiproton shifted by 0.161691 :

$$\rho \ = (0.931854\, sin(65.6465\, (r + 0.161691)^3)\, (r + 0.161691)^{-1.56349}\, e^{-0.43651})^2 .$$

Fig. 3 shows graphically the mass or charge density of the self-interacting antiproton shifted by 0.161691 in the interval $0.0943263 \leq r \leq 0.5$. Accordingly, the mass or charge density of the antiproton expands from $R_{01} = 0.0943263$ up to $R_1 = 0.0951547$. The characteristic feature of this graphic is that the antiproton can be described as a sequence of quantized eigenstates (spherical shells). According to (4.72), zeros of the mass or charge density curve are given by the quantum number (particle or mass number) n in the equation

$$65.6465\,(r + 0.161691)^3 = n\,\pi\;, \qquad n = 1, 2, 3, 4\,... \tag{4.73}$$

$$r_{01} = 0.20137,\; r_{02} = 0.29574,\; r_{03} = 0.36193,\; r_{04} = 0.41463\,...$$

* * *

In the framework of our model concept, we assume that the part of the antiproton that exceeds the limit of $R_{01} = 0.0943263\,fm$ in the direction of the center suddenly changes into photons. Conversely, we assume that the photons in the cavity volume $V_{01} = \frac{4\,\pi\,R_{01}{}^3}{3} = 0.0035155\,fm^3$ suddenly change into an antiproton as soon as they exceed the limit $R_{01} = 0.0943263\,fm$ in the direction of the external vacuum. Thus, in the internal vacuum (in the cavity $V_{01} = \frac{4\,\pi\,R_{01}{}^3}{3}$) the Casimir energy $E_{cas01} = \frac{\pi}{12\,R_{01}}$ reduced by $d = \hbar\,c$ is located [5]. From that, assuming a homogeneous distribution of the photons in the cavity, we obtain the Casimir energy density or the Casimir energy pressure

$$E_{cas01}\,\rho_{Ph} = \frac{\frac{\pi}{12\,R_{01}}}{\frac{4\,\pi\,R_{01}{}^3}{3}} = \frac{1}{16\,R_{01}{}^4}\;. \tag{4.74}$$

The volume integral over the energy density

$$E_{cas01} = \int_0^{2\pi}\int_0^{\pi}\int_0^{R_{01}} \frac{1}{16\,R_{01}{}^4}\,r^2\,sin(\vartheta)\;d\varphi\,d\vartheta\;d\,r = \frac{\pi}{12\,R_{01}} = 2.775465 \tag{4.75}$$

gives then the correct Casimir energy E_{cas01}.

The Casimir pressure P_{cas01}, which acts in the interval $0 \leq r \leq 0.0943263$ on the lower sphere surface $F_{01} = 4\,\pi\,{R_{01}}^2$ of the antiproton, is given by the relation:

$$P_{cas01} = \frac{1}{4\,\pi\,{R_{01}}^2}\frac{d}{dr}\,E_{cas01} = \frac{1}{16\,{R_{01}}^4} = 789.493975\ . \tag{4.76}$$

* * *

Substituting the values in (4.70) into the energy equation (4.28), we get the energy of the antiproton at the point r:

$$E_{11} = -3.50546 - 196.952\,r^2 + \frac{0.495952}{r} - \sqrt{0.43651 - \frac{0.10667}{r^3} + \frac{1.12698}{r^2}} \tag{4.77}$$

Multiplication of the energy (4.77) by the density function (4.72) gives the energy density at the point r:

$$E_{11}\,\rho = \left(-3.50546 - 196.952\,r^2 + \frac{0.495952}{r} - \sqrt{0.43651 - \frac{0.10667}{r^3} + \frac{1.12698}{r^2}}\right) \\ (0.931854\,sin(65.6465\,(r + 0.161691)^3)\,(r + 0.161691)^{-1.56349}\,e^{-0.43651})^2 \tag{4.78}$$

In order to normalize the total energy of the antiproton according to (4.70) and (4.78), we perform the volume integral over the energy density of the antiproton in the definition domain *[0.0943263, 0.0951547]* and equate the result to the total energy after multiplication with the density constant:

$$E = -\,3.009509413 = x\int_{0.0943263}^{0.0951547} E_{11}\,\rho\,r^2 dr = -\,x\ 0.0002484\,, \tag{4.79}$$

where x is the normalization or density constant.

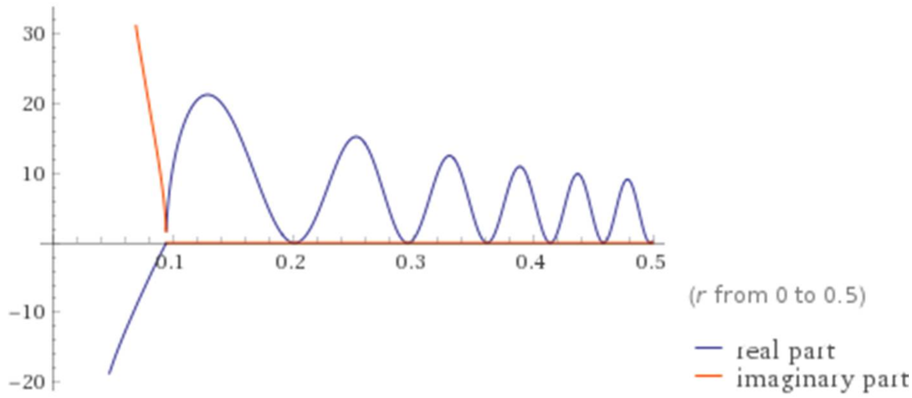

Figure 4: The pressure P/x , which acts on the sphere surface $4\,\pi\,r^2$ of the antiproton.

The pressure P , which acts on the sphere surface $4\,\pi\,r^2$ of the antiproton at the point r , is therefore given by the relation (Fig. 4):

$$P = -x\frac{1}{4\,\pi\,r^2}\frac{d}{dr}E$$
$$= -\frac{x}{4\,\pi}\left(-3.50546 - 196.952\,r^2 + \frac{0.495952}{r} - \sqrt{0.43651 - \frac{0.10667}{r^3} + \frac{1.12698}{r^2}}\right) \tag{4.80}$$
$$(0.931854\,sin(65.6465\,(r+0.161691)^3)\,(r+0.161691)^{-1.56349}\,e^{-0.43651})^2\,.$$

According to Eq. (4.76), at the point $R_{01} = 0.0943263\,fm$ the Casimir energy pressure $P_{cas01} = 789.493975$ acts on the lower sphere surface of the antiproton. According to Eq. (4.80), in hydrodynamic equilibrium the pressure of the antiproton on the same sphere surface must be equal to the Casimir energy pressure P_{cas01} ; therefore, it follows for the density constant

$$P_{cas01} = 789.493975 = P_{01} = x\ \ 0.055438,\ x = 14241.0255582\,. \tag{4.81}$$

Eq. (4.79) gives then:

$$E\,/\,x = -\,0.0002484 = -\,3.009509413\,/\,14241.0255582\ = -\,0.0002113\,. \tag{4.82}$$

That means, normalizing the total energy, the integral in Eq. (4.79) gives approximately the same amount E/x as by determining the density constant using the pressure equation (4.80). This approximately accordance was achieved by initially setting the naked mass-energy arbitrarily $b = 4.6494$, i.e. in order to solve the equation system (4.38-4.44) the energy normalization equation (4.79) acted as the missing eighth equation.

* * *

According to Eq. (4.80), at the position $R_1 = 0.0951547\, fm$ the repulsive pressure $P_1 = 14241.02556 \times 4.02854 = 57370.5411$ acts on the upper sphere surface of the antiproton. Consequently, during the creation of the antiproton a backpressure at the same level must act on the upper surface of the spherical shell so that the antiproton does not suddenly expand and burst at a critical distance from the center. In our opinion, such high pressure values can only be achieved in form of radiation pressure during the cosmological development "shortly after Big Bang". Accordingly, antiprotons can only be spontaneously created in cosmological dimensions at a radiation pressure of $P_{Radiation1} = -57370.5411$. At higher pressure values in an earlier universe, they cannot be stable, because "shortly" after their creation they would be pushed back in form of photons into the singularity in $R_{01} = 0.0943263\, fm$.

5. The inhomogeneously charged proton on the outer spherical shell

Substituting the electromagnetic field variables (3.8) and (3.15) for the inhomogeneously charged proton on the outer spherical shell into the radial differential equation (2.12), we obtain

$$\left(\frac{r^2}{f}\right)\Big[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \left(E + \frac{e^2}{r}\left(-a_0\left(a_1 (r-R_{02}) + a_2 (r-R_{02})^2 + a_3 (r-R_{02})^3\right) + 1\right)\right)^2 - m_0^2c^4$$
$$- e^2 m \frac{(\hbar c)^2}{m_0 c^2}\left(-a_0\left(\frac{2a_3R_{02}^3 - 2a_2R_{02}^2 + 2a_1R_{02}}{r^3} + \frac{-3a_3R_{02}^2 + 2a_2R_{02} - a_1}{r^2} + a_3\right) - \frac{2}{r^2}\right)\Big]\frac{f}{\hbar^2c^2} = l\,(l+1)\,, \qquad (5.1)$$

where R_{02} is the starting point of the proton on the outer spherical shell.

For central symmetric potentials it is usual to make the assumption $f = \frac{z}{r}$. By this way, Eq. (5.1) can be reduced to:

$$z'' + \frac{1}{d^2}[(E + \frac{e^2}{r}(-a_0\,(a_1\,(r - R_{02}) + a_2\,(r - R_{02})^2 + a_3\,(r - R_{02})^3) + 1))^2 - b^2 \tag{5.2}$$
$$- e^2\frac{d^2}{b}m\,(-a_0\Big(\frac{2\,a_3\,R_{02}{}^3 - 2\,a_2\,R_{02}{}^2 + 2\,a_1\,R_{02}}{r^3} + \frac{-3\,a_3\,R_{02}{}^2 + 2\,a_2\,R_{02} - a_1}{r^2} + a_3\Big) - \frac{2}{r^2})]\,z = \frac{l\,(l+1)}{r^2}\,z,$$

where $b = m_o c^2$ and $d = \hbar c$ are two very useful abbreviations.

Dividing the variables E, b, e^2, for simplicity, by $\frac{d}{fm}$, we obtain from (5.2) an equation with dimensionless quantities:

$$z'' + [(E + \frac{e^2}{r}(-a_0\,(a_1\,(r - R_{02}) + a_2\,(r - R_{02})^2 + a_3\,(r - R_{02})^3) + 1))^2 - b^2 \tag{5.3}$$
$$- \frac{e^2}{b}m\,(-a_0\Big(\frac{2\,a_3\,R_{02}{}^3 - 2\,a_2\,R_{02}{}^2 + 2\,a_1\,R_{02}}{r^3} + \frac{-3\,a_3\,R_{02}{}^2 + 2\,a_2\,R_{02} - a_1}{r^2} + a_3\Big) - \frac{2}{r^2})]\,z = \frac{l\,(l+1)}{r^2}\,z$$

A general solution of the differential equation (5.3) is given by

$$z = z_0\,\sin(c_3\frac{e^2\,r^3}{3})\,r^n\,e^{-a\,r}. \tag{5.4}$$

Substituting the solution (5.4) into the differential equation (5.3), a coefficient comparison for the powers of r yields the following seven equations:

$$c_3 = a_0\,a_3\,, \tag{5.5}$$
$$a_2 = 3\,a_0\,R_{02}\,, \tag{5.6}$$
$$(n+1)\,n + e^4\,\big(a_0\,R_{02}\,(a_1 - 2\,a_3\,R_{02}{}^2) + 1\big)^2 - e^2\,\frac{m}{b}\,a_0\,(a_1 - 3\,a_3\,R_{02}{}^2) - l\,(l+1) = 0\,, \tag{5.7}$$
$$a\,(n+1) + \big(a_0\,e^4\,(a_1 - 3\,a_3\,R_{02}{}^2) - e^2\,E\big)\,\big(a_0\,R_{02}\,(a_1 - 2\,a_3\,R_{02}{}^2) + 1\big) = 0\,, \tag{5.8}$$
$$a^2 - b^2 + \big(E - a_0\,e^2\,(a_1 - 3\,a_3\,R_{02}{}^2)\big)^2 - a_0\,a_3\,e^2\,\frac{m}{b} = 0\,, \tag{5.9}$$
$$E = a_0\,e^2\,R_{02}\,(a_1 - 2\,a_3\,R_{02}{}^2) - e^2\,\big(a_0\,R_{02}\,(a_1 - 2\,a_3\,R_{02}{}^2) + 1\big)\,, \tag{5.10}$$
$$a = n + 2\,. \tag{5.11}$$

where according to Eq. (3.7) $a_0 = \frac{1}{a_1\,(R_2 - R_{02}) + a_3\,(R_2 - R_{02})^2\,(2\,R_2 + R_{02})}$ holds.

We obtain from these equations as a special case the solution to the homogeneous charge and mass distribution, if we set $a_1 = 3\, a_3\, R_{02}^2,\ a_3 = 1$ [1]:

$$(n + 1)\, n + e^4 \,(a_0\, R_{02}{}^3 + 1)^2 - l\,(l + 1) = 0\,, \tag{5.12}$$

$$a\,(n + 1) - e^2\,(a_0\, R_{02}{}^3 + 1)\, E = 0\,, \tag{5.13}$$

$$a^2 - b^2 + E^2 - a_0\, e^2\, \frac{m}{b} = 0\,, \tag{5.14}$$

$$E = -\,e^2\,(a_0\, R_{02}{}^3 + 1)\,, \tag{5.15}$$

$$a = n + 2\,, \tag{5.16}$$

$$a_0 = \frac{1}{R_2{}^3 - R_{02}{}^3}\,, \tag{5.17}$$

where R_2 is the end point of the proton on the outer spherical shell.

The special case "homogeneous charge and mass distribution" must be included in the general case "inhomogeneous charge and mass distribution"; from this it follows that for the inhomogeneous charge and mass distribution we can set $a_1 = 3\, a_3\, R_{02}^2 + x$, $a_3 = 1$:

$$(n + 1)\, n + e^4\,(a_0\, R_{02}\,(R_{02}{}^2 + x) + 1)^2 - e^2\, \frac{m}{b}\, a_0\, x - l\,(l + 1) = 0\,, \tag{5.18}$$

$$a\,(n + 1) + (a_0\, e^4\, x - e^2 E)\,(a_0\, R_{02}(R_{02}{}^2 + x) + 1) = 0\,, \tag{5.19}$$

$$a^2 - b^2 + (E - a_0\, e^2\, x)^2 + a_0\, e^2\, \frac{m}{b} = 0\,, \tag{5.20}$$

$$E = a_0\, e^2\, x - e^2\,(a_0\, R_{02}(R_{02}{}^2 + x) + 1)\,, \tag{5.21}$$

$$a = n + 2\,, \tag{5.22}$$

$$a_0 = \frac{1}{R_2{}^3 - R_{02}{}^3 + (R_2 - R_{02})\, x}\,. \tag{5.23}$$

The equation system (5.18)-(5.23) consists of 6 equations for the determination of 8 unknowns ($E, a, n, a_0, x, b, R_{02}, R_2$); therefore, we need two more equations to solve (5.18)-(5.23). We get the first of these equations by using Eq. (5.3) in order to establish the energy balance of the inhomogeneously charged proton ($m = ½$):

$$E_{11} = -\frac{e^2}{r}(-a_0\,(a_1\,(r - R_{02}) + a_2\,(r - R_{02})^2 + a_3\,(r - R_{02})^3 + 1) \tag{5.24}$$

$$- \sqrt{b^2 + \frac{0.75}{r^2} + \frac{e^2}{2\,b}\,(-a_0\,(\frac{2\,a_3\,R_{02}{}^3 - 2\,a_2\,R_{02}{}^2 + 2\,a_1\,R_{02}}{r^3} + \frac{-3\,a_3\,R_{02}{}^2 + 2\,a_2\,R_{02} - a_1}{r^2} + a_3) - \frac{2}{r^2})},$$

$$E_{12} = -\frac{e^2}{r}(-a_0\,(a_1\,(r - R_{02}) + a_2\,(r - R_{02})^2 + a_3\,(r - R_{02})^3 + 1) \tag{5.25}$$

$$+ \sqrt{b^2 + \frac{0.75}{r^2} + \frac{e^2}{2\,b}\,(-a_0\,(\frac{2\,a_3\,R_{02}{}^3 - 2\,a_2\,R_{02}{}^2 + 2\,a_1\,R_{02}}{r^3} + \frac{-3\,a_3\,R_{02}{}^2 + 2\,a_2\,R_{02} - a_1}{r^2} + a_3) - \frac{2}{r^2})}.$$

where E_{12} is the positive energy of the proton exerting pressure in the direction of the center. E_{11} is not possible for the proton because it contains a root expression with a negative sign, even though the magnetic quantum number is $m = ½$; we have already seen that E_{11} is the negative energy of the antiproton ($m = -½$) exerting pressure in the direction of the external vacuum.

Considering the boundary condition $E_{12}(R_{02}) = 0$ at the position $r = R_{02}$, we get from (5.25):

$$b^2 + \frac{0.75}{R_{02}{}^2} - \frac{e^2}{2\,b}\,a_0\left(\frac{x}{R_{02}{}^2} + 3\right) - (\frac{e^2}{R_{02}})^2 = 0\,. \tag{5.26}$$

Reducing, for simplicity, $e^2 a_0$ to a_0, it follows from equations (5.18)-(5.23) and (5.26):

$$(n + 1)\,n + (a_0\,R_{02}\,(R_{02}{}^2 + x) + e^2)^2 - \frac{m}{b}\,a_0\,x - l\,(l + 1) = 0\,, \tag{5.27}$$

$$a\,(n + 1) + (E - a_0\,x)\,(a_0\,R_{02}\,(R_{02}{}^2 + x) + e^2) = 0\,, \tag{5.28}$$

$$a^2 - b^2 + (E - a_0\,x)^2 + a_0\,\frac{m}{b} = 0\,, \tag{5.29}$$

$$E = a_0\,x - (a_0\,R_{02}\,(R_{02}{}^2 + x) + e^2)\,, \tag{5.30}$$

$$a = n + 2\,, \tag{5.31}$$

$$a_0 = \frac{e^2}{R_2{}^3 - R_{02}{}^3 + (R_2 - R_{02})\,x}\,, \tag{5.32}$$

$$b^2 + \frac{0.75}{R_{02}{}^2} - \frac{m}{b}\,(a_0\left(\frac{x}{R_{02}{}^2} + 3\right) + \frac{2\,e^2}{R_{02}{}^3}) - (\frac{e^2}{R_{02}})^2 = 0\,. \tag{5.33}$$

Solving these 7 equations for 8 unknowns, we get the following solutions:

$$\left(\frac{b\,R_{02}\left(4\,R_{02}^{\;2}\,(b^2-n-2)-8\,n-11\right)}{4\,m}+e^2\right)^2+(n+1)(n+2)=0\,, \tag{5.34}$$

$$n=\frac{R_{02}\left(4\,(b^3-3)\,R_{02}^{\;2}-7\right)+2\,e^4\,R_{02}+4\,e^2\,\frac{m}{b}}{6\,R_{02}^{\;3}+4\,R_{02}}\,, \tag{5.35}$$

$$a=n+2\,, \tag{5.36}$$

$$a_0=\frac{b\,(-b^2+n+2)}{m}\,, \tag{5.37}$$

$$x=\frac{b\,(8\,n+11)}{4\,a_0\,m}\,, \tag{5.38}$$

$$E=a_0\,(R_{02}\,(R_{02}^{\;2}+\mathrm{x})-\mathrm{x})\,, \tag{5.39}$$

$$a_0=\frac{e^2}{R_2^{\;3}-R_{02}^{\;3}+(R_2-R_{02})\,x} \tag{5.40}$$

with $e^2 = 0.007297412953$.

Since we have 7 equations for 8 unknowns, we must reduce the number of unknowns by 1. From (5.34)-(5.40) we get the solution for the homogeneous charge and mass distribution if we set $x = 0$ [1] :

$$n = -1.375\,, \tag{5.41}$$
$$a = 0.625\,, \tag{5.42}$$
$$b = 3.66298\,, \tag{5.43}$$
$$R_{02} = 0.171994\,, \tag{5.44}$$
$$R_{12} = 0.172867, \tag{5.45}$$
$$a_0 = 93.7168 \text{ für } m = ½\,, \tag{5.46}$$
$$E = 0.476823\,. \tag{5.47}$$

In order to reduce the number of unknowns in equations (5.34)-(5.40) by 1 for the inhomogeneous charge and mass distribution, we need another equation, which we want to derive from the normalization of the total energy. Therefore, we first arbitrarily set $b = 4.405$ for the naked mass-energy and justify afterwards why we have just made this assumption. From (5.34) and (5.35) it follows:

$$\frac{\left(0.00729741-\left(4.405\,R_{02}\,(n\,(R_{02}^{\;2}+2)-17.404\,R_{02}^{\;2}+2.75)\right)\right)^2}{m^2}+(n+1)(n+2)=0\,, \tag{5.48}$$

$$n=\frac{0.00331324\,m+32.8081\,R_{02}^{\;3}-3.49995\,R_{02}}{3\,R_{02}^{\;3}+2\,R_{02}}\,. \tag{5.49}$$

For $m = -\frac{1}{2}$ we get from these two equations the following solutions with positive starting point R_{02}:

1. $n = -1.99914,\ R_{02} = 0.00332143,\ a_0 = -170.94188365$ (5.50)
2. $n = -1.59489,\ R_{02} = 0.0933945,\ a_0 = -167.38044115$ (5.51)
3. $n = -1.38008,\ R_{02} = 0.142606,\ a_0 = -165.48796505$ (5.52)
4. $n = -1.00167,\ R_{02} = 0.204976,\ a_0 = -162.15417295$. (5.53)

All these four solutions are physically not correct because they lead in (5.37) to a negative value for a_0.

For $m = \frac{1}{2}$ we get from equations (5.34) and (5.40) the following solutions with positive starting point R_{02}:

1. $n = -1.53318,\ R_{02} = 0.105697$ (5.54)
2. $n = -1.41308,\ R_{02} = 0.133614$ (5.55)
3. $n = -1.00161,\ R_{02} = 0.203877$ (5.56)
4. $n = -1.0,\ R_{02} = 0.00110448$. (5.57)

Only the first solution (5.54) leads to a positive energy eigenvalue and thus, to a negative pressure in the direction of the center (“attraction”). The remaining three solutions (5.55)-(5.57), on the other hand, are physically not correct because the energy eigenvalue is negative and leads to a positive pressure in the direction of the external vacuum ("repulsion"). So, it follows from (5.34)-(5.40) for the inhomogeneously charged proton:

$$m=0.5,\ n = -1.53319,\ a = 0.46681,\ E = 2.2884024,\ b =4.405, \quad (5.58)$$
$$a_0 = 166.83686,\ a_0 / e^2 = 22862.46718,\ x = 0.016706786,$$
$$a_1 = 0.0502217,\ a_2 = 0.317088,\ a_3 = 1,\ R_{02} = 0.105696,\ R_2 = 0.106562\ .$$

Substituting these values into Eq. (5.4), we get the density function

$$\rho \ = (sin(55.6123\, r^3)\, r^{-1.53319}\, e^{-0.46681})^2. \tag{5.59}$$

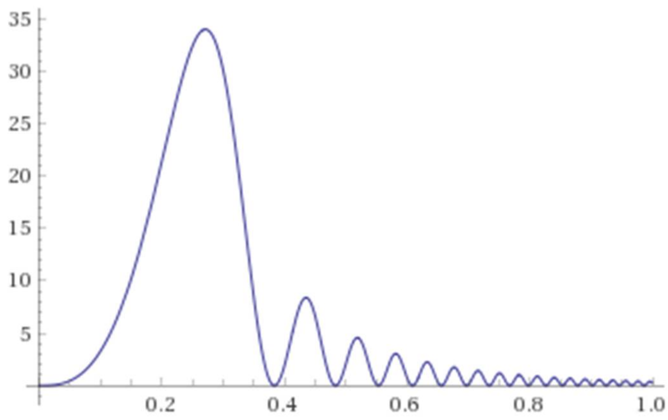

Figure 5: The mass or charge density of the proton:

$\rho \ = (sin(55.6123\ r^3)\ r^{-1.53319}\ exp(-\ 0.46681\ r))^2$

Fig. 5 shows graphically the mass or charge density of the self and external interacting proton in the interval $0 \le r \le 1$. The first maximum of this curve lies at the point $r_{Max2} = 0.2710576$. In order to place the first maximum in the starting point R_{02} , we shift in (5.59) r by $- R_{02} + r_{Max2} = -\ 0.105696 + 0.2710576\ \ = 0.1653616$:

$$\rho \ = (0.925712\, sin(55.6123\, (r + 0.165362)^3)\, (r + 0.165362)^{-1.53319}\, e^{-0.46681})^2\, . \tag{5.60}$$

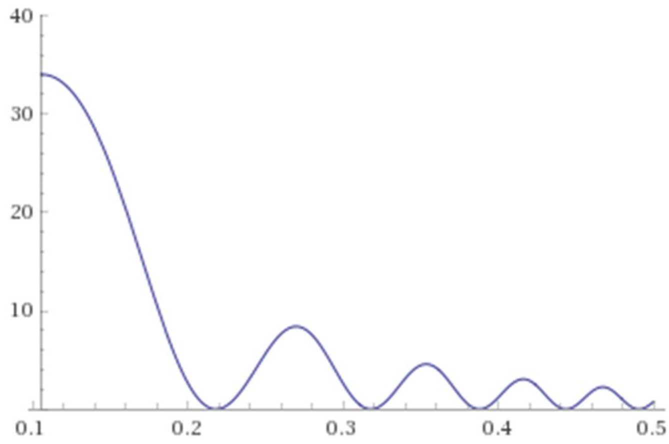

Figure 6: The mass or charge density curve of the proton shifted by *0.1653616* :

$$\rho \ = (0.925712\, sin(55.6123\,(r + 0.165362)^3)\,(r + 0.165362)^{-1.53319}\, e^{-0.46681})^2 \ .$$

Fig. 6 shows graphically the mass or charge density of the proton shifted by *0.1653616* in the interval $0.105696 \leq r \leq 0.5$. Accordingly, the mass or charge density of the proton expands from $R_{02} = 0.105696$ up to $R_2 = 0.106562$. The characteristic feature of this graphic is that the proton can be described as a sequence of quantized eigenstates (spherical shells). According to (5.60), zeros of the mass or charge density curve are given by the quantum number (particle or mass number) n in the equation

$$55.6123\,(r + 0.165362)^3 = n\,\pi \ , \qquad n = 1, 2, 3, 4 \ldots \tag{5.61}$$

$$r_{01} = 0.21834,\ r_{02} = 0.318072,\ r_{03} = 0.388031,\ r_{04} = 0.443726 \ldots$$

* * *

In the framework of our model concept, we assume that the proton is preceded by a cavity (by a buffer zone) that separates the proton from the antiproton and has the thickness $x = R_{02} - R_1 = 0.105696 - 0.0951547 = 0.0105413$. That part of the proton that exceeds the limit $R_{02} = 0.105696\,fm$ in the direction of the antiproton suddenly changes into photons. Conversely, we assume that the photons in the cavity volume $V_{02} = \frac{4\,\pi\,((R_1+x)^3 - R_1{}^3)}{3}$ suddenly change into a proton as soon as they exceed the limit $R_{02} = 0.105696\,fm$ in the direction of the external vacuum. Thus, in the cavity volume

V_{02} the Casimir energy $E_{cas02} = \frac{\pi}{12\,((R_1 + x) - R_1)} = \frac{\pi}{12\,x}$ reduced by $d = \hbar c$ is located [5]. From that, assuming a homogeneous distribution of the photons in the cavity, we obtain the Casimir energy density or the Casimir energy pressure

$$E_{cas02}\,\rho_{Ph} = \frac{\frac{\pi}{12\,x}}{\frac{4\,\pi\,((R_1 + x)^3 - R_1{}^3)}{3}} = \frac{1}{16\,x\,((R_1 + x)^3 - R_1{}^3)} \,. \qquad (5.62)$$

The volume integral over the energy density

$$E_{cas02} = \int_0^{2\pi}\int_0^{\pi}\int_{R_1}^{R_1 + x} \frac{1}{16\,x\,((R_1 + x)^3 - R_1{}^3)}\, r^2\, sin(\vartheta)\, d\varphi\, d\vartheta\, d\,r = \frac{\pi}{12\,x} \qquad (5.63)$$

gives then the correct Casimir energy E_{cas02} .

The Casimir pressure P_{cas02} , which acts in the interval $0.0951547 \leq r \leq 0.105696$ on the lower sphere surface $F_{02} = 4\,\pi\,R_{02}{}^2$ of the proton, is given by the relation:

$$P_{cas02} = \frac{1}{4\,\pi\,R_{02}{}^2}\frac{d}{dr}\,E_{cas02} = \frac{1}{16\,x\,((R_1 + x)^3 - R_1{}^3)} = 18573.1316556\,. \qquad (5.64)$$

* * *

Substituting the values in (5.58) into the energy equation (5.25), we get the energy of the proton at the point r:

$$E_{12} = 2.78731 + 166.837\,r^2 - \frac{0.498905}{r} + \sqrt{19.404 - \frac{0.113259}{r^3} + \frac{1.06638}{r^2}} \qquad (5.65)$$

Multiplication of the energy (5.65) by the density function (5.60) gives the energy density at the point r:

$$E_{12}\,\rho = (2.78731 + 166.837\,r^2 - \frac{0.498905}{r} + \sqrt{19.404 - \frac{0.113259}{r^3} + \frac{1.06638}{r^2}}) \qquad (5.66)$$
$$(0.925712\,sin(55.6123\,(r + 0.165362)^3)\,(r + 0.165362)^{-1.53319}\,e^{-0.46681})^2$$

In order to normalize the total energy of the antiproton according to (5.58) and (5.66), we perform the volume integral over the energy density of the proton in the definition domain *[0.105696, 0.106562]* and equate the result to the total energy after multiplication with the density constant:

$$E = 2.2884024 = x \int_{0.105696}^{0.106562} E_{12}\, \rho\, r^2 dr = x\, 0.00144777\,, \tag{5.67}$$

where x is the normalization or density constant.

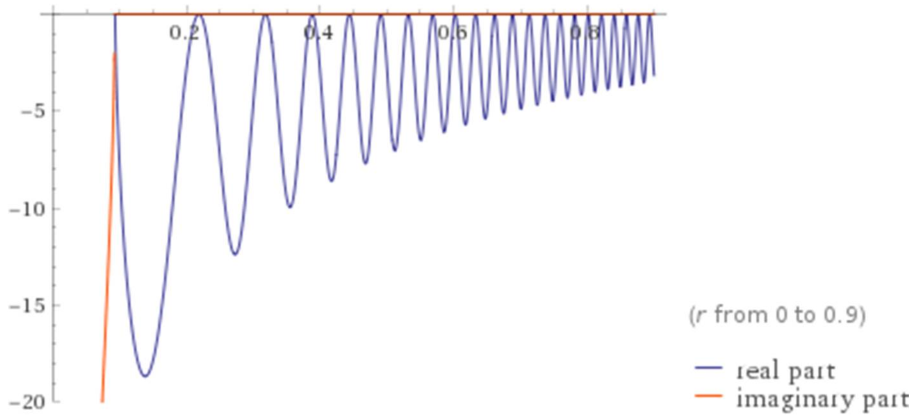

Figure 7: The pressure P/x , which acts on the sphere surface $4\,\pi\, r^2$ of the proton.

The pressure P , which acts on the sphere surface $4\,\pi\, r^2$ of the proton at the point r , is therefore given by the relation (Fig. 7):

$$\begin{aligned} P &= -x \frac{1}{4\,\pi\, r^2} \frac{d}{dr} E \\ &= -\frac{x}{4\,\pi} (2.78731 + 166.837\, r^2 - \frac{0.498905}{r} + \sqrt{19.404 - \frac{0.113259}{r^3} + \frac{1.06638}{r^2}}) \\ &\quad (0.925712\, sin(55.6123\,(r + 0.165362)^3)\,(r + 0.165362)^{-1.53319}\, e^{-0.46681})^2\,. \end{aligned} \tag{5.68}$$

According to Eq. (5.64), at the point $R_{02} = 0.105696\, fm$ the Casimir energy pressure $P_{cas02} = 18573.131658$ acts on the lower sphere surface of the proton. According to Eq. (5.68), in hydrodynamic equilibrium the pressure of the proton on the same sphere surface must be equal to the Casimir energy pressure P_{cas02}; therefore, it follows for the density constant

$$P_{cas02} = 18573.131658 = P_{02} = x \ \ 11.597, \ x = 1601.5462324 \ . \tag{5.69}$$

Eq. (5.67) gives then:

$$E / x = 0.00144777 = 2.2884024 \ / \ 1601.5462324 \ = 0.00142887 \ . \tag{5.70}$$

That means, normalizing the total energy, the integral in Eq. (5.67) gives approximately the same amount E / x as by determining the density constant using the pressure equation (5.68). This approximately accordance was achieved by initially setting the naked mass-energy arbitrarily $b = 4.405$, i.e. in order to solve the equation system (5.34-5.40) the energy normalization equation (5.67) acted as the missing eighth equation.

* * *

According to Eq. (5.68), at the position $R_2 = 0.106562 \ fm$ the attractive pressure $P_2 = - \ 1601.5462324 \ x \ 12.0213 = - \ 19252.66772388$ acts on the upper sphere surface of the proton. Consequently, during the creation of the proton, a backpressure at the same level must act on the upper surface of the sphere so that the proton does not suddenly contract and be annihilated in the singularity below. Solving the fundamental equations of the QEHD for the antiproton, we have seen that such high pressure values can only be achieved in form of radiation pressure during the cosmological development "shortly after Big Bang". Accordingly, protons can only be spontaneously created in cosmological dimensions at a radiation pressure of $P_{Radiation2} = - \ 19252.66772$. At higher pressure values in an earlier universe, they cannot be stable, because "shortly" after their creation they would be pushed back in form of photons into the singularity in $R_{02} = 0.105696 \ fm$.

6. A final remark

In the present work, the fundamental equations of the QEHD were solved with the novel potential approach (3.6) for the inhomogeneously charged proton-antiproton pair. For the sake of simplicity, the Casimir energy that is also *present inside the proton and antiproton* was not taken into account; therefore, the exact solution of the fundamental equations of the QEHD cannot neglect the additional Casimir forces inside the proton and antiproton!

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