

Compton Effect

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Abstract: Compton scattering, discovered by American physicist Arthur Holly Compton, is the scattering of a photon by an electron. It results in a decrease in energy of an X-ray or gamma ray photon, called the Compton effect. In this article, we provide a simple, concise discussion about one of three principle forms of photon interaction "Compton scattering" which demonstrates the particle nature of electromagnetic radiation.

Keywords: photon; electron; momentum; energy; Compton scattering; wavelength; particle behavior; relativistic mechanics.

In empty space, the photon – the basic unit of all light – moves at c (the speed of light) and its energy and momentum are related by $E = pc$, where p is the momentum of the photon. This derives from the following relativistic relation, with $m_0 = 0$:

$$E^2 = p^2c^2 + m_0^2c^4$$

In some situations, photon behaves like a wave, while in others, it behaves like a particle. The photons can be thought of as both waves and particles. In 1924 a French physicist Louis de Broglie developed a formula to relate this dual wave and particle behavior:

$$E = h\nu, \quad c = \lambda\nu, \quad E = \frac{hc}{\lambda} = mc^2,$$

where E and m are the energy and mass of the photon, ν and λ are the frequency and wavelength of the photon, h is the Planck constant, c is the speed of light. From this we obtain the definition of the photon wavelength through the Planck constant and the momentum of the photon:

$$\lambda = \frac{h}{mc}$$

This equation is used to describe the wave properties of matter, specifically, the wave nature of the electron:

$$\lambda_e = \frac{h}{m_e v}$$

where λ_e is wavelength, h is Planck's constant, $m_e = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the relativistic mass of the

electron, moving at a velocity v .

$$p_e = \frac{h}{\lambda_e}$$

From this it follows that,

$$\frac{dp_e}{dt} = \frac{p_e^2}{h} \times - \frac{d\lambda_e}{dt}$$

Sir Isaac Newton first presented his three laws of motion in the "Principia Mathematica Philosophiae Naturalis" in 1686. His second law defines a force exerted by a photon on the electron to be equal to the rate of change in momentum of the electron:

$$F = \frac{dp_e}{dt}$$

$$F = \frac{p_e^2}{h} \times - \frac{d\lambda_e}{dt}$$

$$E^2 = p_e^2 c^2 + E_0^2$$

$$E^2 - E_0^2 = p_e^2 c^2$$

$$(E - E_0) (E + E_0) = p_e^2 c^2$$

$$E_K = \frac{m_e^2 v^2}{(m_0 + m_e)}$$

For non-relativistic case:

$$m_e = m_0$$

$$E_K = \frac{m_0 v^2}{2}$$

$$F = E_K \frac{(m_0 + m_e)}{h} \times - \frac{d\lambda_e}{dt}$$

$$E_K = \frac{hF}{(m_0 + m_e) \times - \frac{d\lambda_e}{dt}}$$

For non-relativistic case:

$$m_e = m_0$$

$$F = m_0 a$$

$$E_K = \frac{ha}{2 \times -\frac{d\lambda_e}{dt}}$$

$$E_K = \frac{3}{2} k_B T = \frac{ha}{2 \times -\frac{d\lambda_e}{dt}}$$

$$a = \frac{3k_B T}{h} \times -\frac{d\lambda_e}{dt}$$

$$E_K = eV = \frac{ha}{2 \times -\frac{d\lambda_e}{dt}}$$

$$a = K_J V \times -\frac{d\lambda_e}{dt}$$

where K_J is the Josephson constant.

An effect published in the Physical Review that explained the x-ray shift by attributing particle-like momentum to light quanta – discovered by American physicist Arthur Compton in early 1920s at Washington University in St. Louis, which amply confirmed the particle behavior of photons at a time when the corpuscular nature of light suggested by photoelectric effect was still being debated. This effect is suggested that when an x-ray quantum of energy $h\nu$ and a momentum $\frac{h}{\lambda}$ interacts with an electron in an atom, which is treated as being at rest with momentum = 0 and energy equal to its rest energy, m_0c^2 . The symbols h , ν , and λ are the standard symbols used for Planck's constant, the photon's frequency, its wavelength, and m_0 is the rest mass of the electron. In the interaction, the x-ray photon is scattered in the direction at an angle θ with respect to the photon's incoming path with momentum $\frac{h}{\lambda_s}$ and energy $h\nu_s$. The electron is scattered in the direction at an angle ϕ with respect to the photon's incoming path with

momentum $m_e v$ and energy $m_e c^2$ (where m_e is the total mass of the electron after the interaction). The phenomenon of Compton scattering may be analyzed as an elastic collision of a photon with a free electron using relativistic mechanics. Since the energy of the photons (661.6 keV) is much greater than the binding energy of electrons (the most tightly bound electrons have a binding energy less than 1 keV), the electrons which scatter the photons may be considered free electrons. Because energy and momentum must be conserved in an elastic collision, we can obtain the formula for the wavelength of the scattered photon, λ_s as a function of scattering

angle θ : $\lambda_s = \frac{h}{m_0 c} \{(1 - \cos\theta) + \lambda\}$ where λ is the wavelength of the incident photon, c is

the speed of light in vacuum and $\frac{h}{m_0 c}$ is λ_C the Compton wavelength of the electron (which

characterizes the length scale at which the wave property of an electron starts to show up. In an interaction that is characterized by a length scale larger than the Compton wavelength, electron behaves classically (i.e., no observation of wave nature). For interactions that occur at a length scale comparable than the Compton wavelength, the wave nature of the electron begins to take over from classical physics).

Compton postulated that photons carry momentum; thus from the conservation of momentum, the momenta of the particles are related by

$$\mathbf{p} = \mathbf{p}_s + \mathbf{p}_e$$

$$\mathbf{p}_e = \mathbf{p} - \mathbf{p}_s$$

Making use of the scalar product yields the square of its magnitude,

$$p_e^2 = (\mathbf{p} - \mathbf{p}_s) \cdot (\mathbf{p} - \mathbf{p}_s)$$

$$p_e^2 = p^2 + p_s^2 - 2p p_s \cos\theta$$

$$F = \frac{p^2 + p_s^2 - 2pp_s \cos\theta}{h} \times -\frac{d\lambda_e}{dt}$$

The conservation of energy merely equates the sum of energies before and after scattering:

$$m_0c^2 + h\nu = h\nu_s + (m_0c^2 + E_K)$$

$$(h\nu - h\nu_s) = E_K$$

$$(\nu - \nu_s) = \frac{F}{(m_0 + m_e) \times -\frac{d\lambda_e}{dt}}$$

$$\Delta\nu = \frac{F}{(m_0 + m_e) \times -\frac{d\lambda_e}{dt}}$$

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