

Determination of the Photon Force and Power

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Abstract: Photons, although lacking in mass, do have momentum — so when they hit a particle like an electron, they apply a force onto it. How much force does photon exert on the particle with which it interacts? It's a problem that's confounded physicists for nearly 150 years – and it may now have a solution. In this article, we derive the photon power and force of the electromagnetic radiation photon on material particles by a direct application of the quantum theory of electromagnetic radiation. In this derivation, Planck's quantum hypothesis, the Newtonian mechanics, the Einsteinian relativity, and the law of conservation play central roles.

Keywords: photon; electron; energy; momentum; force; power; acceleration; Newton's laws of motion.

In empty space, the photon – the basic unit of all light – moves at c (the speed of light) and its energy and momentum are related by $E = pc$, where p is the momentum of the photon. This derives from the following relativistic relation, with $m_0 = 0$:

$$E^2 = p^2c^2 + m_0^2c^4.$$

In some situations, photon behaves like a wave, while in others, it behaves like a particle. The photons can be thought of as both waves and particles. In 1924 a French physicist **Louis de Broglie** developed a formula to relate this dual wave and particle behavior:

$$E = h\nu, \quad c = \lambda\nu, \quad E = \frac{hc}{\lambda} = mc^2,$$

where E and m are the energy and mass of the photon, ν and λ are the frequency and wavelength of the photon, h is the Planck constant, c is the speed of light. From this we obtain the definition of the photon wavelength through the Planck constant and the momentum of the photon:

$$\lambda = \frac{h}{mc}$$

This equation is used to describe the wave properties of matter, specifically, the wave nature of the electron:

$$\lambda_e = \frac{h}{m_e v}$$

where λ_e is wavelength, h is Planck's constant, $m_e = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the relativistic mass of the electron, moving at a velocity v .

$$p_e = \frac{h}{\lambda_e}$$

From this it follows that,

$$\frac{dp_e}{dt} = -\frac{d\lambda_e}{dt} \times \frac{h}{\lambda_e^2}$$

$$\frac{dp_e}{dt} = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}$$

Sir Isaac Newton first presented his three laws of motion in the "**Principia Mathematica Philosophiae Naturalis**" in 1686. His second law defines a force exerted by a photon on the electron to be equal to the rate of change in momentum of the electron:

$$F = \frac{dp_e}{dt}$$

$$F = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}$$

The work W done by the photon force on the electron equals the change in the electron's kinetic energy E_k :

$$W = dE_k = F dx$$

$$\frac{dE_k}{dt} = F v = \frac{p_e^2 v}{h} \times -\frac{d\lambda_e}{dt}$$

When a photon with energy E collides with an electron associated with the energy E_e . Some of the energy and momentum is transferred to the electron, but both energy and momentum are conserved in this elastic collision. After the collision the photon has energy E' and the electron has acquired an energy $E_e' > E_e$. One of the most powerful laws of physics is the **Law of conservation of energy**. For a collision occurring between photon and particle in an isolated system, the total energy of the two particles before the collision is equal to the total energy of the two particles after the collision. That is,

$$\begin{aligned} E + E_e &= E' + E_e' \\ - (E' - E) &= (E_e' - E_e) \\ - (E' - E) &= m_0c^2 + E_k' - (m_0c^2 + E_k) \\ - (E' - E) &= (E_k' - E_k) \\ - \Delta E &= \Delta E_k \end{aligned}$$

where ΔE_k = change in kinetic energy of the electron and ΔE = change in energy of the photon.

For an infinitely small change in the energy, the equation takes the form:

$$\begin{aligned} - dE &= dE_k \\ -\frac{dE}{dt} &= \frac{dE_k}{dt} = \frac{p_e^2 v}{h} \times -\frac{d\lambda_e}{dt} \end{aligned}$$

Given that the power of the photon is the rate of energy loss of the photon:

$$P = -\frac{dE}{dt}$$

$$P = \frac{p_e^2 v}{h} \times -\frac{d\lambda_e}{dt}$$

Thus, we have the formula for the calculation of the photon power and force of the electromagnetic radiation photon on material particles.

Discussion:

According to the law that nothing may travel faster than the speed of light – i.e., according to the Albert Einstein's law of variation of mass with velocity (the most famous formula in the world. In the minds of hundreds of millions of people it is firmly associated with the menace of atomic weapons. Millions perceive it as a symbol of relativity theory):

$$m_e = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_e^2 c^2 - m_e^2 v^2 = m_0^2 c^2$$

That the electron's mass m_e in motion at speed v is the mass m_0 at rest divided by the factor

$\sqrt{1 - \frac{v^2}{c^2}}$ implies: the mass of the electron is not constant; it varies with changes in its velocity.

Differentiating the above equation, we get:

$$m_e v dv + v^2 dm_e = c^2 dm_e$$

$$dm_e (c^2 - v^2) = m_e v dv$$

In relativistic mechanics (the arguably most famous cult of modern physics, which has a highly interesting history which dates back mainly to Albert Einstein and may be a little earlier to H. Poincaré), we define the energy $m_e c^2$ which a moving electron possess to be $= m_0 c^2 + E_k$.

$$m_e c^2 = m_0 c^2 + E_k$$

$$\frac{dm_e c^2}{dt} = \frac{dE_k}{dt} = Fv$$

$$F = \frac{dp_e}{dt} = \frac{d(m_e v)}{dt}$$

$$F = \frac{dm_e}{dt} v + \frac{dv}{dt} m_e$$

$$F = F \frac{v^2}{c^2} + m_e a$$

$$F = \frac{m_e a}{1 - \frac{v^2}{c^2}}$$

So as v approaches c , the bottom term approaches zero and therefore the force approaches infinity. It requires an infinite amount of force to accelerate the electron to the speed of light.

Because:

$$m_e = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore:

$$F = \frac{m_e^3 a}{m_0^2}$$

For non-relativistic case ($v \ll c$), the above equation reduces to $F = m_0 a$.

$$\frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt} = \frac{m_e^3 a}{m_0^2}$$

From this it follows that,

$$a = \frac{m_0^2 v^2}{hm_e} \times -\frac{d\lambda_e}{dt}$$

Thus, we have the formula for the calculation of the acceleration of the electron. For non-relativistic case ($v \ll c$), the above equation reduces to:

$$a = \frac{m_0 v^2}{h} \times -\frac{d\lambda_e}{dt}$$

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