Halbklassische Berechnung der Schalenenergien der gg-Kerne
mit Hilfe der Casimir-Kräfte –
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is slightly corrected and translated by Halil Güveniş:

Semi-classical calculation of the shell energies of even-even nuclei with the help of Casimir forces

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Abstract

In the present paper we calculate in semi-classical approximation the shell energies of even-even nuclei with the help of Casimir forces. We first demonstrate that for the total binding energy of the helium nucleus, essentially tangential and radial Casimir forces, introduced by us, are responsible. In a second step, we carry out the same calculation for the beryllium nucleus and in this case we also find out that for the total binding energy of the beryllium nucleus, essentially tangential and radial Casimir forces, introduced by us, are responsible. Finally, we calculate some characteristic shell quantities of selected even-even nuclei – ^{8}Be, ^{16}O, ^{24}Mg, ^{32}S, ^{40}Ca, ^{48}Cr, ^{56}Ni, ^{64}Ge, ^{96}Cd – and we show that with increasing shell radius a strongly declining shell energy spectrum follows. We give the shell energy spectrum of the exemplary selected nucleus ^{96}Cd and we show how the shell energies are composed of tangential and radial acting Casimir energies. Altogether, we obtain a fascinatingly simple and intuitive model of even-even nuclei; its correctness however can only be verified by solving the fundamental equations of quantum hydrodynamics in the interior of the nucleons.

Key words: shell energies and the total binding energy of even-even nuclei, tangential and radial acting Casimir forces, shell energy spectrum of even-even nuclei, fundamental equations of quantum hydrodynamics.

1 Introduction

In order to explain the stability of elementary particles and atomic nuclei, many attempts have been made to describe nuclear forces by Casimir forces [1-3]. However, all these attempts ended with the statement that regarding their size Casimir forces are indeed comparable to nuclear forces, but they lead to wrong sign, i.e., to repulsive nuclear forces. – In the following we want to show that this statement is correct only in the context of a particular geometry; if one changes the geometry, then the Casimir forces in question change, too. In a previous work we have demonstrated in detail that by suitable photon pressure assumptions certain spatial symmetries are given and based on these symmetries, the geometry of atomic nuclei can be, step by step, determined [4]. The aim of the present work is, based on this geometry, to calculate the shell energies of even-even nuclei with the help of Casimir forces in semi-classical approximation.

The paper is organized as follows: In Section 2, the total binding energy of the helium nucleus is calculated in semi-classical approximation with the help of Casimir forces. In Section 3, we carry out the same calculation for the beryllium nucleus. In Section 4, we calculate some characteristic shell quantities of selected even-even nuclei and discuss the results of calculation for various higher nuclear shells. Section 5 is provided for conclusions.

2 The helium nucleus

The simplest even-even-nucleus is ^{4}He. We therefore begin our semi-classical calculation of the shell energies of even-even nuclei with the helium nucleus.

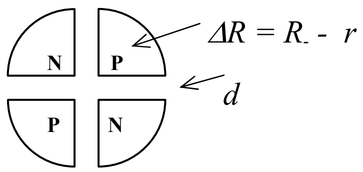


Figure 1: Geometry of the helium nucleus

^{4}He consists of two protons and two neutrons which are placed, because of the charge symmetry, diagonally to each other (Fig. 1). Each proton and each neutron makes up one quarter of a spherical shell with the volume

$$V_{He} = (4\pi/3)\ (R^3 - r^3)\ , \tag{1}$$

where r is the radius of the inner and R the radius of the outer spherical shell surface. Protons and neutrons are separated on two sides by the gap distance d and have the common surface

$$S_{He} = \pi\,(R^2 - r^2)\,/\,2\ . \tag{2}$$

The mass or charge center of individual nucleons is located, calculating from the center of the whole sphere, on the axis of symmetry at a distance of

$$\underline{r} = (3/4)(\,R^4 - r^4)/(R^3 - r^3) + r + d\,/\,2^{1/2}. \tag{3}$$

Since protons and neutrons have the spin ½ $\hbar$ laying in line with their center of mass or charge, they always revolve around their own axis of symmetry. Because of this rotation each proton-neutron pair has the rotational energy

$$E_{Rot} = \hbar^2 I(I+1)/2\theta$$
$$= \hbar^2 (2 \cdot 1/2 \cdot 0{,}7071)(2 \cdot 1/2 \cdot 0{,}7071 + 1)/[2(M_P + M_N)((2/5)R^2 + (d/2)^2)] \quad (4)$$
$$= 12{,}5148 \;\; MeV\, fm^2 / (0{,}4R^2 + 0{,}25\, d^2)\, ,$$

where $\theta = (M_P + M_N)((2/5)R^2 + (d/2)^2)$ is the moment of inertia of the proton-neutron pair. However, it must be noted at this point that the rotational energy does not contribute to the total binding energy of the helium nucleus. Although on an individual gap each proton-neutron pair has a non-vanishing spin, the total spin of the helium nucleus gives zero because of the opposite directions on four various gaps; so, the total rotational energy gives zero, too.

The centrifugal force in d-direction caused by the rotational energy (4) is

$$F_{Rot} = -\,\partial E_{Rot}/\partial d = 6{,}2574\; MeV\, fm^2\, d/(0{,}4\, R^2 + 0{,}25\, d^2)^2\, , \quad (5)$$

i. e., this force must be compensated by nuclear binding forces, so that the nucleus does not break apart. We assume that for the compensation two types of Casimir energies are suitable:

$$E_{Cas}\,(d) = -\,2\,(\pi^2\, \hbar c / 720)\, S_{He}/ d^3 - 2\,(\pi^3\, \hbar c / 720)\, R\, [1/s_1{}^2 - 1/s_2{}^2]$$
$$= -\,(\pi^3\, \hbar c / 720)\, [(R^2 - r^2)/d^3 + 2/R - 2\,R/(R+d)^2] \quad (6)$$
$$= -\,8{,}4977\, [(R^2 - r^2)/d^3 + 2/R - 2\,R/(R+d)^2]\; MeV\, fm\, ,$$

with $s_1 = R$, $s_2 = R + d$.

The corresponding nuclear binding forces in d-direction are:

$$F_{Cas}\,(d) = -\,\partial E_{Cas}/\partial d = -\,8{,}4977\, [3\,(R^2 - r^2)/d^4 - 4\,R/(R+d)^3]\; MeV\, fm\, . \quad (7)$$

The first of these forces occurs when two plane-parallel plates (walls of the nucleons) are situated vis-à-vis at a distance of d (Fig. 1). The second force occurs when the said plane-parallel walls of nucleons are placed in front of a curvature radius R at a distance $s_1 = R$ or $s_2 = R + d$ (Fig. 1). In balance of forces the two Casimir forces in (7) compensate the centrifugal force (5) caused by the rotational movement:

$$F_d \,/\, MeV\,fm = 6{,}2574\, fm\; d \,/(0{,}4\, R^2 + 0{,}25\, d^{\,2})^2$$
$$- 8{,}4977\, [3\,(R^2 - r^2)\,/\,d^{\,4} - 4\,R\,/(R{+}d)^3] = 0\,. \quad (8)$$

Further, the cavity in the center of the nucleus generates Casimir forces depending on r or R :

$$F_{Cas}\,(r, R) = -\,(\pi^3\,\hbar c\,/720)\; R\,[2\,r\,/((R + r)s_3^{\,3}) - 2\,r\,/((R + r)s_4^{\,3})] \quad (9)$$
$$= -\,(\pi^3\,\hbar c\,/720)\; R\,[2\,r\,/((R + r)\,(R - r\,)^3) - 2\,r\,/((R + r)(\,R + r + 2^{1/2}d)^3)]$$
$$= -\,8{,}4977\; R\,[2\,r\,/((R + r)\,(R - r\,)^3) - 2\,r\,/((R + r)(\,R + r + 2^{1/2}d)^3)]\; MeV\,fm$$

with $s_3 = R - r\,,\ s_4 = R + r + 2^{1/2}d\,.$

The in total attractive, radial Casimir forces in (9) are counteracted by the repulsive Coulomb force

$$F_C = (1/4\pi\varepsilon_o)\; e^2\,/\,(2\,\underline{r}\,)^2$$
$$= 1{,}44\; MeV\,fm\,/\,[(3/2)\,(R^4 - r^4)/(R^3 - r^3) + 2r + 2^{1/2}d]^2. \quad (10)$$

In balance of forces, the radial equilibrium condition gives

$$F_r \,/\, MeV\,fm = -\,8{,}4977\; R\,[2\,r\,/((R + r)\,(R - r\,)^3) - 2\,r\,/((R + r)(\,R + r + 2^{1/2}d)^3)]$$
$$+\,1{,}44\,/\,[(3/2)\,(R^4 - r^4)/(R^3 - r^3) + 2r + 2^{1/2}d]^2 = 0\,. \quad (11)$$

For 4 gaps and 4 nucleons in helium nucleus, we obtain the total binding energy

$$E = 4\,E_{Cas}\,(d) + 4\,E_{Cas}\,(r, R) + E_{\mathbf{C}}$$
$$= -\,4\cdot 8{,}4977\,[(R^2 - r^2)\,/\,d^{\,3} + 2\,/\,R\; - 2\,R\,/(R{+}d)^2]\; MeV\,fm \quad (12)$$
$$-\,4\cdot 8{,}4977\; R\,[r\,/((R + r)\,(R - r\,)^2) - r\,/((R + r)(\,R + r + 2^{1/2}d)^2)]\; MeV\,fm$$
$$+\,1{,}44\; MeV\,fm\,/\,[(3/2)\,(R^4 - r^4)/(R^3 - r^3) + 2r + 2^{1/2}d].$$

Introducing the new variables *(d/R)* and *(r/R)* in (8), (11) and (12), we obtain three equations for determining the total binding energy:

$$F_d R^2/ MeV\, fm = 6{,}2574\, fm\, (d/R) / [R\,(0{,}4 + 0{,}25\,(d/R))^2)^2] \qquad (13)$$
$$- 3 \cdot 8{,}4977\,(1 - (r/R)^2) / (d/R)^4 + 4 \cdot 8{,}4977 / (1 + (d/R))^3 = 0\,,$$

$$F_r R^2 / MeV\, fm = - 2 \cdot 8{,}4977\,(r/R) / [(1 + (r/R))\,(1 - (r/R))^3]$$
$$+ 2 \cdot 8{,}4977\,(r/R) / [(1 + (r/R))(1 + (r/R) + 2^{1/2}(d/R))^3] \qquad (14)$$
$$+ 1{,}44 / [(3/2)\,(1 - (r/R)^4)/(1 - (r/R)^3) + 2(r/R) + 2^{1/2}(d/R)]^2 = 0\,,$$

$$E = - 4 \cdot 8{,}4977\,[(1 - (r/R)^2) / (R\,(d/R)^3) + 2 / R - 2 / (R\,(1 + (d/R))^2)]\ MeV\, fm$$
$$- 4 \cdot 8{,}4977\,[(r/R) /(R((1 + (r/R))(1 - (r/R))^2) - (r/R)/(R(1 + (r/R))(1 + (r/R))$$
$$+ 2^{1/2}(d/R))^2)]\ MeV\, fm + 1{,}44\ MeV\, fm / [R((3/2)\,(1 - (r/R)^4)/(1 - (r/R)^3)$$
$$+ 2(r/R) + 2^{1/2}(d/R))] = - 28{,}28\ MeV. \qquad (15)$$

With the help of the conditional equations (13), (14) and (15) we want to show now that for the total binding energy of the helium nucleus essentially tangential and radial Casimir forces, introduced by us, are responsible. The Eqs. (13), (14) and (15) are so compact that the variables *(d/R), (r/R), R* and the single force and energy values can only be determined by numerically computing:

<u>^{4}He</u>

R [fm]	r /R	d/R	r [fm]	d [fm]	E [MeV]	E_{exp} [MeV]
2,4351	0,007275	1,37169	0,017715	3,3402	- 28,2819	- 28,28

$$F_d R^2/ MeV\, fm = 4{,}6528 - 7{,}2007 + 2{,}5479 = 0$$
$$F_r R^2/ MeV\, fm = - 0{,}1256 + 0{,}0048 + 0{,}1207 = 0$$

$4 E_{Cas}(d)$		$4 E_{Cas}(r, R)$		E_C	$= E$ [MeV]
- 5,4082	-	0,1023	+	0,1712	
- 27,9174	+	0,0116			
+ 4,9632					
- 28,3624	-	0,0907	+	0,1712	= - 28,2819

Remarkable about this result is that the total binding energy of the helium nucleus consists exclusively of tangentially acting Casimir energies $4\ E_{Cas}\ (d)$. The radially acting Casimir energies $4\ E_{Cas}\ (r,\ R)$ play only a minor role because the Coulomb energy E_C is poorly developed. Nevertheless, the sum of radial energies gives $E_r = 4\ E_{Cas}\ (r,\ R) + E_C = 0{,}0805\ MeV$, i.e. when two helium nuclei try to come closer to each other and to form a beryllium nucleus, so they first meet in the radial direction a repulsive Coulomb force. Only if they manage by supplying additional energy to transform their walls to plane-parallel plates and to come so close to each other as the gap distance $d = 3{,}3402\ fm$, then they can merge to a beryllium nucleus and reduce their total binding energy about $0{,}0805\ MeV$.

If we calculate the volume of the helium nucleus according to (1), we obtain $V_{He} = 60{,}4838\ fm^3$. This is a surprising result, because the volume of four free nucleons is about $V_{o4} = 4\ (4\pi/3)\ (1{,}3\ fm)^3 = (4\pi/3)\ (2{,}0636\ fm)^3 = 36{,}81\ fm^3$. Although according to (8) and (11) the nucleons in the helium nucleus are, in contrast to free nucleons, under tangential and radial pressure, their radius has not fallen below $2{,}0636\ fm$, but rose to $2{,}4351\ fm$; this is the weak point of our semi-classical model. Since in (8) and (11) no quantum theoretical term for the reaction of the nucleon on an external force is present, for bounded nucleons we systematically calculate a larger radius, as it is actually the case. In the present work we are primarily interested in the contribution of Casimir forces to the total binding energy of even-even nuclei. Therefore, we let aside the systematic error by the calculation of radii of nuclei and go over to the next even-even nucleus, to beryllium nucleus.

3 The beryllium nucleus

^8Be consists of four protons and four neutrons which are placed, because of the charge symmetry, diagonally to each other. Each proton and each neutron makes up one eighth of a spherical shell with the volume

$$V_{Be} = (4\pi/3)\ (R^3 - r^3)\ , \tag{16}$$

where r is the radius of the inner and R the radius of the outer spherical shell surface. Protons and neutrons are separated on three sides by the gap distance d and have the common surface

$$S_{Be} = \pi (R^2 - r^2) / 4 . \tag{17}$$

The mass or charge center of individual nucleons is located, calculating from the center of the whole sphere, on the axis of symmetry at a distance of

$$\underline{r} = (3/4)(R^4 - r^4)/(R^3 - r^3) + r + d / 2^{1/2}. \tag{18}$$

While the helium nucleus is symmetrical in two dimensions, the beryllium nucleus has three-dimensional symmetry, e. i., the geometry and the charge and the spin distribution of the beryllium nucleus remain unchanged turning it in any direction. This establishes for centrally symmetric potentials the basis to structure more protons and neutrons above the beryllium nucleus in form of spherical shells. So, the beryllium nucleus forms the first shell, above which further nuclear shells can be placed by repeated application of the beryllium nucleus in form of concentric spherical shells according to the magic number 8 (Fig. 2). The total binding energy of even-even nuclei is then given as the sum of their shell energies. Therefore, in the fallowing we want to calculate the characteristic quantities of two successive nuclear shells ("beryllium nuclei") representing all other spherical shells.

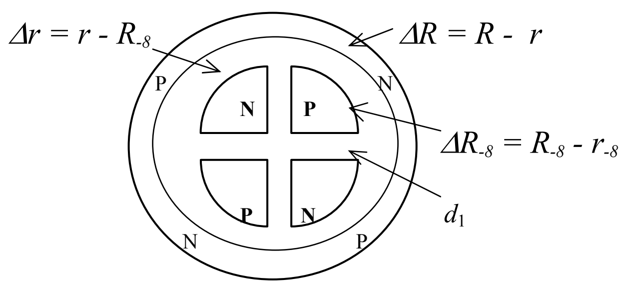


Figure 2: Geometry of the first and second shell

Since protons and neutrons have the spin ½ $\hbar$ laying in line with their center of mass or charge, they always revolve around their own axis of symmetry. Because of this rotation each proton-neutron pair has the rotational energy

$$E_{Rot} = \hbar^2 I(I+1)/2\theta$$
$$= \hbar^2 (2 \cdot 1/2 \cdot 0{,}7071)(2 \cdot 1/2 \cdot 0{,}7071 + 1)/[2(M_P + M_N)((2/5)R^2 + (d/2)^2)] \quad (19)$$
$$= 12{,}5148 \;\; MeV\, fm^2 / (0{,}4R^2 + 0{,}25\, d^2).$$

The centrifugal force in d-direction caused by the rotational energy (19) is

$$F_{Rot} = - \partial E_{Rot} / \partial d = 6{,}2574\; MeV\, fm^2\; d /(0{,}4\, R^2 + 0{,}25\, d^2)^2 \,, \quad (20)$$

i. e., this force must be compensated by nuclear binding forces, so that the nucleus does not break apart. We assume that for the compensation two types of Casimir energies are suitable:

$$E_{Cas}\,(d) = - 2\,(\pi^2\, \hbar c / 720)\, S_{Be} / d^3 - 2\,(\pi^3\, \hbar c / 720)\, R\, [1/s_1^{\,2} - 1/s_2^{\,2}]$$
$$= - (\pi^3\, \hbar c / 720)\, [(R^2 - r^2) / d^3 + 2 / R - 2\, R / (R+d)^2] \quad (21)$$
$$= - 8{,}4977\, [(R^2 - r^2) / d^3 + 2 / R - 2\, R / (R+d)^2]\; MeV\, fm\,,$$

with $s_1 = R\,,\; s_2 = R + d\,.$

The corresponding nuclear binding forces in d-direction are:

$$F_{Cas}\,(d) = - \partial E_{Cas} / \partial d = - 8{,}4977\, [3\,(R^2 - r^2) / d^4 - 4\, R / (R+d)^3]\; MeV\, fm\,. \quad (22)$$

In balance of forces the two Casimir forces in (22) compensate the centrifugal force (20) caused by the rotational movement:

$$F_d / MeV\, fm = 6{,}2574\, fm\; d /(0{,}4\, R^2 + 0{,}25\, d^2)^2$$
$$- 8{,}4977\, [3\,(R^2 - r^2) / d^4 - 4\, R / (R+d)^3] = 0\,. \quad (23)$$

Further, the cavities between nuclear shells generate in radial direction Casimir forces depending on curvature radii r, R und R_{-8} (Fig. 2)

$$F_{Cas}(r, R) = - (\pi^3 \hbar c /720)\, R\, [2 r /((R + r)s_3^3) - 2 r /((R + r)s_4^3)]$$
$$+ (\pi^3 \hbar c /720)\, R_{-8}\, [2 r /((R_{-8} + r)s_5^3) - 2 r /((R_{-8} + r)s_6^3)] \qquad (24)$$
$$= - 8{,}4977\, [2 r R /((R + r) (R - r)^3) - 2 r R /((R + r)(R + r + 2^{1/2}d_1)^3)$$
$$- 2 r R_{-8} /((R_{-8} + r) (r - R_{-8})^3) + 2 r R_{-8} /((R_{-8} + r)(r + R_{-8} + 2^{1/2}d_1)^3)]\ MeV\,fm$$

with $s_3 = R - r$, $s_4 = R + r + 2^{1/2}d_1$,

$s_5 = r - R_{-8}$, $s_6 = r + R_{-8} + 2^{1/2}d_1$,

d_1 := gap distance of the first shell.

The in total attractive, radial Casimir forces in (24) are counteracted by the repulsive Coulomb force

$$F_C = (1/4\pi\varepsilon_o)\, Z e^2 / \underline{r}^{\,2}$$
$$= 1{,}44\, Z\, MeV\,fm / [(3/4)\, (R^4 - r^4)/(R^3 - r^3) + r + d_1/2^{1/2}]^2. \qquad (25)$$

In balance of forces, the radial equilibrium condition gives

$$F_r / MeV\,fm = - 2 \cdot 8{,}4977\, [\, r R /((R + r) (R - r)^3) - r R /((R + r)(R + r + 2^{1/2}d_1)^3)$$
$$- r R_{-8} /((R_{-8} + r) (r - R_{-8})^3) + r R_{-8} /((R_{-8} + r)(r + R_{-8} + 2^{1/2}d_1)^3)]$$
$$+ 1{,}44\, Z / [(3/4)\, (R^4 - r^4)/(R^3 - r^3) + r + d_1/2^{1/2}]^2 = 0\,. \qquad (26)$$

For 12 gaps, 8 nucleons and 4 protons in beryllium nucleus, we obtain the shell energy

$$E = 12\, E_{Cas}(d) + 8\, E_{Cas}(r, R) + 4\, E_C$$
$$= - 12 \cdot 8{,}4977\, [(R^2 - r^2) / 2d^3 + 2 / R - 2 R /(R+d)^2]\ MeV\,fm$$
$$- 8 \cdot 8{,}4977\, (r R /(R + r))[1/(R - r)^2 - 1/(R + r + 2^{1/2}d_1)^2]\ MeV\,fm \qquad (27)$$
$$+ 8 \cdot 8{,}4977\, (r R_{-8} /(R_{-8} + r))[1/(r - R_{-8})^2 - 1/(r + R_{-8} + 2^{1/2}d_1)^2]\ MeV\,fm$$
$$+ 4 \cdot 1{,}44\, Z / [(3/4)\, (R^4 - r^4)/(R^3 - r^3) + r + d_1/2^{1/2}]\ MeV\,fm\,.$$

Introducing the new variables *(d/R)* and *(r/R)* in (23), (26) and (27), we obtain three equations for determining the shell energies:

$$F_d R^2 / MeV\, fm = 6{,}2574\, fm\, (d/R) /[R\,(0{,}4 + 0{,}25\,(d/R))^2)^2] \qquad (28)$$
$$- 3 \cdot 8{,}4977\, (1 - (r/R)^2) /(2\, (d/R)^4) + 4 \cdot 8{,}4977 / (1 + (d/R))^3 = 0\,,$$

$$F_r / MeV\, fm = - 16{,}9954\, [(r /R)/(1 + (r /R))(1/R^2(1- (r /R))^3$$
$$- 1/R^2((r /R) + 1 + 2^{1/2}\, (d_1 /R))^3]$$
$$+ 16{,}9954\, [(r /R)b_{-8} /(b_{-8} + (r /R))(1 /R^2((r /R) - b_{-8})^3 \qquad (29)$$
$$- 1/R^2((r /R) + b_{-8} + 2^{1/2}\, (d_1/R))^3]$$
$$+ 1{,}44\, Z / R\, (((3/4)\, (1 - (r/R)^4)/(1 - (r/R)^3) + (r/R) + 2^{1/2}(d_1 /2R))^2 = 0\,,$$

$$E = [- 12 \cdot 8{,}4977\, [(1 - (r/R)^2)/2(d/R)^3 + 2 - 2/(1+(d/R))^2]$$
$$- 8 \cdot 8{,}4977\, ((r /R) /(1 + (r /R))[1 / (1- (r /R))^2$$
$$- 1/((r /R) + 1 + 2^{1/2}\, (d_1/R))^2]$$
$$+ 8 \cdot 8{,}4977\, ((r /R)\, b_{-8} /(b_{-8} + (r /R))[1 / ((r /R) - b_{-8})^2 \qquad (30)$$
$$- 1/((r /R) + b_{-8} + 2^{1/2}\, (d_1/R))^2]$$
$$+ 4 \cdot 1{,}44\, Z /(((3/4)\, (1 - (r/R)^4)/(1 - (r/R)^3) + (r/R) + (d_1/R)/2^{1/2})]\, MeV\, fm / R\,,$$

mit $R_{-8} = b_{-8}\, R = (R_{-8} / R)\, R$.

With the help of the conditional equations (28), (29) and (30) we want to show now that for the total binding energy of the beryllium nucleus essentially tangential and radial Casimir forces, introduced by us, are responsible. The Eqs. (28), (29) and (30) are so compact that the variables *(d/R), (r/R), R* and the single force and energy values can only be determined by numerically computing:

<u>^8Be</u>

R [fm]	*r /R*	*d/R*	*r [fm]*	*d [fm]*	*E [MeV]*	E_{exp} *[MeV]*
3,5123	*0,2216*	*1,13132*	*0,778326*	*3,9735*	*- 56,4805*	*- 56,48*

$F_d R^2 / MeV\, fm \;=\; 3{,}8883 - 7{,}3992 + 3{,}5109 = 0$

$F_r / MeV\, fm \;=\; -0{,}5299 + 0{,}0111 + 0{,}5188 = 0$

$12\, E_{Cas}\,(d)$		$8\, E_{Cas}\,(r, R)$		$4E_C$	$= E\,[MeV]$
$-\;9{,}5332$	$-$	$5{,}7948$	$+$	$3{,}6895$	
$-\;58{,}0659$	$+$	$0{,}4410$			
$+\;12{,}7827$					
$-\;54{,}8163$	$-$	$5{,}3537$	$+$	$3{,}6895$	$= -\,56{,}4805$

Thus, the total binding energy of the beryllium nucleus consists almost exclusively of the tangentially acting Casimir energies in amount of - *54.8163 MeV*. The radially acting energies in amount of $E_r = 8\ ECAS\ (r,\ R) + 4EC = -\ 1.6642\ MeV$ make, in percentage terms, only a small contribution to the total binding energy of the beryllium nucleus. But they are entirely sufficient that the beryllium nucleus, in contrast to the helium nucleus, is particularly attractive for passing neutrons.

According to (16) the volume of the beryllium nucleus is $V_{Be} = 179{,}5195\ fm^3$. This is too large compared to the volume of eight free nucleons $V_{o8} = 8\ (4\pi/3)\ (1{,}3\ fm)^3 = (4\pi/3)\ (2{,}6\ fm)^3 = 73{,}6222\ fm^3$. Although the nucleons in the beryllium nucleus are under tangential and radial pressure, their radius has not been depressed below *2,6 fm*, but rose to *3.5123 fm*; this is the weak point of our semi-classical model. Since in (23) and (26) we have not taken into account the internal quantum hydrodynamic force reaction of nucleons which occurs by an external force exertion, we obtain systematically a larger radius than it is allowed. As already announced we first want to let aside this systematic error, and we are only interested in the contribution of the Casimir forces to the shell energies of even-even nuclei.

4 Higher nuclear shells

While ^{8}Be consists of a single nuclear shell, all other even-even nuclei possess A / 8 nuclear shells, so for example ^{16}O 2, ^{56}Ni 7 und ^{96}Cd 12 shells. With the help of the conditional equations (28), (29) and (30) we want to show now that for the shell energies of the gg-nuclei essentially tangential and radial Casimir forces, introduced by us, are responsible. We have numerically calculated some characteristic shell quantities of selected even-even nuclei – ^{8}Be, ^{16}O, ^{24}Mg, ^{32}S, ^{40}Ca, ^{48}Cr, ^{56}Ni, ^{64}Ge, ^{96}Cd – and presented them in tabular form in Appendix. The most important criterion by determining the shell energies is that their sum gives the experimentally known total binding energy. In order to fulfill this criterion we could not exactly calculate the radii of nuclear shells and the shell distances. Therefore, our specifications to radii of nuclear shells R and to shell distances $\Delta r = r - R_{-8}$ (Fig. 2) are always to be regarded as a trend analysis. An exact calculation can only be performed by solving the fundamental equations of quantum hydrodynamics [6] in the interior of the nucleons.

The first striking fact at the tables in Appendix is the relatively high binding energy concentrated in the first shell of even-even nuclei:

Nuclide	R_1 [fm]	E_1 [MeV]
^{8}Be	3,5123	- 56,4805
^{16}O	2,4647	- 80,5141
^{24}Mg	1,7000	- 118,5011
^{32}S	1,3532	- 151,5412
^{40}Ca	1,1500	- 182,4450
^{48}Cr	1,0113	- 209,6230
^{56}Ni	0,8280	- 263,8511
^{64}Ge	0,7620	- 290,8420
^{96}Cd	0,5498	- 431,0106

Responsible for this relatively high increase is mainly the tangential acting Casimir energy - *203,9448 MeV fm / R* in Eq. (30). Since the first shell of each nuclide is under maximum pressure, it decreases most radically as expressed in the table above by reduced shell radii R_1. Only below $R_1 = 1\ fm$, the tangentially acting second Casimir energy $-\ 50{,}9862\ MeV\ fm\ (1 - (r/R)^2)/(R(d/R)^3)$ in Eq. (30) also makes a significant contribution to the shell energy. Already by the transition to the second shell, the shell energy becomes considerably lower because for the contraction of the radius the first shell stands in the way; this is a fact which is even more true for higher shells, so that with increasing shell radius a strongly declining shell energy spectrum follows. In order to see how the shell energies are composed of tangential and radial components, we give below the shell energy spectrum of the exemplary selected nucleus ^{96}Cd :

<u>^{96}Cd</u>

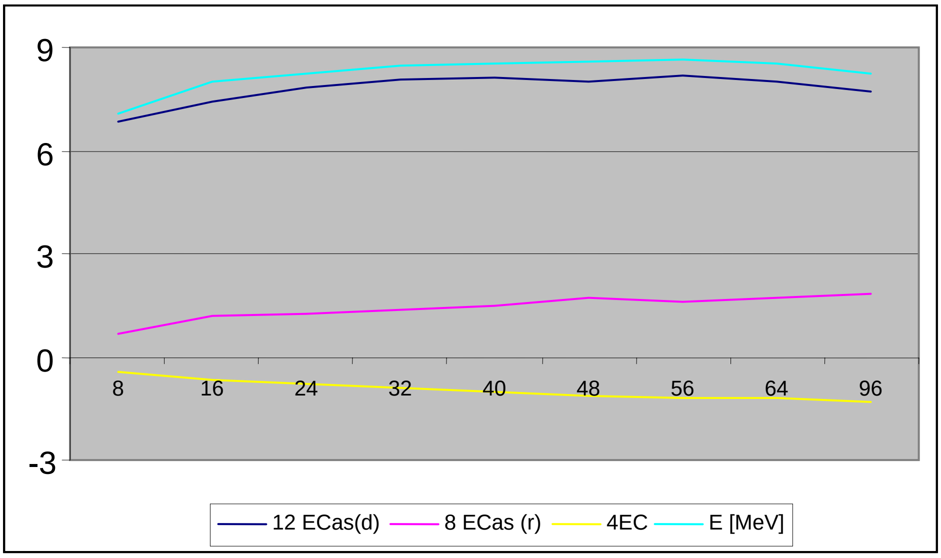
	$12\ E_{Cas}(d)$	+	$8\ E_{Cas}(r, R)$	+	$4E_C$	=	E_i [MeV]
1. Shell:	- 451,5083	-	9,7679	+	30,2656	=	- 431,0106
2. Shell:	- 101,6868	-	22,0131	+	15,1162	=	- 108,5837
3. Shell:	- 55,3924	-	20,8931	+	12,0985	=	- 64,1870
4. Shell:	- 33,9606	-	15,9007	+	10,1687	=	- 39,6926
5. Shell:	- 22,5977	-	13,4574	+	8,5391	=	- 27,5160
6. Shell:	- 17,0275	-	14,2032	+	7,7689	=	- 23,4618
7. Shell:	- 13,7458	-	14,0067	+	7,3997	=	- 20,3528
8. Shell:	- 11,4798	-	13,8762	+	7,1452	=	- 18,2108
9. Shell:	- 9,8255	-	13,7839	+	6,9591	=	- 16,6503
10. Shell:	- 8,5673	-	13,7150	+	6,8171	=	- 15,4652
11. Shell:	- 7,5798	-	13,6618	+	6,7052	=	- 14,5364
12. Shell:	- 6,7853	-	13,6194	+	6,6147	=	- 13,7900
Sum:	-740,1568	-	178,8984	+	125,5980	=	-793,4572

Thus, the total binding energy of the cadmium nucleus is composed almost exclusively of tangentially acting Casimir energies of - *740,1568 MeV*, where the binding energy of the first shell is - *451,5083 MeV* falling until twelfth shell on - *6,7853 MeV*. The radially acting energies in amount of $E_r = 8\ E_{Cas}\ (r,\ R) + 4E_{\mathbf{C}} = -\ 53{,}3004\ MeV$, however, contribute less to the total binding energy, but with - *7,0047 MeV* on the twelfth shell they are entirely sufficient that the cadmium nucleus is particularly attractive not only for passing neutrons, but at a sufficient distance also for individual protons.

In order to better overlook the contribution of tangentially and radially acting Casimir energies to the total binding energy, we divide the values for the total binding energy of nuclei given in the table at the end of Appendix by the mass number *A*, and we obtain

<u>The total binding energy per nucleon</u>

Nuclide	- $(12\ E_{Cas}\ (d)/A$ +	$8\ E_{Cas}\ (r,\ R)/A$	$+ 4E_{\mathbf{C}}/A)$	$= -\ E_B\ [MeV]/A$
^{8}Be	*6,8520* +	*0,6692*	- *0,4612*	= *7,0601*
^{16}O	*7,4562* +	*1,1863*	- *0,6546*	= *7,9879*
^{24}Mg	*7,8354* +	*1,2352*	- *0,8107*	= *8,2599*
^{32}S	*8,0414* +	*1,3770*	- *0,9214*	= *8,4970*
^{40}Ca	*8,1058* +	*1,4827*	- *1,0381*	= *8,5503*
^{48}Cr	*8,0086* +	*1,6915*	- *1,1280*	= *8,5722*
^{56}Ni	*8,1907* +	*1,6262*	- *1,1714*	= *8,6455*
^{64}Ge	*8,0377* +	*1,7045*	- *1,2133*	= *8,5289*
^{96}Cd	*7,7100* +	*1,8635*	- *1,3083*	= *8,2652*



The main contribution to the total binding energy per nucleon is made almost exclusively by the tangentially acting Casimir energies. The contribution of the radially acting energies remains by the specified gg-nuclei under *0,57 MeV* per nucleon. Even the shape of the curve "the total binding energy per nucleon" is largely determined by the tangentially acting Casimir energies. After a relatively sharp increase the maximum of the curve is achieved between the nuclides ^{56}Ni und ^{64}Ge. After that, the curve falls less sharp in accordance with the tangentially acting Casimir energies per nucleon until their final trend.

In order to show the final trend of tangential and radial components of the shell energy, we have continued the semi-classical calculation of the shell energy spectrum of the nuclide ^{96}Cd , and we arrived at the 29th shell to an estimation of the energy values of ^{232}Lv:

^{232}Lv

29th shell

R [fm]	*r /R*	*d/R*	*r [fm]*	*d [fm]*	*E [MeV]*
55,5	0,9819319	0,446562	*54,4976*	*24,7842*	*- 9,5866*

$12\, E_{Cas}\,(d)$	+ $8\, E_{Cas}\,(r, R)$	+ $4E_C$	$= E\,[MeV]$
- 0,3694	- 1858,9344	+ 6,0853	
- 3,6747	+ 0,1528		
+ 1,7561	+ 1845,5533		
	- 0,1556		
- 2,2880	- 13,3839	+ 6,0853	= - 9,5866

As we see, the tangentially acting Casimir energies on the 29th shell are only *- 2,2880 MeV*. This is not a surprising result, because the shell in question consists only of an extremely thin strip of $\Delta R = R - r = 1{,}0024\ fm$ and has a radius of $R = 55{,}5\ fm$. But the really paradox of the 29th shell is its high binding energy of *- 9,5866 MeV*, although it is generally assumed that at sufficiently high nuclear shells the repulsive Coulomb force exceeds the nuclear binding forces and thus, the nucleus becomes unstable. Obviously, our semi-classical model can not explain the experimental evidence of the instability of nuclei at sufficiently high nuclear shells. – In order to find an explanation for this phenomenon, we assume that with increasing shell number the shell height $\Delta R = R - r$ becomes extremely small, and therefore, the photons can penetrate the shell in question more and more. Thus, the radially acting Casimir energies are much weaker than *- 9,5866 MeV*, so that the Coulomb energy can easily exceed the sum of the tangentially and radially acting Casimir energies. – However, whether this statement is true, cannot be shown in the present work because our starting point is a semi-classical model, and we may not give more detailed information about shell heights, radii and distances. An adequate solution for this problem can only be found in the context of a quantum hydrodynamic treatment.

5 Conclusions

Finally, we can make the statement that the Casimir forces can excellently be used as nuclear binding forces. The shell and the total binding energy of even-even nuclei can be calculated by tangentially and radially acting Casimir energies in accordance with experiment. The shape of the curve "total binding energy per nucleon" can be interpreted in detail as a trend of tangentially and radially acting Casimir energies. In view of these successes we can ask now, what a model we should prefer in the future for the description of nuclear forces – the Casimir force model or the particle exchange model [7] with which modern physics describe the nuclear forces?

This question cannot be answered hastily. The Casimir force model has still to pass a critical test: In the context of quantum hydrodynamic treatment of nuclear forces it must be still shown that the Casimir force is also in the interior of nuclei the only possible nuclear binding force, and starting with this force the shell energies, heights, radii and distances can be exactly calculated. Until this is done, we cannot say that the Casimir force model is preferable to the particle exchange model.

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Appendix

Some characteristic shell quantities of selected even-even nuclei

^{8}Be	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	3,5123	0,2216	1,13132	0,7783257	-56,4805	-56,4800

^{16}O	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	2,4647	0,18949	1,0615	0,4670	-80,5141	
2	3,5123	0,7631716	0,889587	3,4343	-47,2919	
					-127,806	-127,619

^{24}Mg	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	1,7	0,159037	0,986685	0,2704	-118,5011	
2	3,8	0,706706	0,92184	2,6855	-54,7241	
3	9,861	0,706353	1,026513	6,180589	-25,0128	
					-198,238	-198,257

^{32}S	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	1,3532	0,141983	0,941198	0,1921	-151,5412	
2	3	0,7084214	0,88707	2,1253	-69,3323	
3	6,4	0,7222333	0,97454	4,622293	-32,6253	
4	13	0,742546	1,02233	9,653098	-18,4063	
					-271,9051	-271,7800

^{40}Ca	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp}[MeV]
1	1,144	0,130237	0,908316	0,1490	182,445	
2	2,81	0,6828807	0,897028	1,9189	-73,5727	
3	5	0,7718783	0,89227	3,8593915	-41,2961	
4	8	0,8067889	0,894865	6,4543112	-26,4059	
5	12,2	0,824045	0,900413	10,053349	-18,2935	
					-342,013	-342,052

^{48}Cr	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp}[MeV]
1	1,011289	0,122042	0,884601	0,123400	-209,623	
2	2,6	0,672042	0,892947	1,747300	-79,4127	
3	4,5	0,780291	0,870663	3,511310	-45,8371	
4	7	0,815993	0,869867	5,711949	-29,8597	
5	9	0,886797	0,76452	7,981176	-25,1547	
6	11	0,907720	0,726189	9,984924	-21,5785	
					-411,466	-411,466

^{56}Ni	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	0,828	0,1095260	0,847039	0,0906875	-263,8511	
2	2,5	0,6373103	0,909441	1,5932758	-82,1764	
3	4,4	0,7750956	0,873911	3,4104206	-46,7153	
4	6,65	0,8258649	0,850847	5,4920016	-31,4936	
5	9,09	0,8628401	0,812973	7,8432168	-23,8641	
6	11,6	0,8899495	0,771054	10,323414	-19,5686	
7	14,3	0,9042228	0,745901	12,930386	-16,4803	
					-484,1494	-483,9910

^{64}Ge	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp}[MeV]
1	0,762	0,1046040	0,831793	0,07970825	-290,842	
2	2,3	0,6370567	0,896195	1,46523041	-89,4595	
3	4	0,7786815	0,858739	3,11472600	-51,2974	
4	5,9	0,8343747	0,827499	4,92281055	-35,4368	
5	8,5	0,8433197	0,842999	7,16821745	-24,8104	
6	11	0,8843180	0,780847	9,72749833	-20,2540	
7	13,5	0,9060661	0,738957	12,2318928	-17,3845	
8	15,7	0,9291536	0,678865	14,5877109	-16,3655	
					-545,850	-545,879

^{96}Cd	R [fm]	r/R	d/R	r [fm]	E [MeV]	E_{Exp} [MeV]
1	0,549800	0,08680800	0,774089	0,0477	-431,0107	
2	1,9	0,61010350	0,880057	1,1592	-108,5837	
3	3,2	0,78867870	0,821204	2,5238	-64,1870	
4	5,1	0,80751160	0,851117	4,1183	-39,6927	
5	7,5	0,83585410	0,845587	6,2689	-27,5160	
6	9,5	0,89291093	0,754174	8,4827	-23,4619	
7	11,5	0,91181577	0,717579	10,4859	-20,3527	
8	13,5	0,92504391	0,686729	12,4881	-18,2108	
9	15,5	0,93481961	0,660319	14,4897	-16,6502	
10	17,5	0,94233888	0,637400	16,4909	-15,4652	
11	19,5	0,94830233	0,617277	18,4919	-14,5365	
12	21,5	0,95314766	0,599430	20,4927	-13,7901	
					-793,4575	-793,3980

Nuclide	$12E_{Cas}(d)$	$8\ E_{Cas}(r)$	$4E_C$	E [MeV]
^{8}Be	-54,8163	-5,3537	3,6895	-56,4805
^{16}O	-119,298	-18,9805	10,4730	-127,806
^{24}Mg	-188,049	-29,6455	19,4567	-198,238
^{32}S	-257,326	-44,0627	29,4840	-271,905
^{40}Ca	-324,232	-59,307	41,5256	-342,013
^{48}Cr	-384,414	-81,1941	54,1420	-411,466
^{56}Ni	-458,678	-91,0683	65,5965	-484,150
^{64}Ge	-514,411	-109,088	77,6489	-545,850
^{96}Cd	-740,157	-178,898	125,598	-793,458