Inertial propulsion Part II: How far away could a hydrogen molecule fly?

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Abstract
In this paper we show that, based on Rutherford–Bohr’s elementary model for the hydrogen atom, under certain initial conditions, a hydrogen molecule that could be isolated from its surroundings could escape and fly over 72 km far away. However, the aforementioned initial conditions are rather difficult to occur in reality.

1. Introduction
In a previous paper [1] we have reported on the topic of ‘inertial propulsion’. This type of propulsion was known to ancient Greeks who used halteres/dumbbells to perform the Greek long jump (http://en.wikipedia.org/wiki/Long_jump), also is met in one Munchausen’s story (http://en.citizendium.org/wiki/Reactionless_propulsion) and finally in the notorious Dean’s space drive (http://en.wikipedia.org/wiki/Dean_drive).

Inertial propulsion through rotating eccentric masses has been thoroughly investigated by Provatidis [2,3]. Not only a numerical procedure but also approximate closed formed solutions have appeared [3]. Also, an electromagnetic equivalent of Dean’s drive has been recently presented [4].

This paper applies the above-mentioned closed formed solutions to a Rutherford–Bohr’s elementary model of hydrogen molecule and calculates the maximum distance this molecule could potentially ‘fly’ far away, under certain ideal conditions that have not reported so far but could only theoretically occur, of course provided the aforementioned model was valid.

2. Closed formed solutions for inertial propulsion, in the vertical direction
For the sake of simplicity, let us consider a molecule of hydrogen that consists of two atoms, that is two protons (nuclei) and two rotating electrons. Let us also assume that the set of two protons remains horizontal at a constant distance (like a rigid object) and the two electrons contra-rotate in synchronization, i.e. they form equal angles with the horizontal line as shown in Figure 1.

When both electrons are found on their horizontal position, as shown in Figure 1, the molecule is assumed to become isolated (no internal forces) from its surrounding. From this position and further, the position of the molecule can be analytically determined in time.
Let \( M = 2m_p \) be the total mass of the two protons (object), \( m_e \) the mass of every electron, \( \omega \) the common angular velocity (initially it is considered to be constant), and \( r \) the common radius of electron’s orbit.

Following [2,3], let us also designate by \( \phi_0 \) the angle that is formatted by one radius and the horizontal (the other radius forms 180 degrees). According to [3], the maximum altitude the object could reach is given by the approximate formula (\( g \) = gravitational acceleration):

\[
z_{\text{max}} \cong \left( \frac{2}{g} \right) \left[ \frac{m_e \omega r}{(2m_e + M)} \right]^2 \cos^2 \phi_0
\]

(1)

Also, the time required for the molecule to reach the abovementioned upper point is obtained by setting the nuclei velocity equal to zero, that finally is [3]:

\[
t_{\text{max}} \cong \frac{\mu \omega \cos \phi_0}{g},
\]

(2)

where

\[
\mu = \frac{2m_e r}{(2m_e + M)}
\]

(3)

Equation (1) depicts that the maximum altitude is achieved for

\[
2 \cos \phi = 1
\]

\[
\phi = 0
\]

The key point is to determine the radius \( r \); note that \( z_{\text{max}} \) is proportional to \( r \). This is easily treated using the Rutherford–Bohr model.

3. Rutherford–Bohr model of hydrogen atom

The above model requires two assumptions, that is:

(i) **Classical mechanics**: the centripetal forces is equal to the Coulomb force:

\[
\frac{m_e v^2}{r} = \frac{k_e e^2}{r^2}
\]

(4)

where \( m_e \) is the electron's mass, \( e \) is the charge of the electron, and \( k_e \) is Coulomb's constant. This equation determines the electron's speed at any radius:

\[
v = \sqrt{\frac{k_e e^2}{m_e r}}
\]

(5)
and

(ii) **The quantum rule**, according to which the angular momentum \( L = m_e v_r r \) is an integer multiple of \( \hbar (=\hbar/2\pi) \):

\[
m_e v_r r = n\hbar, \quad n = 1, 2, 3, \ldots
\]

Substituting (5) into (6) gives an equation for \( r \) in terms of \( n \):

\[
\sqrt{k_e e^2 m_e r} = n\hbar
\]

so that the allowed orbit radius at any \( n \) is:

\[
r_n = \frac{n^2\hbar^2}{k_e e^2 m_e}
\]

4. **Numerical results**

Consider the following established data:

\[
\begin{align*}
\hbar & = 6.63 \times 10^{-34} \text{/(2}\pi) \quad \text{[joule.sec]} \\
\varepsilon_0 & = 8.85 \times 10^{-12} \quad \text{[farad/m]}, \quad k_e = 1/4\pi\varepsilon_0 \\
e & = -1.602 \times 10^{-19} \quad \text{[Coulomb]} \\
m_e & = 9.109 \times 10^{-31} \quad \text{[kg]} \\
m_p & = 1.672 \times 10^{-27} \quad \text{[kg]} \\
g & = 9.81 \quad \text{[m/s²]}
\end{align*}
\]

Then the smallest possible value of \( r \) in the hydrogen atom, called the *Bohr radius*, is equal to:

\[
r_1 = \frac{\hbar^2}{k_e e^2 m_e} \approx 5.29 \times 10^{-11} \text{m}
\]

Moreover, Equation(5) implies that the electron velocity is calculated equal to:

\[v = 2.1869 \times 10^6 \text{ m/s},\]

while its angular velocity ( \( \omega = v/r \) ) is:

\[\omega = 4.1287 \times 10^{16} \text{ s}^{-1}\]

Also, Eq(1) and Eq(2) imply that:

\[z_{\text{max}} = 7.2273 \times 10^4 \text{ m} \approx 72 \text{ km}, \quad \text{and} \quad t_{\text{max}} = 121.39 \text{ s}.
\]

Provided the angular velocity is constant, the number of rotation until the molecule reaches the upper point is estimated by:

\[
\frac{\omega t_{\text{max}}}{2\pi} = 7.9763 \times 10^{17} \text{ revolutions}
\]
5. Discussion

It is well known (http://en.wikipedia.org/wiki/Reduced_mass) that the so-called reduced mass is the "effective" inertial mass appearing in the two-body problem of Newtonian mechanics. This is a quantity which allows the two-body problem to be solved as if it were a one-body problem. Note however that the mass determining the gravitational force is not reduced. In the computation one mass can be replaced by the reduced mass, if this is compensated by replacing the other mass by the sum of both masses. In our case, the mass of the proton is much larger, about 1836.1 times the mass of the electron.

In accordance to the above general remark, but starting from either second Newton’s law or Lagrange equations, we have achieved to predict the maximum altitude the hydrogen molecule could raise. The only restriction is that we assumed that both the angular velocity and the gravitational acceleration are constant (for 72km altitude, the latter actually reduces from 9.81 to 9.59 m/s²).

Since the molecule moves upward, a part of the kinetic energy is transformed into gravitational potential energy \(2\left(m_p + m_e\right)gz_{\text{max}}\), a fact that somehow reduces the angular velocity. Actually, for the current Rutherford–Bohr model the initial kinetic energy \(K_0 = 2 \left(\frac{1}{2} m_e v^2\right)\) equals to 4.3566×10⁻¹⁸ Joules, while the potential energy at the altitude of 72km is only 2.3722×10⁻²¹ Joules. As the latter corresponds to a very small relative variation of the kinetic energy, only 0.05% with respect to \(K_0\), the angular velocity of the electron does not practically change.

References


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