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The Fine-Structure Constant in Unexplored Relations

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Abstract:
Formulas for the fine-structure constant are presented: speculative, well-known, and original — five of which are related to quarks.

1. Introduction

If physical constants are truly invariable, then the relationships between them must also be stable. Any such relationship must be derived mathematically and supported by experimental data.

A large number of published papers express the fine-structure constant using various formulas that yield approximate — and sometimes even more precise — values than those currently established by experiments. The authors of these works often believe that "there is something to it."

This paper presents original expressions which, although not yet sufficiently supported by theory, yield values that exhibit a high degree of consistency. All expressions are based on the same theoretical framework — the concept of the ***'unity of the whole and its parts***' — and/or ***Bošković's Theory of Natural Philosophy*** [2].

All calculations can be performed based on the input data presented in the first segment of the table, while interpretations arising from other theoretical frameworks are welcome and encourage further discussion.

The primary goal of this work is to explore the possibility of varying the structure of the expression for the fine-structure constant depending on physical parameters. Should this approach prove fruitful, there is a realistic chance of discovering other significant formulas that have not been known until now.

2. The Fine-structure constant in speculative formulas

The idea that the fine-structure constant ***α,*** or its inverse ***ά=1/ α,*** can be expressed solely through mathematical constants is widespread in both the literature and online sources. There exists a large number of speculative and simplistic examples, contributed by both amateurs and recognized scientific authorities. Here, we cite just one such example [1].

$$\alpha' = 4\pi^3 + 4\pi^2 + \pi^1 = \pi\left(4\pi^2 + 4\pi + 1\right) = 137{,}0363037 \quad \textit{(1)}$$

Below are my approximate values obtained using formulas that rely exclusively on the mathematical constants ***e*** and ***π***. Although they appear elegant, neither these formulas nor the previous ones have solid mathematical or physical foundations. They are presented as speculative illustrations—examples of how approximate values of this constant can be reached through various speculative approaches.

$$\alpha' = \left[e^{2\pi} + 4\pi + 1/4\pi + 1/(4\pi)^2 + 1/(8\pi)^3\right]/4 = 137{,}035999794 \quad \textit{(2)}$$

$$\alpha' = \left[e^{2\pi} + 4\pi + 1/4\pi + 1/(4\pi)^2 + 1/129^2\right]/4 = 137{,}035999069 \quad \textit{(3)}$$

$$\alpha' = \left[e^{2\pi} + 4\pi + 1/4\pi + 1/(4\pi)^2 + 1/(8\pi + \pi/8 - 1/e^{2\pi})^3\right]/4 = 137{,}035999075 \quad \textit{(4)}$$

3. The Fine-structure constant in celations among physical constants

The fine-structure constant is a dimensionless ratio between physical quantities and, after ***2π***, is one of the most frequently used constants in physics. Thanks to the concept applied in all my works — where all physical quantities are inextricably linked and presented within a unified Excel table — the derivation of formulas involving the fine-structure constant has been significantly simplified.

For this reason, all tables in this work are labeled as Segments ***1*** through ***4***, as they represent parts of the same fundamental table. There is a large number of formulas involving this constant, and here we present some of them. Those highlighted in blue font are, according to available sources, previously unknown.

The input constants are presented in the MKS unit system, and for brevity, the masses of the proton and neutron are denoted by the abbreviations ***pr*** and ***ne***, respectively. Constants highlighted in red font are those taken in this work as the fundamental reference values for defining the MKS system of units and other defined constats:

ά – Inverzna konstanta fine-strukture,
h – Plankova konstanta,
c – Brzina svetlosti,
pr – Masa Protona,
μ – Proton/elektron odnos masa,

$\mathbf{k_B}$ – Bolcmanova konstanta,
ħ – Redukovana Plankova konstanta,
G – Njutnova gravitaciona konstanta,
λ_p – Komptonova talasna dužina protona,
m_{pl} – Plankova masa,

T – Tau/Mion odnos masa,
$m_{R\infty}$ – Ekvivalentna masa Ridbergovog koh. radijusa,
m_{el} masa elektrona,
r_{ce} – Klasični radijus elektrona,
ne – masa Neutrona,
M_u – Masa Univerzuma,
γ – Neutron/proton odnos masa,
β – Odnos $r_{ce}/\lambda_p = \mu/2\pi\acute{\alpha}$,
r_f – Fundamentalni limit,
m_f – Masa fundamentalne čestice,
m_H – Energija(masa) osnovnog stanja atoma vodonika,
q_{pl} – Plankovo naelektrisanje,
e – jedinično naelektrisanje,
$1/R_\infty$ – Inverzna Ridbergova konstanta,
a_0 – Borov radijus,
l_{pl} – Plankova dužina
$\acute{\alpha}_G$ – gravitaciona sprega elektrona,
K_j – Josepsonova konstanta,
R_k – von Klizing konstanta – (el.otpor),
σ – Štefan–Bolcmanova konstanta,
σ_R – Radijaciona konstanta za zračenje crnog tela.
q, Δp, ξ, S – Izvedene bezdimenzionalne konstante da se pojednostave formule.

The second segment of the table shows values of the inverse fine-structure constant obtained using known formulas. The third segment, presented both in tabular and mathematical form, contains previously unknown formulas. The fourth segment represents values of $\acute{\alpha}$ derived from quark masses using previously unknown formulas.

Tabela Konstante i formule za Alfa

1 – konstante		konstante	
$2\pi =$	6,28318531	$e^{2\pi}=$	535,491655525
$\acute{\alpha} =$	137,035999084	$k_B =$	1,38065E-23
$\mu =$	1836,152673430	$\hbar = h/2\pi$	1,05457182E-34
$h =$	6,62607015E-34	$G = c^2 * l_{pl} / m_{pl}$	6,67383542E-11
$c =$	2,99792458E+08	$\lambda_p = h/(pr*c)$	1,32140986E-15
$pr =$	1,67262192369E-27	$m_{pl} =$	2,17651009E-08
$T = m_\tau / mu =$	16,8167350604176	$m_{R\infty} =$	8,65787664E-14
$m_{el} = pr/\mu$	9,10938370155E-31	$r_{ce} =$	2,81794033E-15
$ne =$	1,67492749787E-27	$M_u =$	1,73944927E+53
		$q=3*e^{\wedge 2\pi}/4+3\log_2(2\pi)/2-\Delta p/2$	404,628455366
$\gamma = ne/pr =$	1,00137841920	$\Delta p=2-1/(2\pi\beta+2)$	1,93506094
$\beta = r_{ce}/\lambda_p = \mu/2\pi\acute{\alpha}$	2,13252559E+00	$\xi=2^{\wedge(4/3-1/(3\pi\beta+3))}/\beta$	1,14669171435
$S=(2\pi)^{\wedge 3}*\beta^2/2^{\wedge(2*\Delta p/3)}$			461,303958296
$r_f = \lambda p\ 2^{\wedge(2\Delta p/3)}$	3,23130882E-15	$m_f = pr * 2^{\wedge(-2\Delta p/3)} / 2\pi$	1,08862171E-28
	Vodonik (13.6 eV)	$m_H = m_e/2\acute{\alpha}^2$	2,42543510E-35
$q_{pl}=(r_f*m_f)^{0.5}*c$	1,77806827E-13	$e=c(m_e r_e)^{0.5}$	1,51890670E-14
$1/R_\infty=4\pi*\acute{\alpha}*a_0=$	9,1126705058E-08	$a_0=\acute{\alpha}*r_f*m_f/m_{el}$	5,29177211E-11
$l_{pl} = r_f*2^{\wedge q/6}$	1,61619877204E-35	$\acute{\alpha}_G=G*m_e^{\wedge 2}/(\hbar c)$	1,75168746E-45
$K_j=2e/h =$	4,584638E+19	$R_k=h/e^2$	2,87206217E-06
		$\acute{\alpha}_R=1/e^2(4\sigma/ck_B^4)^{1/3}$	157,554873613
		$\sigma=(2\pi)^5 k_B^4/(16*15*c^2h^3)$	5,6703744E-08

2 – **Poznate formule za ά**

		Gm_{pl}^2/e^2=	137,035999084
$\sqrt{(c^2 m_e a_0/e^2)}$	137,035999084	$4c/(2\pi h K_j^2)$ =	137,035999084
$\hbar/(c*m_{el}*r_{ce})$	137,035999084	$c\hbar/e^2$ =	137,035999084
q_{pl}^2/e^2	137,035999084	$\alpha_G^{1/2} a_0/l_{pl}$ =	137,035999084
$1/(4\pi R_\infty a_0)$	137,035999084	$m_e c a_0/\hbar$ =	137,035999084
$l_{pl} m_{pl}/(r_e m_e)$	137,035999084	$ά_R \pi^{2/3} /15^{1/3}$ =	137,035999084

To avoid errors that may occur during input into mathematical format, the results are presented in tables and calculated directly. Afterwards, the expressions are also provided in mathematical form.

4. The Fine-structure constant in previously unknown rlations

For the following formulas, which use input data from the previous tabular presentation, we will not delve into the domain of their applicability. Such divisions are essentially artificial. The universe does not recognize boundaries between phenomena — it operates as a '***unity of the whole and its parts***' which is the fundamental idea of this approach.

Segment ***3*** contains a series of formulas based on values from CODATA 2018 [4], organized by ascending exponent — from 1 to 9 (or the root from 1 to 1/9). Exponent 4 appears in Segment ***4***, which relates to the top (***t***) and bottom (***b***) quarks, while expressions for exponents 5 and 8 have not yet been formulated.

3 – **Nepoznate formule za ά** — **korenovano**

(5)	$ά = r_f*m_f / (r_{ce}*m_{el})$	*bez korena*	137,035999084
(6)	$ά = \mu / (2\pi\beta)$		137,035999084
(7)	$ά^{\wedge 2} = m_{el}/(2*m_H)$	18778,865045	137,035999084
(8)	$ά^{\wedge 2} = a_0/r_{ce}$	18778,865045	137,035999084
(9)	$ά^{\wedge 2} = (q/\log_2\gamma-1)/\log_2\mu$	18778,865045	137,035999084
(10)	$ά^{\wedge 3} = 1/(4\pi*R_\infty r_{ce})$	2573380,5331	137,035999084
(11)	$ά^{\wedge 6} = \xi^{\wedge 3}*\beta*m_{R\infty}/(8\pi*pr)$	6,622287E+12	137,035999084
(12)	$ά^{\wedge 7} = \xi^{\wedge 3} * m_{R\infty}/(16\pi^{\wedge 2}*m_{el})$	9,074918E+14	137,035999084
(13)	$ά^{\wedge 9} = \xi^{\wedge 3}*m_{R\infty}/(32\pi^{\wedge 2}* m_H)$	1,704167E+19	137,035999084
(14)	$2^{\wedge -f}*\{[(2^{\wedge -el}+2^{\wedge -mu}+2^{\wedge -\tau})/(2/3+1/(f*\pi))]^{\wedge 0,5} - 2^{\wedge -mu/2}-2^{\wedge -\tau/2}\}^{\wedge -2}$		137,035999084

Based on the knowledge of the fundamental particle, here is the basic formula for ***ά*** connected to it, and all formulas from the second segment can be successively reduced to (***5***):

$$ά = r_f * m_f / \left(r_{ce} * m_{el}\right) \qquad (5)$$

Essentially, formula ***(6)*** is known, but it is not used because $\boldsymbol{\beta}$, which is the ratio of the classical electron radius to the Compton wavelength of the proton, cannot be determined experimentally.

$$\acute{\alpha} = \mu / \left(2\pi\beta\right) \qquad \textit{(6)}$$

In ***(7),*** $\boldsymbol{m_H}$ is the energy of the ground state of the hydrogen atom (13.6 eV) converted into mass:

$$\acute{\alpha}^2 = m_{el} / \left(2 * m_H\right) \qquad \textit{(7)}$$

Formula ***(8)*** relates the Bohr radius and the classical electron radius.

$$\acute{\alpha}^2 = a_0 / \mathrm{r}_{ce} \qquad \textit{(8)}$$

The significance of formula ***(9)*** lies in the fact that it connects the neutron, proton, and electron, which are the essential—and nearly the only—constituents of the universe.

$$\acute{\alpha}^2 = (q / \log_2 \gamma - 1) / \log_2 \mu \qquad \textit{(9)}$$

Formula ***(10)*** connects the Rydberg constant and the classical electron radius.

$$\acute{\alpha}^3 = 1 / \left(4\pi * R_\infty * r_{ce}\right) \qquad \textit{(10)}$$

Formula ***(11)*** connects the Rydberg constant and the proton mass.

$$\acute{\alpha}^6 = \xi^3 * \beta * m_{R\infty} / (8\pi * pr) \qquad \textit{(11)}$$

Formula ***(12)*** connects the Rydberg constant and the electron mass.

$$\acute{\alpha}^7 = \xi^3 * m_{R\infty} / (16\pi^2 * \mathrm{m}_{el}) \qquad \textit{(12)}$$

Thanks to the concept of the '***unity of the whole and its parts***' formula ***(13)*** yields the highest exponent of alpha, "9" which does not appear in any known physics formula. The equivalent mass of the Rydberg (Bošković's non-cohesive limit) radius, $\boldsymbol{m_{R\infty}}$, was derived by the Theory [2]:

$$\acute{\alpha}^9 = \xi^3 * m_{R\infty} / \left(32 * \pi^2 * \mathrm{m}_H\right) \qquad \textit{(13)}$$

Formula ***(14)*** is essentially a rewritten, improved Koide [3] formula, using the CODATA 2018 value for the muon mass and the tau mass $\boldsymbol{m_\tau} = \mathbf{3{,}1674852349 * 10^{-27}}$kg. By solving this formula for ***ά*** using logarithmic values (see the following table), the application of formula ***(14)*** is enabled.

Mase leptona	3,167485E-27	1,883532E-28	9,10938370E-31
f=log2(Mu/mlf)	***τ=log2(Mu/mτ)***	***mu=log2(Mu/mmu)***	***el=log2(Mu/mel)***
269,752304	***264,889540303***	***268,961366034***	***276,653237125***

$$\alpha' = \left\langle \left[\left(2^{-el} + 2^{-mu} + 2^{-\tau}\right) / \left(2/3 + \frac{1}{f * \pi}\right) \right]^{1/2} - \left[2^{-mu/2} + 2^{-\tau/2}\right] \right\rangle^{-2} \quad \textbf{(14)}$$

We will not discuss in detail the significance and properties of these formulas here, as we believe this deserves a separate study. However, it is worth noting that the formulas do not only change the constants but also the mathematical operations. From the table, one can observe how well the formulas correspond to the input data. Neutrinos are not included in the analysis, nor is their connection to the fine-structure constant clear, while such a connection does not exist for bosons. Additionally, this constant appears in all complex structures and likely also within cosmological frameworks.

5. The fine-structure constant in quark relations

The fourth segment of the table is dedicated to deriving formulas in which the inverse fine-structure constant **ά** is expressed as a dependent variable, continuing the approach developed in my previous works. In these formulas, the quark mass values—shown in the penultimate column—play a central role, enabling such derivations. These values can be directly compared with the reference experimental data presented in the last column.

For clarity and conciseness, the constant S, introduced in Segment ***1***, is also used.

	4 – Nepoznate formule sa kvarkovima za ά		***m[MeV/c²]***	
(15)	***(2*(t/pr)^3* S / T)^1/4+1***	**137,035999084**	172759,564	172760 ± 300
(16)	***(2^9*(b/pr)^3*S * T)^1/4***	**137,035999084**	4185,77053	4180+40; −30
(17)	***2 * (c/pr)^3* S / T***	**137,035999084**	1273,058973	1275 +25;−35
(18)	***2^4* (s/pr)^3* S * T***	**137,035999084**	96,97446792	95+9; −3
(19)	***Kombinovana iz 4 kvarka***	**137,035999084**		

It is possible to formulate a more elegant expression for the top quark mass, but only the value from the table satisfies the condition from equation *(19)*, which was obtained by combining four expressions for quark masses. This serves as an indicator that the form involving (***ά -1***) is the correct solution related to the top quark.

$$\alpha' = 2^{5/4} * \left[S^{1/2} * (\mathrm{t*b})^{3/4} / pr + S * (\mathrm{c*s})^{3/2} / pr^3 \right]^{1/2} \qquad \textbf{(19)}$$

The formulas for quarks involve key constants such as ***2, π, ά*** and ***β***, as well as a hypothetical constant ***Δp***, which is also linked to ***β*** and here demonstrates its practical value. The terms within the parentheses highlight a distinction between the components corresponding to second- and third-generation quarks. This formula could be the subject of deeper discussion, raising questions about the relationship between quark and lepton masses, as well as potential symmetries that may underlie it.

Conclusion

Nearly twenty previously unknown formulas for determining the fine-structure constant have been presented, derived from relationships between parameters at various levels of the organization of the physical world. All formulas stem from the same theoretical approach — the concept of the **'*unity of the whole and its parts*'** — and are grounded in Ruđer Bošković's *Theory of Natural Philosophy* [2].

References:

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