PHYSICAL REPRESENTATION

Of

IMAGINARY NUMBERS

Philosophical Meditation

$$\sqrt{-1}$$

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Abstract

Imaginary numbers are imaginary and not real in the Argand diagram. But, they can be aligned with real numbers. There are many dimensions of imaginary numbers, current math has ignored. Also, unimaginary numbers can be identified. Research paper demonstrates, the physical representations of imaginary numbers and their imaginary relationships (with transactions and physical objects like apples, oranges and bananas). Mathematical field calculations are done with imaginary numbers (in new methods). This ground breaking knowledge will change the scope to improve mathematics.

Keywords:

Imaginary, Complex, Numbers, Mathematics, Math, Field, Philosophy, Relationship, Physics, Square root, Square, Real, Engineering, Argand, 3D

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1. Introduction

Imaginary numbers are useful, but nobody knows what they really are. Science accepts only **empirical** evidence (the record of one's direct observations or experiences). In the mainstream belief, imaginary numbers exist only in the imagination like fractions do. But their applications are empirical. If mathematics is closely related to science (mathematics used in science), imaginary numbers make **science mystical** (belief in the spiritual apprehension of truths that are beyond the intellect).

When there is a problem, there is no answer. When there is answer to the problem, there is no problem. When there is a problem, fools deliberately ignore the problem while nescient have no clue (unaware). On contrary, the wise solve the problem and share knowledge (solution), sincerely.

When a student asks: “Do Imaginary numbers really exist like apples and oranges?” The teacher **changes the question** and answers: “Yes, imaginary numbers have real world applications.” This is the common situation in math classes and in internet expert answers. (Read Fact Finding section to learn the general consensus / knowledge of imaginary numbers). Teacher is **confused** at the nature of imaginary numbers and **confident** with the application of imaginary numbers in real world. Therefore, he/she **chooses to ignore** the problem. Problem remains without solution.

Most intellects **do not like** to accept that they do not know the true nature of imaginary numbers. Simply, they are **lazy to think**, and they just keep on **parroting** what’s on text books. Author has taken the **historical challenge** to show you the physical existence of Imaginary number representations. Author knows imaginary numbers. [17] Therefore, he says what he does, and does what he says, truthfully. Author **confirms** that imaginary number **physical representations are not imaginary**, but they have **imaginary relationships** in nature. Thus, he can touch and show imaginary number physical representations. Both imaginary relationships and imaginary number physical representations exist. [26]

Brief history of imaginary numbers (included) is necessary to evaluate author's contribution. Considering the hypothesis, fact finding and analysis in this research paper, if a gap in knowledge is found, author will contribute new knowledge to mathematics. Paper ends with conclusions and recommendations.

An average personal with exposure to imaginary/complex numbers in collage math class, can easily understand everything written in this paper. (Common sense and not much jargon) Advanced knowledge touch the basics and is not complicated.

Most readers are the product of standard academic system. Author's philosophical ideas (many subjects) are radical and do not directly fit into the standard mathematics. He did some processing (calculations) to present this paper, where the reader can understand author's work, from known to unknown.

The terms and mathematical methods used in this paper are under the author's interpretation. Author reserves his right to express beyond standards in mathematics. This paper shall be categorized under 'Philosophy of Mathematics'.

2. History of Imaginary Numbers [1]

2.1. The very first mention of trying to use imaginary numbers dates back to the 1st century.

2.2. In 50 A.D., Heron of Alexandria studied the volume of an impossible section of a pyramid. Impossible was when he had to take $\sqrt{81-114}$. Then he **gave up**.

2.3. After negative numbers were invented, mathematicians tried to find, "could a squared number equal to a negative one?" Not finding an answer, they **gave up**.

2.4. In 1500 A.D., to solve 3rd and 4th degree polynomial equations, sometimes mathematicians required square roots of negative numbers. But, they too **gave up**.

2.5. In 1545 A.D., the first major work with imaginary numbers. Girolamo Cardano wrote a book titled Ars Magna. He solved the equation $x(10-x)=40$, finding the answer to be 5 plus or minus $\sqrt{-15}$. Although he found the answer, he **disliked** imaginary numbers. He said that work with them would be, "*as subtle as it would be useless*", and referred to working with them as "*mental torture*."

2.6. In 1637 A.D., Rene Descartes came up with the standard form for complex numbers, which is **a+bi**. He termed "**imaginary**". But, he too didn't like complex numbers. He assumed that *if Imaginary numbers were involved, you couldn't solve the problem*.

2.7. Issac Newton agreed with Rene Descartes, and Albert Girad. He even went as far as to call these, "*solutions impossible*".

2.8. Rafael Bombelli was a firm believer in complex numbers. He had a wild idea that you could use imaginary numbers to get the real answers. Today, this is known as **conjugation**.

2.9. In 1685 A.D., John Wallis said that a complex number was just a point on a plane, where X-axis would be real numbers, and the Y-axis would be imaginary numbers. But he was **ignored**.

2.10. More than a century later, Caspar Wessel published a paper showing how to represent complex numbers in a plane, but was also **ignored**.

2.11. In 1777 A.D., Euler made the **symbol i stand for √-1**, which made imaginary numbers a little easier to understand.

2.12. In 1804 A.D., Abbe Buee thought about John Wallis's idea to **graph** complex numbers, and agreed with him.

2.13. In 1806, Jean Robert Argand wrote how to plot complex numbers in a plane, and today that plane is called **Argand diagram**.

2.14. In 1831 A.D., Carl Friedrich Gauss made **Argand's idea** popular to mathematical community, took Descartes' **a+bi notation**, and termed it **complex number**.

2.15. In 1833 A.D., William Rowan Hamilton expressed complex numbers as pairs of real numbers (such as **4**+**3**i being expresses as **(4,3)**).

2.16. Karl Weierstrass, Hermann Schwarz, Richard Dedekind, Otto Holder, Henri Poincare, Eduard Study, and Sir Frank Macfarlane Burnet all studied the general theory of complex numbers.

2.17. Augustin Louis Cauchy and Niels Henrik Able made a **general theory** about complex numbers accepted.

2.18. August Mobius made many notes about how to apply complex numbers in **geometry**.

2.19. In 1843 A.D., a mathematical physicist William Rowan Hamilton extended the idea of an axis of imaginary numbers in the plane to a **three-dimensional** space of quaternion imaginaries.

2.20. So far mathematicians and philosophers all over the world **could not** understand the philosophy of Imaginary numbers. They understood Imaginary numbers are **not real** numbers, because they could not plot them on a real number line. Though they couldn't explain the physical existence of Imaginary number representations, they were confident that

Imaginary numbers **exist in mathematics**, because the applications of Imaginary numbers exist in real world.

2.21. In 2016 Mar 30, Lakshan Bandara published a Youtube video titled **Untold Story of Imaginary numbers**, explaining the story of Imaginary Numbers. But, **no one took it seriously**, because intellectual society members are stubborn and **upish** to neglect **knowledge outside their system**.

2.22. In 2018 Aug 12, Lakshan Bandara republished the philosophy of Imaginary numbers in eBook titled "**Untold stories of Existence and Nonexistence True False**"; book available at amazon.com

2.23. In 2018 Aug 26, Lakshan Bandara republished an eBook titled "**Imaginary Numbers: With Apples, Oranges and Bananas**", a detailed version of the **Philosophy of Imaginary Numbers** chapter in previous eBook ; book available at amazon.com

3. Hypothesis

3.1. There is confusion in the nature of imaginary numbers

3.2. Applications of Imaginary numbers in real world (calculation) available.

3.3. No philosophical explanation of the **relationship of Imaginary numbers** available using real word objects

4. Fact Finding

Following facts found by reading through research work of others (example: Google scholar) and internet discussions (example: quora.com) relevant to imaginary numbers:

4.1. We show how the definition of a "geometric product" of vectors in 2-and 3-dimensional space provides **precise geometrical interpretations of the imaginary numbers** often used in conventional methods. [2]

4.2. For those of us who think that fractions are not abstract, substitute imaginary numbers for fractions. I recall that, in **grade school, when I first encountered imaginary numbers, they were very mysterious**. What, I wondered, made some numbers imaginary and others real? Reflection on this question helped me see that despite the suggestive language **real numbers weren't so real and imaginary numbers weren't so imaginary**. Later, a high school student told me: "a **mathematician is someone for whom imaginary numbers are just as real as real numbers**." Not a bad definition. [3]

4.3. If only real humans were so blessed. Each year a new crop of high school students becomes acquainted with the Quadratic Formula and, along with it, negative discriminants. The **number i is magically invoked to resolve the difficulty**. Is it a real number? The correct answer is Clintonesque: "It depends on how you define 'real'." So we compromise**: we say that it is an imaginary number, but we make sure that it is on the exam**—that will make it seem real enough. Happily **for most students, imaginary numbers are often no more than a fleeting nuisance**. Those who do continue beyond high school algebra pass into an imaginary-free zone called calculus. Because we are often successful at extending this respite through linear algebra and ordinary differential equations, many mathematics majors never see an imaginary number during their entire college careers. [4]

4.4. To understand the necessity, consider the charming words (unfortunately not found in Nahin's book) of Descartes, in coining the term imaginary: "For any equation one can imagine as many roots [as its degree would suggest], but in many cases **no quantity exists which corresponds to what one imagines**." As late as 1770 **Euler stated that imaginary numbers are impossible**. If these are the **opinions of an ordinary genius and a magician, then how can we deny the necessity of constructing such quantities**? [4]

4.5. It is demonstrated that regardless of the results of the MINOS, OPERA and ICARUS experiments, the current interpretation of the **special theory of relativity (STR) has been refuted by the theoretical and experimental investigation of processes in linear electric circuits that allowed proving the general scientific principle of physical reality of imaginary and complex numbers**. The **physical reality of imaginary and complex numbers is explained**. [5]

4.6. in the **special theory of relativity** (STR), these dogmata have it, in particular, that **physical bodies cannot move at superluminal speed**, whereupon **imaginary (this is where their name stems from) and complex numbers have no physical meaning**. [5]

4.7. When it came to physicists, they decided to close the issue by alleging that **imaginary numbers have no physical meaning**. [5]

4.8. Since **physics is an integral science, the principle of physical reality of imaginary and complex numbers proved above both theoretically and experimentally is a general physical principle**, i.e., **it is true not only for the linear electric circuit theory** [5]

4.9. Hidden dimensions of the Universe: However, **convincing evidence of the physical reality of imaginary and complex numbers is obviously not enough**. [5]

4.10. Above, we have **proven the principle of physical reality of not only imaginary, but complex numbers**, as well; **due to this, any intermediate parallel Universes can exist**: [5]

4.11. The **fact that imaginary numbers have physical meaning is supported by a theoretical and experimental investigation of oscillation processes in linear electric circuits**. Since mathematics is a common and unambiguous language of science, the principle of the **physical reality of imaginary numbers must be accepted as the general scientific one**, true, in particular, for the STR. It is demonstrated that the Multiverse hypothesis suggested herein explains the phenomenon of the dark matter and the dark energy. [6]

4.12. **Theoretical and experimental studies of oscillation processes in linear electric circuits described in the article prove physical reality of concrete imaginary numbers**. Since these experimental studies are available for verification at any electronic laboratory, they are quite reliable and still not refuted, in contrast to MINOS and OPERA experiments having the similar goal. Since **nature is unified and consistent, the principle of physical reality of imaginary numbers is generally scientific**. Therefore, theories and hypotheses of all exact sciences such as theory of relativity, quantum mechanics, optics, radio electronics, etc. should be adjusted in accordance with this principle. It is shown how it can be done, for example, in the special theory of relativity. The article presents **relativistic formulas adjusted according to the principle of physical reality of imaginary numbers**. They are the **basis of the conception of**

the hidden Multiverse which consists of mutually invisible parallel universes, coexisting in different dimensions. [7]

4.13. The **concept of imaginary numbers and complex math can be difficult for some engineers and physicists**. On the other hand, **they are extremely useful for solving many types of equations that engineers and physicists encounter**. [8]

4.14. **Complex numbers**, particularly **imaginary numbers**, sometimes seem mysterious **and unreal**. [9]

4.15. Many **futile attempts** have been made to ascribe some **physical meaning** to **imaginary numbers**. [9]

4.16. It is widely known statement about complex numbers belonging to Leibniz (1646-1716): "The **Spirit of God has found an outlet in subtle analysis of this miracle, freak of the world of ideas, the dual essence, located between being and non-being, which we call the imaginary root of negative uni**t". Or like this: "**Complex numbers are a fine and wonderful refuge of the divine spirit, as if it were an amphibian of existence and non-existence**". [10]

4.17. In 1759, Francis Mather (1731-1824) published an article entitled "Discourse on the use of the algebra of the minus sign". Here is what he wrote in it about **imaginary numbers**: "As far as I can tell, **they serve only to make confusion in the whole theory of equations and make a vague and mysterious** in that that **by its very nature is particularly clear and simple ... It is highly desirable so to avoid negative roots in algebra, and if they still arise, cast them religiously**. There are good reasons to believe that if we could get rid of the negative roots, thus would be removed objections raised by many scholars and men of witty against the algebraic calculations as being too complex and endowed with almost incomprehensible to the mind concepts. Algebra, or universal arithmetic, by their very nature, of course, is the science of not less than a simple, clear and useful for evidence than geometry" [10]

4.18. Leonhard Euler in his fundamental work "Complete introduction to algebra" (1770), noting the **mysterious unreal nature of imaginary numbers, regarded them as a product of the imagination**: "**Square roots of negative numbers are not equal to zero, are not less than zero, and are not greater than zero**. From this it is clear that the square roots of negative numbers cannot be among the possible (actual, real) numbers. Hence, **we have no another way except to acknowledge these numbers as impossible ones**. This leads us to **the notion of numbers, impossible in essence, which are usually called imaginary (fictitious) numbers, because they exist only in our imagination.**" [10]

4.19. Not surprising, physicists, faced with complex numbers in their theoretical schemes, **perceived imaginary components in them as unrealistic entities** (numbers, parameters). So deeply into and fixed in their minds (along with mathematicians), as a matter of course, the **unreality of numbers that were called imaginary**. Apparently, the **mystery of imaginary numbers we have managed to solve**, and this report is a brief review of publications on the subject. **How did we have come to this discovery**, and on which conceptual basis we have based? It will not be difficult to understand, if we look on a sad result with far-reaching consequences to which came physicists because of ignorance of the physical meaning of imaginary numbers and taking the way of mathematical abstractions. **As a bright example, it is very instructive to turn to quantum mechanics** (QM) [1]. Let us to analyze the very foundations of this theory. [10]

4.20. **Ignorance of the physical meaning of imaginary** unit i and, hence, **imaginary numbers in complex wave functions, led physicists to attributing to the polar-azimuthal functions a fictitious, erroneous, physical meaning**. Evidently, they were so confident in the imaginary nature (unreality) of "imaginary" components of complex numbers, regarding this as a dogma, that **none of them even began to doubt, that maybe these numbers are not imaginary, but are real**? We think that for this reason physics are not even looked into the reference books on mathematics, in which they would have found that the complex polar-azimuth functions determine the coordinates of the nodes and antinodes of the standing waves in the three-dimensional field-space. This means **that both components of the complex function are real**. As a result, physicists-theorists unreasonably (arbitrary) attributed to polar-azimuth functions of the solutions of the wave equation the meaning of electron atomic orbitals. [10]

4.21. Then, realizing that after all they cannot do without "imaginary" components of polarazimuth functions, physicists, making **combinations from the "real" and "imaginary" constituents, introduced the operation called hybridization of atomic orbitals**. From that time **the mixing of "real" and "imaginary" constituents of complex wave functions became a routine operation in quantum mechanics (QM) and quantum chemistry (QC)**. [10]

4.22. Absolute **lack of understanding of the real physical meaning of complex wave Y-functions with their "imaginary" components** led in result, as we can see, to the **construction of an abstractmathematical theory – quantum mechanics**. [10]

4.23. In fact, as shown **by the experience of physics and follows from our analysis, as set out in books and articles, starting from 1995, "real" and "imaginary" parts of complex wave functions are both real.** [10]

4.24. Thus, **'imaginary' numbers are not actually imaginary**. All **conjugate ("real" and "imaginary") numbers are real**. In particular**, the wave function, called in modern physics as a "complex" function comprising real and imaginary terms, in actual fact, is contained only real components, reflecting thus the potential-kinetic essence of rest-motion**.

4.25. Taking into account all of the above, we believe that the **word "imaginary" as not reflecting reality** will eventually **be removed from mathematics and physics**, **harmonizing the mathematical structures with the laws of the Universe, and thus expanding the horizons of knowledge.** [10]

4.26. Presently, **factorials** of real negative numbers and **imaginary numbers**, except for zero and negative integers are interpolated using the **Euler's gamma function**. In the present paper, the concept of factorials has been **generalised as applicable to real and imaginary numbers**, and multifactorials. **New functions** based on Euler's factorial function have been proposed for the factorials of real negative and imaginary numbers. As per the present concept, the **factorials of real negative numbers, are complex numbers**. The factorials of real negative integers have their **imaginary part equal to zero, thus are real numbers**. Similarly, the **factorials of imaginary numbers are complex numbers**. The moduli of the complex **factorials of real negative numbers, and imaginary numbers are equal to their respective real positive number factorials**. Fractional factorials and multifactorials have been defined in a new perspective. The proposed concept has also been extended to Euler's gamma function for real negative numbers and imaginary numbers, and beta function. [11]

4.27. In a model of a renewable resource no significance can be attached to negative or **imaginary numbers** of organisms. [12]

4.28. **The time projection, since it never actually happened, is an "imaginary number"**. It is a number that **never really occurs** – the **schedule consultant only imagines** what would have occurred, "but for" the acceleration efforts. [13]

4.29. **Imaginary numbers run contra to common sense on a basic level**, but you must **accept them as a system, and then they make sense**.; M B Drennan, Oxford UK [14]

4.30. purely **real numbers only describe a perfect, simplified world in physics** while **imaginary numbers must be used to include the myriad complicating factors found in the "real" world**.; Mark Lewney, Cardiff EU [14]

4.31. Complex numbers (the sum of real and **imaginary numbers**) **occur quite naturally in the study of quantum physics**.; Gareth Owen, Crewe UK [14]

4.32. Complex numbers have essential concrete **applications** in a variety of scientific and related areas such as **signal processing**, **control theory**, **electromagnetism**, **fluid dynamics**, **quantum mechanics**, **cartography**, and **vibration analysis**. [15]

4.33. **Imaginary Numbers were once thought to be impossible**, and so they were **called "Imaginary" (to make fun of them)**. But then people researched them more and discovered they were actually **useful and important because they filled a gap in mathematics** ... but the "imaginary" name has stuck. And that is also how the name "Real Numbers" came about (**real is not imaginary**). [16]

4.34. I've been **using imaginary numbers for over a decade**, but **never really questioned where they came from, why we need them, or why they're so ubiquitous in engineering**. What I found really fascinated me, and really served to remind me **how deep and profound the connection between mathematics and reality** is. **Imaginary numbers are just an abstract concept that basically fall out of algebra, but turn out be essential in describing real world processes.** It's ironic that zero, negative, and imaginary numbers were resisted for so long precisely because they don't seem directly connected to anything in the real world - but once we take the leap of faith and accept these guys, we find ourselves with incredible tools are essential in describing complex real world phenomena. [18]

4.35. An imaginary number is a number that, when squared, has a negative result. Essentially, an **imaginary number is the square root of a negative number and does not have a tangible value**. While **it is not a real number** — that is, **it cannot be quantified on the number line** — **imaginary numbers are "real" in the sense that they exist and are used in math.** [19]

4.36. Imaginary numbers, also called complex numbers, are **used in real-life applications**, such as **electricity**, as well as **quadratic equations**. In quadratic planes, imaginary numbers show up in equations that don't touch the x axis. Imaginary numbers become particularly useful in **advanced calculus**. [19]

4.37. Usually denoted by the symbol i, **imaginary numbers are denoted by the symbol j in electronics** (because i already denotes "current"). Imaginary numbers are particularly applicable in **electricity, specifically alternating current (AC) electronics**. AC electricity changes between positive and negative in a sine wave. Combining AC currents can be very difficult because they may not match properly on the waves. Using **imaginary currents and real numbers helps those working with AC electricity do the calculations and avoid electrocution**. [19]

4.38. **Imaginary numbers can also be applied to signal processing**, which is useful in **cellular technology and wireless technologies**, as well as **radar and even biology (brain waves)**. Essentially, if what is being measured relies on a **sine or cosine wave, the imaginary number is used**. [19]

4.39. Imaginary numbers have also made an appearance in **pop culture**. In **Dan Brown's "The Da Vinci Code," protagonist Robert Langdon refers to Sophie Neveu's belief in the imaginary**

number. **Isaac Asimov has also used imaginary numbers in his short stories, like "The Imaginary**," where **imaginary numbers and equations describe the behavior of a species of squid**.

4.40. From: **Doctor** Steven; **Imaginary numbers may not have any true meaning in the real world** (i.e., **it is hard to have 3i dollars, or even to be 240i degrees east**). But in many continuous applications where the state of a model at one point in time is dependent on the state of the model at a previous point in time, these **imaginary values can affect the value**. [20]

4.41. As mentioned, **imaginary numbers are not physical things, just like real numbers are not physical**. They are a concept designed to ease description of certain things. However, the question I think you are trying to ask is a bit more **nuanced than that**. We live in the real world, where real numbers are prevalent. **You can very easily see if you have four cookies, that you have a physical example of the number 4**. However, you probably **will never see a person with 4+3i cookies, because i does not really exist in physical quantities in our world.** So where DOES it exist this way? [21]

4.42. Whether or not you can have 3i dollars in your hand, the ways that imaginary numbers are used can give us real world results that are incredibly unexpected. [21]

4.43. How can **one show that imaginary numbers really do exist**? One does it in **exactly the same way one would show that fractions exist**. [22]

4.44. **Complex numbers are not found in reality**, but that is **not because so-called Imaginary numbers do not exist**. It is **because no numbers are found in reality**. In fact no mathematical objects at all are found in reality. At best **they exist in some kind of Platonic Realm**. [23]

4.45. That's what i does. It rotates pictures a quarter-turn (by convention, it rotates counterclockwise, and **−i rotates clockwise**, but it could just as easily have been defined the opposite way). [24]

4.46. Imaginary numbers (and a generalization of them, the quaternions) are **essential objects in graphics**, because they can be used to encode rotations of an image. If you think of them as "ways to alter an image," **they don't feel nearly as abstract as if you try to think of them as "numbers" in the usual sense.** [24]

4.47. I don't know why we confuse it: **Imaginary numbers should not be called imaginary so do "irrational" numbers. They depict real world.** In that sense, whole maths is imaginary! Call them iota numbers. [24]

4.48. What about √-1 ? Just like 0, negative numbers, √2, π, it is a valid number that can't be expressed in terms of the prior numbers. It is as "real" as any of them. Students are bothered by the fact that these numbers were given the name "imaginary". They are no more imaginary than the number 0. They are no more imaginary than the claim that √2 is "irrational". **Mathematicians (because of their lack of imagination) steal ordinary words (such as irrational and imaginary) and then use them for their own purposes.** √-1 may be "imaginary" but not in the sense of fantasy or fiction. It is just the same word being used in a different way. [24]

4.49. I don't think it is possible to **explain the imaginary number i to a layman so easily**. [24]

4.50. **If the new object plays by all the rules, then we say that it "exists"**, though in a quite different sense than we say that Mount Everest exists, or New York exists, or Sagittarius A* exists, or the sun exists, or the mug presently on my desk exists. Even the humble and uncontentious **number 1 only exists in the same abstract sense that i is said to exist**. [24]

4.51. I think Steven Strogatz did an excellent job explaining imaginary numbers in his Finding your roots NYT column: http://opinionator.blogs.nytimes.com/2010/03/07/finding-your-roots/. Here complex **doesn't mean complicated**; it means that **two types of numbers, real and imaginary, have bonded together to form a complex**, a hybrid number like 2 + 3i.". He also has a beautiful conclusion: "Complex numbers are **magnificent, the pinnacle of number systems...they're the end of the quest, the holy grail**. They are the culmination of the journey that began with 1...Some **imaginary friends you never outgrow**." [24]

4.52. Armed with these two sets of numbers, I can point you to anywhere on the **map**. Going **20 miles east and 15 miles north of here would be "20 + 15i"**. "11 - 5i" would mean go 11 miles east and 5 miles south. And so on. And instead of writing "20 + 15i", we can also just **write them as pairs of numbers**, like **(20, 15)**, or (11, -5). Once we agree on the convention, I would know exactly what you mean when you tell me to make a (30, 11) move. [24]

4.53. This has many **applications** in addition to points on a map. We can use them in any situation where you have a two-dimensional quantity, ranging from **locations of boats in a game of Battleship**, to **radio signals that have both "amplitude" and "phase"**. [24]

4.54. So, in school, we are taught that the **first set of numbers are "real numbers"**, and the **second set are "imaginary"**. But you can see from this discussion that **"imaginary" numbers aren't imaginary**. At all. **They're as real compared to real numbers, just like north-south is as real compared to east-west.** The **nomenclature is really, really unfortunate, and this fake mystery keeps getting propagated through schools**. [24]

4.55. You can think about this fake money as Monopoly bills: you can print as much as you can from them, but they are not real at the end of the day. To quantify "fake" money bills now, we can agree on the sign "i" for example, and **five fake bills would be presented as 5i**. Now, if these **five bills would ever grow with a rate of two**, the fact of being malicious makes the economy sinks and hence **growing in the reverse path which makes the economy in "debt"** which we agreed earlier in this post to note it as a "negative sign". Thus, the **growth of five fake bills i.e. 5i with a rate equals to two would be -25**. Congratulations, you just invented a new set of numbers to quantify fake money growth. [24]

4.56. It turns out that all **physics and engineering uses of these complex numbers** is to do with things involving rotation. Indeed one can show that all pure maths involving them can be interpreted by this **geometric visualisation**. [24]

4.57. We don't really have a problem to understand the non-imaginary numbers because their samples are abundant. A fraction 2/5 is when I take 2 pieces of cakes from your 5 pieces of cakes. Negative number -1,000 is when I owe you a 1,000, then my money is -1,000. It's not so hard to understand. But **imaginary number**? Other than some mathematicians found it fun to play with, **we have no day-to-day concrete idea of what it is**. Even physicists before Quantum Mechanics era never use imaginary number more than merely a mathematical trick. [24]

4.58. The major application of imaginary numbers will be in **vector based application such as application dealing with directional element such as Forces, Momentum, Electric, magnetical Fields etc and images** too. But when you try to **apply the concept to real world scenarios** like counting and object quantities **it doesn't make any senses**. [24]

4.59. Algebra was created a long time ago to solve real world problems. But sometimes, when solving a problem, mathematicians would find themselves needing to find the square root of a negative number, which can't be done with ordinary (real) numbers. And so they just had to **throw their hands in the air and give up**. But then they said, "why don't we simply create an abstract, imaginary, mathematical tool that **doesn't have a real world analogue** but would allow us to continue with our calculations?". In the end the **abstract imaginary parts cancelled out leaving a pure real world answer**. This **cheat allowed them to solve some hitherto unsolvable problems**. [24]

4.60. Now, since **you cannot find the root of minus 1 or even prove that it exists, simply accept that the root of minus 1 exists** because, **logically, it must exist since the root of +1 exists**. **Don't ask for logic here.** The logic is that since logic cannot prove or disprove the existence of the root of minus 1, the root of minus 1 must, in fact, exist. It would be **very inconvenient for the root of minus 1 not to exist** since mathematicians can only solve certain (entirely abstract and theoretical) equations given that the root of minus 1 exists. **Therefore, the root of minus 1 exists and it is not imaginary.** [24]

4.61. Since the question only asks about the particular and specific number i, and not the complex number system based on it, I wouldn't bother going into all that unless the layman asked, "**why bother?**" I would simply say that **once the idea is fully explored it turns out to be a very useful concept in several areas of applied math, and I'd email him a link to a great fractal geometry image-rich website**. First, the **layman would need to understand the notion of a paradox**. I'd use examples from **ordinary language to get that idea established: "a square circle"**, for instance, or other **oxymorons** ("military intelligence" and "honest politician" are usually good for a chuckle). The **point is to get across the idea** that in languages, it's **possible to build a phrase or sentence that is self-contradictory, paradoxical, or just nonsense**. [24]

4.62. I work as a **teacher** (sort of). This is how I **explain i to kids** if they happen to ask. In the real world, we happen to have things that **oscillate side to side or up and down in a straight line**, like a weight on a spring, or sledgehammer, or guitar string when you pluck it. Let's take a look at **weight on a spring**. It goes up and down in a straight line. What you can do, is imagine that weight in not rocking up and down, but traveling in the "imaginary" circle casting a shadow into the reality. Real number is a position of weight in the real world (on that up and down line). **Imaginary number, however, is a position on the "imaginary" side to side line**. Why would you want this? Well, mathematically, it is sometimes more convenient to view oscillation as rotation. **It makes calculation easier**. [24]

4.63. There are certainly many other things to know about complex numbers, which are all the numbers that look like 23+2i. They can be **added, subtracted, multiplied and divided** just like other numbers you may know. They can be **represented geometrically** and that can be sometimes very useful. They **can be used to easily solve problems in engineering**, and they are **completely fundamental to our understanding of the physical world via quantum mechanics**. They are as **essential** as they are **beautiful**, but really everything they do follows directly from the simple assumption that **i is a thing whose square is -1**, and our willingness to suspend any qualms and reservations about what such a creature might "actually" be. [24]

4.64. I know it's the square root of -1. Isn't it easy to say **it's a number which doesn't really exist**? But then go on to point out that really **no numbers exist**. 4 is a description of a certain set of objects. It might be four cars or four dollars or four komodo dragons. But **nobody's ever seen a "four" itself**. But **numebrs like 4 are very useful** for describing every day things. [24]

4.65. In a discussion with Prof. Udaya Jayathilaka at Moratuwa university, he showed physical representations of complex numbers (a+bi) are possible by putting “a” and “b” into two separate boxes. He demonstrated this by **putting two books into two boxes**. With no disrespect to professor’s thinking, author questions. Why did he put real part and imaginary part into two boxes, not just into a single box? The reason is, professor **assumes the real numbers are different from imaginary numbers**. Note: Author’s contribution is to demonstrate the **imaginary relationship** of “a” and “b”, **unknown to mathematics**. [25]

4.66. Prof. Udaya Jayathilaka at Moratuwa university **challenged** that imaginary numbers in standard mathematics is **different** to author’s version of imaginary relationship. Note: **Calculations** of imaginary and complex numbers (even **3d complex number multiplications**) using the **fundamental imaginary relationships** suggests otherwise to professor’s challenge. Note: Authors sees the professor as a high ranker of the **academic system**, which he should be proud of. But, author appreciates free thinkers, who can think out of the box and expand boundaries in a radical manner. [25]

5. Analysis

Based on hypothesis, the facts found are categorized in to three as follows:

5.1. Fact finding 4.2, 4.3, 4.4, 4.6, 4.7, 4.9, 4.10, 4.13, 4.14, 4.15, 4.16, 4.17, 4.18, 4.19, 4.20, 4.23, 4.24, 4.25, 4.27, 4.29, 4.30, 4.33, 4.34. 4.35, 4.40, 4.41, 4.42, 4.43, 4.44, 4.45, 4.46, 4.47, 4.48, 4.49, 4.50, 4.51, 4.54, 4.55, 4.57, 4.58, 4.59, 4.60, 4.61, 4.64, 4.65, 4.66 show, the **confusion** in the nature of Imaginary numbers.

5.2. Fact finding 4.1, 4.5, 4.6, 4.8, 4.11, 4.12, 4.13, 4.19, 4.21, 4.22, 4.24, 4.26, 4.27, 4.28, 4.31, 4.32, 4.34, 4.35, 4.36, 4.37, 4.38, 4.39, 4.40, 4.46, 4.52, 4.53, 4.56, 4.62, 4.63 show, the **applications** of Imaginary numbers in real world (calculation).

5.3. Lack of fact findings show, **no philosophical explanation of the relationship of Imaginary numbers** available using real word objects.

6. Gap in knowledge

Considering the analysis (5.1 and 5.2),

Hypothesis 3.1 is true with facts

Hypothesis 3.2 is true with facts

Hypothesis 3.3 is true with lack of facts

Analysis of facts with reference to hypothesis suggests, the current mathematics calculate using imaginary numbers for real world applications without the true knowledge of its nature. Mathematics has blind belief in imaginary numbers.

Therefore, there is a gap (requirement) in mathematics for a philosophical explanation of the relationship of Imaginary numbers using real word objects (like apples, oranges and bananas). Author will contribute the solution to fulfill the identified gap in knowledge.

7. Author's Contribution

7.1 Questions and Answers

1. Do imaginary numbers **really exist**?
   Yes

2. Do imaginary numbers exist the **same way fractions exist**?
   No

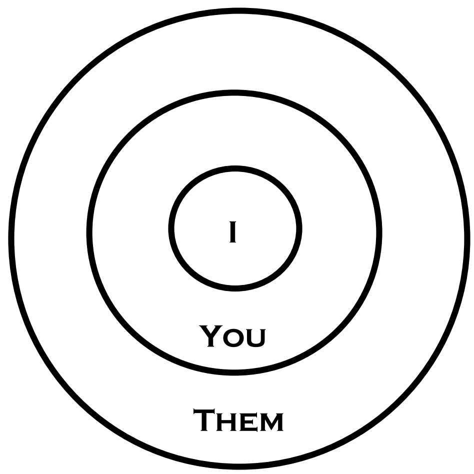
3. Are there **physical representations** of imaginary numbers?
   Yes.

4. Are **physical representations** of imaginary numbers, imaginary?
   No.

5. What is **imaginary** about imaginary numbers?
   The relationship.

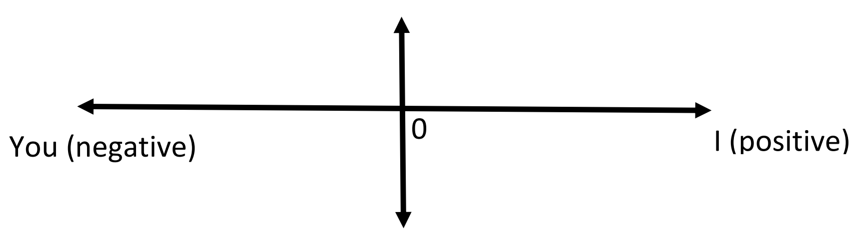
7.2 Philosophy of Imaginary Numbers

Mathematics is **self-centered**.



"I" is viewed to myself as positive, and I view "you" as negative.

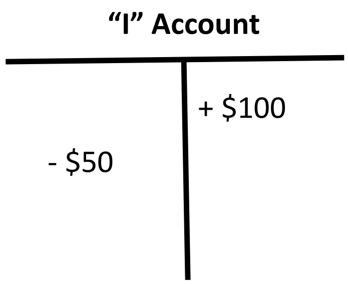
The real axis of Argand diagram shows "I" and "you" with two dimensions.

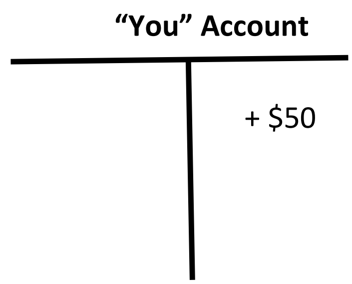


"I" and "You" are real to me. Therefore, "+Re" and "-Re" are in **real line**.

"I" and "you" **relationship** can be shown using **transaction**.

Suppose I have $100. I give you $50.



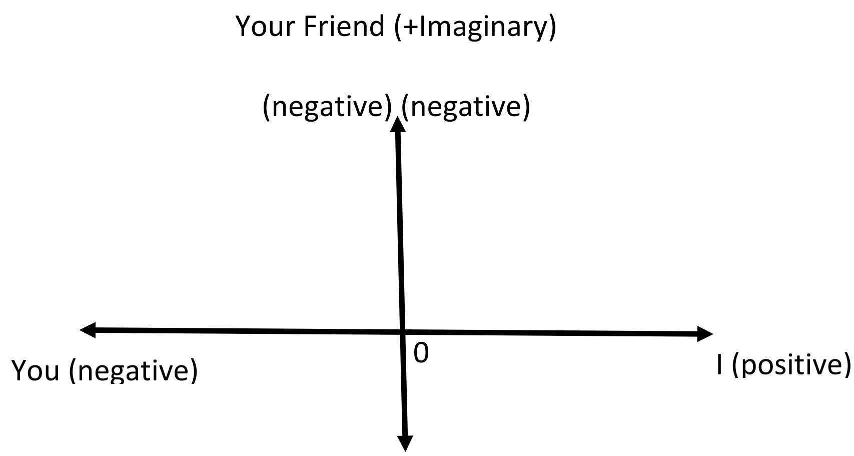


The **relationship** is important. When I give you money, I view "you" as negative.

(+100) + (**-50**) = (+50), in my view.

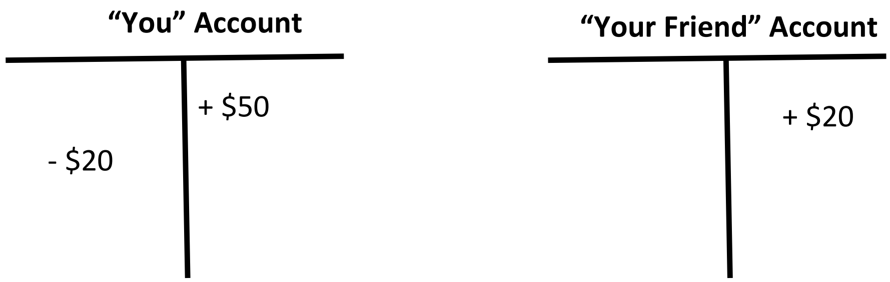
In mathematics, I view “your friend” as +imaginary. Your friend physically exists. My **relationship** to your friend is +imaginary. Relationship is the key, here. The Argand diagram shows “your friend” as +imaginary.

In other words, “you” view “your friend” as negative. Therefore, “I” view “your friend” as (negative)(negative).



“I” and “you” and “+imaginary” relationships can be shown using transactions.

I already gave you $50. Suppose you give $20 to your friend.



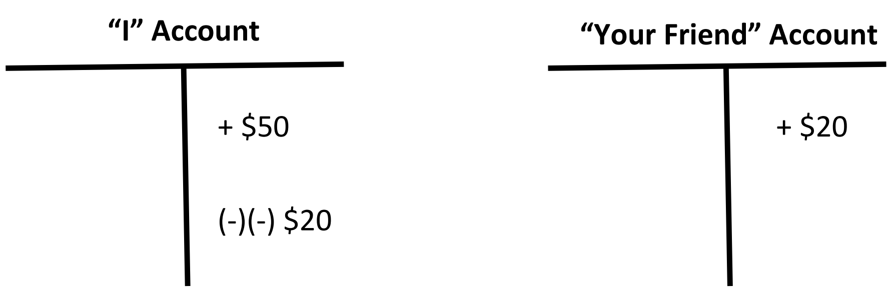
You are self-centered. Therefore, "you" see "your friend" as negative.

(+50) + (**-20**) = (+30), in your view.

You should know that you gave my money to your friend.

"I" have an "+**imaginary" relationship** with "your friend".

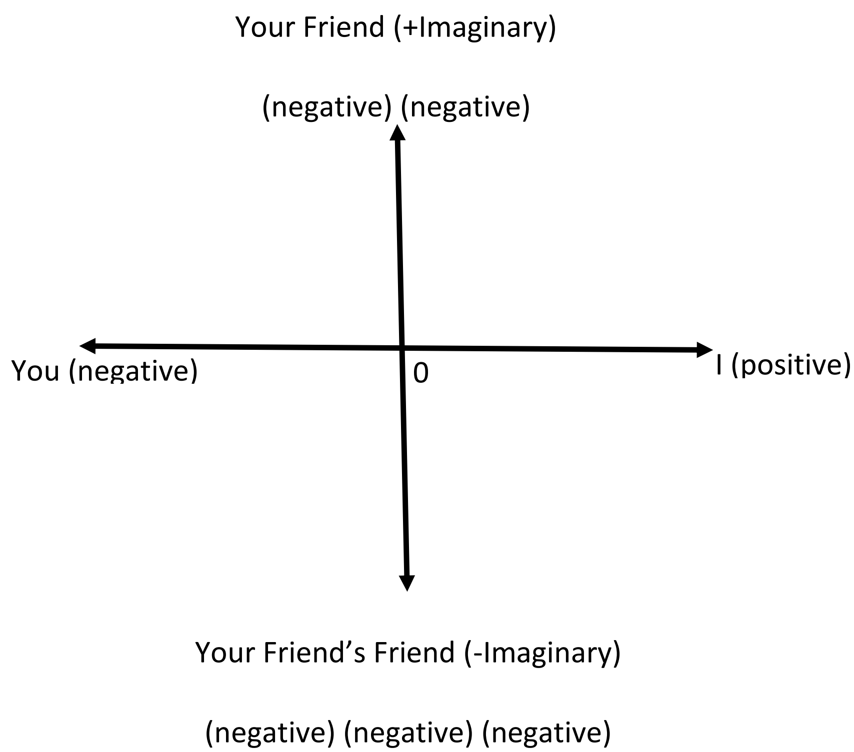
A → B; B → C; Therefore, A → C.



When the operators collapse (-) x (-) = (+), in **my view**, **your friend** is be **me**. "I" am "+imaginary" to me. Though **collapsed operators** are **easy to manage**, the **expanded operators** give **details without information loss** of relationships.

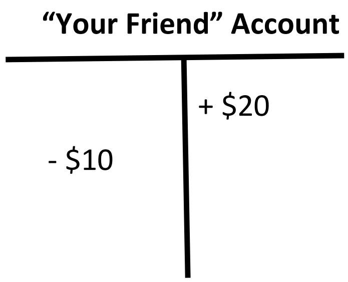
In mathematics, “I” view “your friend’s friend” as “-imaginary”. “Your friend’s friend” **physically exists**. My **relationship** to your friend’s friend is -imaginary. Author repeatedly emphasize the importance of Relationship in mathematical calculations. The Argand diagram shows your friend’s friend as -imaginary.

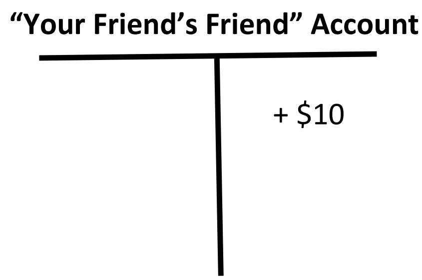
In other words, “your friend” view “your friend’s friend” as (negative). “You” view “your friend’s friend” as (negative)(negative). Therefore, “I” view “your friend’s friend” as (negative)(negative)(negative).



“I” and “you” and “+imaginary” and “-imaginary” relationships can be shown using transactions.

You already gave my money to your friend. Suppose “your friend” give $10 to “your friend’s friend”.



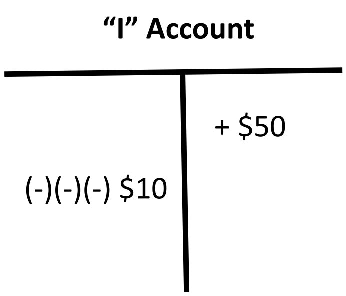


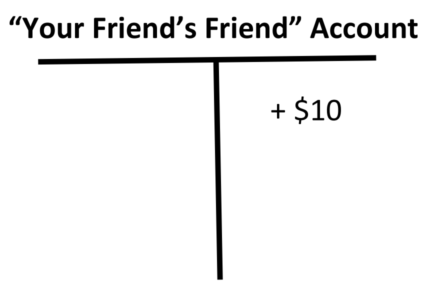
“Your friend’s friend” has my money. Therefore, “your friend’s friend” is related to “me”.

I am self-centered. “I” see “your friend’s friend” as (negative)(negative)(negative).

“I” have an “-**imaginary” relationship** with “your friend”.

A → B; B → C; C → D; Therefore, A → D.





As previously mentioned, operators can collapse. (-) x (-) x (-) = (-).

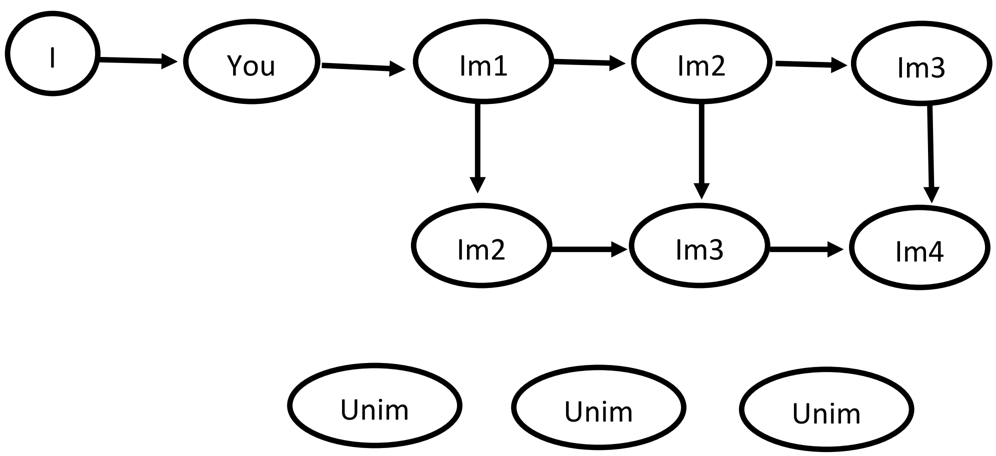
In my view, “your friend’s friend” will be “you”. “**You**” will be “**-imaginary**” to me.

There can be **many imaginary relationships** along a **linear path**. In a system, relationships work as a **network (mesh)**. Therefore, many linear paths of imaginary relationships can be found in a transaction system.

I am self-centered.

In my view, **imaginary relationships** exist.

Also, there can be transactions in a system, which are not related to me. They are called **unimaginary relationships**.



I = (+) real; You = (-) real

Im1 = (+) Imaginary; Im2 = (-) Imaginary; Im3 = (-)(-) Imaginary; Im4 = (-)(-)(-) Imaginary

Unim = Unimaginary

7.3 Imaginary Numbers in mathematical calculations

The standard mathematics use following notation to calculate square and square root.

$(+2)^2 = (+4)$; → $(+4)^{1/2} = (+2)$

$(-2)^2 = (+4)$; → $(+4)^{1/2} = (-2)$

When calculating the square root of a negative number,

$(-4)^{1/2} = (+4)^{1/2} \times (-1)^{1/2}$

$(-1)^{1/2}$ is symbolized by i, the imaginary number.

Author's method of calculating the square root of a negative number is as follows:

$(-4)^{1/2} =$ **(-)** $[(+4)^{1/2}]$

= **(-)**(-)2 and **(-)**(+)2

= **(-)**(-)2 and **(-)**(-)(-)2

The bolded **(-)** refers to "You" and, it is not used in imaginary relationship calculations.

I am self-centered, therefore in my view,

(-)(-) is +Imaginary relationship

(-)(-)(-) is -Imaginary relationship.

Author does not intend to challenge the current stable system of complex number calculations, though information loss is obvious as pointed out in ‘Limitations of Argand diagram” section.

This paper is meant to demonstrate the behavior of imaginary number relationship.

The standard mathematics calculate with one operator.

For example:

1 - 3 = -2

Imaginary number relationships have **multiple operators**. Therefore, direct calculations of real and imaginary numbers is a bit tricky.

If Imaginary is (-)(-)(-),

Re(+5) + (-Im)(+6) // (-Im) can be You

= (+5) + (-)(-)(-)(+6) // Collapse (-)x(-)x(-) = (-)

= (+5) + (-6)

= (-1)

In standard mathematics,

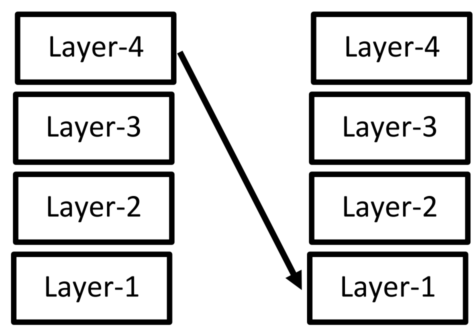
$i^1 = (-1)^{1/2}$

$i^2 = -1$

$i^3 = -1 \text{ x } i = -i = (-)(-1)^{1/2}$

$i^4 = i^2 \text{ x } i^2 = (-1) \text{ x } (-1) = (+1)$

Author uses his own method of calculation to see the behavior of multiplication. He pays attention to detail regarding layers in mathematics. First (-) layer (you relationship) is not used in imaginary calculations. Cross layer calculations can be done if needed.



Cross Layer Calculations

i = +imaginary number = $(-)\ \mathbf{(-)}\ 1$ and

i = -imaginary number = $(+)\ (-)\ 1 = (-)(-)\ \mathbf{(-)}\ 1$

$i^2 = (-)^2\ \mathbf{(-)}\ 1 = (+)\ \mathbf{(-)}1 = -1$ and

$i^2 = \{(-)(-)\}^2\ \mathbf{(-)}\ 1 = (+)\ \mathbf{(-)}1 = -1$

$i^3 = i^2 \text{ x } i = (-)1 \text{ x } (-)\mathbf{(-)} = -i$ and

$i^3 = i^2 \text{ x } i = (-)1 \text{ x } (-)(-)\ \mathbf{(-)} = (-)(-i) = +i$

$i^4 = i^2 \text{ x } i^2 = (-)1 \text{ x } (-)1 = (+)1$

7.4 Mathematics Field with Imaginary Numbers

Let a = +Im(2); b = +Im(3); c = +Im(4)

<u>Associativity of addition and multiplication</u>

a + (b + c) = (a + b) + c

a · (b · c) = (a · b) · c

a + (b + c) = (a + b) + c

Application:

// bolded **(-1)** refers to “You”, and not used in imaginary calculations

(-)(-)2 + { **(-)**(-)3 + **(-)**(-)4 }

= **(-)** [(-)2 + { (-)3 + (-)4 }]

= **(-)** [(-)2 + (-)7]

= (-)(-)9

{ **(-)**(-)2 + **(-)**(-)3 } + **(-)**(-)4

= **(-)** [{ (-)2 + (-)3 } + (-)4]

= **(-)** [(-)5 + (-)4]

= (-)(-)9

Result: pass

a · (b · c) = (a · b) · c

Application:

(-)(-)2 x { **(-)**(-)3 x **(-)**(-)4 }

= **(-)** [(-)2 x { (-)3 x (-)4 }]

= **(-)** [(-)2 x (+)12 }]

= (-)(-)24

{ **(-)**(-)2 x **(-)**(-)3 } x **(-)**(-)4

= **(-)** [{ (-)2 x (-)3 } x (-)4]

= **(-)** [(+)6 x (-)4]

= (-)(-)24

Result: pass

<u>Commutativity of addition and multiplication</u>

a + b = b + a

a · b = b · a

a + b = b + a

Application:

(-)(-)2 + **(-)**(-)3

= **(-)** { (-)2 + (-)3 }

= **(-)**(-)5

(-)(-)3 + **(-)**(-)2

= **(-)** { (-)3 + (-)2 }

= **(-)**(-)5

Result: pass

a · b = b · a

Application:

(-)(-)2 x **(-)**(-)3

= **(-)** { (-)2 x (-)3 }

= **(-)**(-)(-)6

(-)(-)3 x **(-)**(-)2

= **(-)** { (-)3 x (-)2 }

= **(-)**(-)(-)6

Result: pass

<u>Additive and multiplicative identity</u>

a + 0 = a

a · 1 = a

a + 0 = a

Application:

(-)(-)2 + 0

= **(-)**(-)2

Result: pass

a · 1 = a

Application:

(-)(-)2 * 1

= **(-)**(-)2

Result: pass

Additive inverses

$a + (-a) = 0$

$a + (-a) = 0$

Application:

(-)(-)2 + [**(-)**{ (-)(-)2 }]

= **(-)** { (-2) + (+2) }

= 0

Result: pass

Multiplicative inverses

$a \cdot a^{-1} = 1$; where $a \neq 0$

$a \cdot a^{-1} = 1$; where $a \neq 0$

Application:

(-)(-)2 x {**(-)**(-)2}$^{-1}$

= **(-)** [(-)2 x {(-)2}$^{-1}$]

= 1

Result: pass

Distributivity of multiplication over addition

a · (b + c) = (a · b) + (a · c)

a · (b + c) = (a · b) + (a · c)

Application:

(-)(-)2 x { **(-)**(-)3 + **(-)**(-)4 }

= **(-)** [(-)2 x { (-)3 + (-)4 }]

= **(-)** [(-)2 x (-)7]

= **(-)** (-)(-)14

{ **(-)**(-)2 x **(-)**(-)3 } + { **(-)**(-)2 x **(-)**(-)4 }

= **(-)** [{ (-)2 x (-)3 } + { (-)2 x (-)4 }]

= **(-)** [{ (+)6 } + { (+)8 }]

= **(-)** (+) 14

= (-) (-)(-) 14

Result: pass

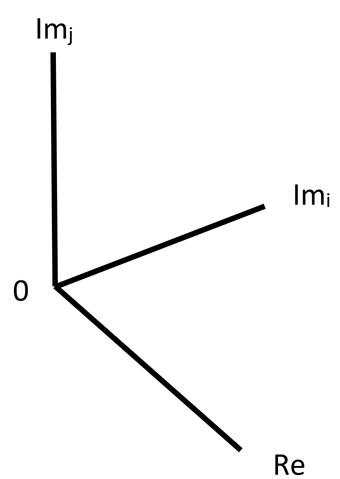
7.5 3 x 2D Planes in 3D Complex Numbers

3D object is a visual capability of geometry. A three dimensional object has 2 planes.

When extending the Argand diagram to x,y,z axis, the maximum imaginary number presentations are four dimensions (+i, -i, +j, -j), resulting three complex planes (one complex world).

Three complex planes:

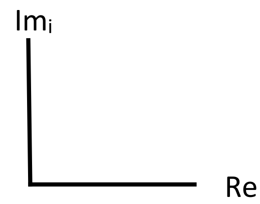
$Re+Im_i$, $Re+Im_j$, Im_i+Im_j



(Re, Im_i) is a 2D plane

$i^2 = -1_i$ is in this plane

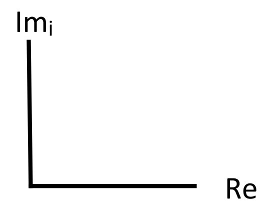
Relative to Re, $(-1_i) = -1$ (real number)



(Re, Im_j) is a 2D plane

$j^2 = -1_j$ is in this plane

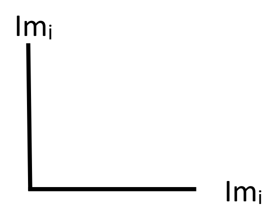
Relative to Re, $(-1_j) = -1$ (real number)



(Im_i, Im_j) is a 2D plane

$i \times j = -1_{ij}$ is in this plane

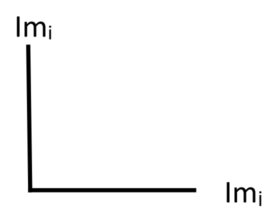
Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)



(Im_i, Im_j) is a 2D plane

$i \times j = -1_{ij}$ is in this plane

Relative to Im_j, $(-1_{ij}) = -1$ (imaginary number)



Collapse operators

+i = **(-)** (-) = (+) // imaginary relationship

-i = **(-)** (-)(-) = (-) // imaginary relationship

+j = **(-)** (-)(-)(-) = (+) // imaginary relationship

-j = **(-)** (-)(-)(-)(-) = (-) // imaginary relationship

Relative to Re, +i = (+); Relative to Re, +j = (+)

Relative to Re, -i = (-); Relative to Re, -j = (-)

$(+i)^2 = (-)^2$ **(-)** 1 = (+) **(-)**1 = -1 and

$(-i)^2 = \{(-)(-)\}^2$ **(-)** 1 = (+) **(-)**1 = -1

Therefore, $i^2 = -1$

$(+j)^2 = \{(-)(-)(-)\}^2$ **(-)** 1 = (+) **(-)**1 = -1 and

$(-j)^2 = \{(-)(-)(-)(-)\}^2$ **(-)** 1 = (+) **(-)**1 = -1

Therefore, $j^2 = -1$

(Im_i, Im_j) is a 2D plane

(-1_{ij}) is in this plane

Relative to Im_i, (-1_{ij}) = -1 (imaginary number)

Relative to Im_j, (-1_{ij}) = -1 (imaginary number)

Relative to Re, (-1_{ij}) = **(+)**(-1) = -1 (real number) // (+i), (+j) = (+); Im_i , Im_j, = (+)

Multiplication of two 3D complex numbers:

(a+bi+cj) X (x+yi+zj)

= a(x+yi+zj)+bi(x+yi+zj)+cj(x+yi+zj)

=ax+ayi+azj + bxi+byi^2+bzij + cxj+cyij+czj^2

= ax+ i(ay+bx) + j(az+cx) + ij(bz+cy) + $(-1_i)$by + (-1_j)cz

= **[ax + $(-1_i)$by + (-1_j)cz + (-1_{ij})(bz+cy)]** + **(ay+bx)**i + **(az+cx)**j --------(1)

= [ax + $(-1_i)$by + (-1_j)cz - (bz+cy)] + (ay+bx)i + (az+cx)j

and

Relative to Im_i, (-1_{ij}) = -1 (imaginary number)

= **[ax + $(-1_i)$by + (-1_j)cz]** + **[(ay+bx)+ (-1_{ij})(bz+cy)]**i + **(az+cx)**j --------(2)

= [ax + $(-1_i)$by + (-1_j)cz] + [(ay+bx) - (bz+cy)]i + (az+cx)j

and

Relative to Im_j, (-1_{ij}) = -1 (imaginary number)

= **[ax + $(-1_i)$by + (-1_j)cz]** + **(ay+bx)**i + [**(az+cx)+ (-1_{ij})(bz+cy)**]j --------(3)

= [ax + $(-1_i)$by + (-1_j)cz] + (ay+bx)i + [(az+cx) - (bz+cy)]j

Three answers are correct.

Different answers, because the of different views.

(Relative to **Im_i** and Relative to **Im_j** and Relative to **Re**)

They are in P+Qi+Rj form.

7.6 Mathematics Field with 3D Complex Numbers

a = (2+3i+4j); b = (5+6i+7j); c = (4+7i+5j)

// bolded **(-1)** refers to "You", and not used in imaginary calculations

<u>Associativity of addition and multiplication</u>

a + (b + c) = (a + b) + c

a · (b · c) = (a · b) · c

a + (b + c) = (a + b) + c

Application:

(2+3i+4j) + [(5+6i+7j) + (4+7i+5j)]

= (2+3i+4j) + [(5+4)+(6+7)i+(7+5)j]

= (2+3i+4j) + [9+13i+12j]

= (2+9)+(3+13)i+(4+12)j

= 11 + 16i + 16j

[(2+3i+4j) + (5+6i+7j)] + (4+7i+5j)

= [(2+5)+(3+6)i+(4+7)j] + (4+7i+5j)

= [7+9i+11j] + (4+7i+5j)

= (7+4)+(9+7)i+(11+5)j

= 11+16i+16j

Result: pass

a · (b · c) = (a · b) · c

Application-1

(2+3i+4j) x [(5+6i+7j) x (4+7i+5j)]

= (2+3i+4j) x [5(4+7i+5j)+6i(4+7i+5j)+7j(4+7i+5j)]

= (2+3i+4j) x [20+35i+25j + 24i+42i^2+30ij + 28j+49ij+35j^2]

// i^2= j^2= ij=-1; (Relative to **Re**)

= (2+3i+4j) x [(20+42i^2+30ij+49ij+35j^2)+(35i+24i)+(25j+28j)]

= (2+3i+4j) x [(20-42-30-49-35)+(35+24)i+(25+28)j]

= (2+3i+4j) x (-136+59i+53j)

= 2(-136+59i+53j)+3i(-136+59i+53j)+4j(-136+59i+53j)

= -272+118i+106j + -408i+177i^2+159ij + -544j+236ij+212j^2

// i^2= j^2= ij=-1; (Relative to **Re**)

= (-272+177i^2+159ij+236ij+212j^2)+(118i-408i)+(106j-544j)

= (-272-177-159-236-212)+(118-408)i+(106-544)j

= (-1056)+(-290)i+(-438)j

$[(2+3i+4j) \times (5+6i+7j)] \times (4+7i+5j)$

$= [2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j)] \times (4+7i+5j)$

$= [10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2] \times (4+7i+5j)$

// $i^2= j^2= ij=-1$; (Relative to **Re**)

$= [(10+18i^2+21ij+24ij+28j^2)+(12i+15i)+(14j+20j)] \times (4+7i+5j)$

$= [(10-18-21-24-28)+(12+15)i+(14+20)j] \times (4+7i+5j)$

$= (-81+27i+34j) \times (4+7i+5j)$

$= -81(4+7i+5j)+27i(4+7i+5j)+34j(4+7i+5j)$

$= -324-567i-405j + 108i+189i^2+135ij + 136j+238ij+170j^2$

// $i^2= j^2= ij=-1$; (Relative to **Re**)

$= (-324+189i^2+135ij+238ij+170j^2)+(-567i+108i)+(-405j+136j)$

$= (-324-189-135-238-170)+(-567+108)i+(-405+136)j$

$= (-1056)+(-459)i+(-269)j$

Result: fail

a · (b · c) = (a · b) · c

Application-2

(2+3i+4j) x [(5+6i+7j) x (4+7i+5j)]

= (2+3i+4j) x [5(4+7i+5j)+6i(4+7i+5j)+7j(4+7i+5j)]

= (2+3i+4j) x [20+35i+25j + 24i+42i^2+30ij + 28j+49ij+35j^2]

// Relative to Im_i, (-1_{ij}) = -1 (imaginary number)

= (2+3i+4j) x [(20+42i^2+35j^2)+(35i+24i+30ij+49ij)+(25j+28j)]

= (2+3i+4j) x [(20-42-35)+(35+24+30+49)i+(25+28)j]

= (2+3i+4j) x (-57+138i+53j)

= 2(-57+138i+53j)+3i(-57+138i+53j)+4j(-57+138i+53j)

= -114+276i+106j + (-57)i+414i^2+159ij + (-228)j+552ij+212j^2

// Relative to Im_i, (-1_{ij}) = -1 (imaginary number)

= (-114+414i^2+212j^2)+(276i-57i+159ij+552ij)+(106j-228j)

= (-114-414-212)+(276-57-159-152)i+(106-228)j

= -740-92i-122j

$[(2+3i+4j) \times (5+6i+7j)] \times (4+7i+5j)$

$= [2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j)] \times (4+7i+5j)$

$= [10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2] \times (4+7i+5j)$

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

$= [(10+18i^2+28j^2)+(12i+15i+21ij+24ij)+(14j+20j)] \times (4+7i+5j)$

$= [(10-18-28)+(12+15-21-24)i+(14+20)j] \times (4+7i+5j)$

$= (-36-18i+34j) \times (4+7i+5j)$

$= -36(4+7i+5j)-18i(4+7i+5j)+34j(4+7i+5j)$

$= -144-252i-180j -72i-126i^2-90ij +136j+238ij+170j^2$

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

$= (-144-126i^2+170j^2)+(-252i-72i-90ij+238ij)+(-180j+136j)$

$= (-144+126-170)+(-252-72+90-238)i+(-180+136)j$

$= -188-472i-44j$

Result: fail

a · (b · c) = (a · b) · c

Application-3

(2+3i+4j) x [(5+6i+7j) x (4+7i+5j)]

= (2+3i+4j) x [5(4+7i+5j)+6i(4+7i+5j)+7j(4+7i+5j)]

= (2+3i+4j) x [20+35i+25j + 24i+42i^2+30ij + 28j+49ij+35j^2]

= (2+3i+4j) x [(20+42i^2+35j^2)+(35i+24i)+(25j+28j)+(30ij+49ij)]

// i^2 = j^2 =-1; (Relative to **Re**)

= (2+3i+4j) x [(20+42i^2+35j^2)+(35i+24i)+(25j+28j)+(30ij+49ij)]

= (2+3i+4j) x [(20-42-35)+(35+24)i+(25+28)j+(30+49)ij]

= (2+3i+4j) x (-57+59i+53j+79ij)

= 2(-57+59i+53j+79ij)+3i(-57+59i+53j+79ij)+4j(-57+59i+53j+79ij)

= -114+118i+106j+158ij + (-171)i+177i^2+159ij+237i^2j + (-228)j+236ij+212j^2+316ij^2

// i^2 = j^2 =-1; (Relative to **Re**)

= (-114+177i^2+212j^2)+(118i-171i+316ij^2)+(106j-228j+237i^2j)+(158ij+159ij+236ij)

= (-114-177-212)+(118i-171i-316i)+(106j-228j-237j)+(158ij+159ij+236ij)

= (-114-177-212)+(118-171-316)i+(106-228-237)j+(158+159+236)ij

=(-503)+(-369)i+(-359)j+(553)ij → LHS

[(2+3i+4j) x (5+6i+7j)] x (4+7i+5j)

= [2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j)] x (4+7i+5j)

= [10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2] x (4+7i+5j)

// $i^2 = j^2 = -1$; (Relative to **Re**)

= [10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2] x (4+7i+5j)

= [(10+18i^2+28j^2)+(12i+15i)+(14j+20j)+(21ij+24ij)] x (4+7i+5j)

= [(10-18-28)+(12+15)i+(14+20)j+(21+24)ij] x (4+7i+5j)

= [-36+27i+34j+45ij] x (4+7i+5j)

= -36(4+7i+5j)+27i(4+7i+5j)+34j(4+7i+5j)+45ij(4+7i+5j)

= -144-252i-180j +108i+189i^2+135ij + 136j+238ij+170j^2 + 180ij+315i^2j+225ij^2

// $i^2 = j^2 = -1$; (Relative to **Re**)

= (-144+189i^2+170j^2)+(-252i+108i+225ij^2)+(-180j+136j+315i^2j)+(135ij+238ij+180ij)

= (-144-189-170)+(-252+108-225)i+(-180+136-315)j+(135+238+180)ij

= (-503)+(-369)i+(-359)j+(553)ij → RHS

LHS=RHS

Result: pass

Note: Order of calculation is important.

$(-503)+(-369)i+(-359)j+(553)ij$

// Relative to Re $(-1_{ij}) = -1$ (real number)

$= (-503+553ij)+(-369)i+(-359)j$

$= (-503-553)+(-369)i+(-359)j$

$= -1056-369i-359j$ // 1st answer

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

$= (-503)+(-369+553ij)i+(-359)j$

$= (-503)+(-369-553)i+(-359)j$

$= -503-922i-359j$ // 2nd answer

// Relative to Im_j, $(-1_{ij}) = -1$ (imaginary number)

$= (-503)+(-369)i+(-359+553ij)j$

$= (-503)+(-369)i+(-359-553)j$

$= -503-369i-912j$ // 3rd answer

All 3 answers are correct, based on perspective.

<u>Commutativity of addition and multiplication</u>

a + b = b + a

a · b = b · a

a + b = b + a

Application:

(2+3i+4j) + (5+6i+7j)

(2+5)+(3+6)i+(4+7)j

= 7+9i+11j

(5+6i+7j) + (2+3i+4j)

(5+2)+(6+3)i+(7+4)j

= 7+9i+11j

Result: pass

a · b = b · a

Application:

(2+3i+4j) x (5+6i+7j)

= 2(5+6i+7j) + 3i(5+6i+7j) + 4j(5+6i+7j)

= (10+12i+14j) + (15i+18i^2+21ij) + (20j+24ij+28j^2)

= [(10) + (18)i^2+(28)j^2+24ij] +(12+15)i+(14+20)j

// i^2= j^2= ij=-1; (Relative to **Re**)

= [10+(18)(-1)+(28)(-1)+24(-1)] +(12+15)i+(14+20)j

= -60 + 27i + 34j

(5+6i+7j) x (2+3i+4j)

= 5(2+3i+4j) + 6i(2+3i+4j) + 7j(2+3i+4j)

= (10+15i+20j) + (12i+18i^2+24ij) + (14j+21ij+28j^2)

= [(10)+18i^2+28j^2+24ij] + (15+12)i + (20+14)j

// i^2= j^2= ij=-1; (Relative to **Re**)

= [10-18-28-24] + (27)i +(34)j

= -60 + 27i + 34j

Result: pass

Additive and multiplicative identity

a + 0 = a

a · 1 = a

a + 0 = a

Application:

(5+6i+7j) + (0+0i+0j)

= (5+0)+(6+0)i+(7+0)j

= 5+6i+7j

Result: pass

a · 1 = a

Application:

(5+6i+7j) x 1

= (5+6i+7j)

Result: pass

Additive inverses

$a + (-a) = 0$

$a + (-a) = 0$

Application:

(5+6i+7j) + (-)(5+6i+7j)

= (5-5)+(6-6)i+(7-7)j

= 0

Result: pass

Multiplicative inverses

$a \cdot a^{-1} = 1$; where $a \neq 0$

$a \cdot a^{-1} = 1$; where $a \neq 0$

Application:

$(5+6i+7j) \times (5+6i+7j)^{-1}$

= 1

Result: pass

Distributivity of multiplication over addition

a · (b + c) = (a · b) + (a · c)

a · (b + c) = (a · b) + (a · c)

Application-1:

(2+3i+4j) x [(5+6i+7j) + (4+7i+5j)]

= (2+3i+4j) x [(5+4)+(6+13)i+(7+5)j]

= (2+3i+4j) x (9+13i+12j)

= 2(9+13i+12j)+3i(9+13i+12j)+4j(9+13i+12j)

= (18+26i+24j)+(27i+39i^2+36ij)+(36j+52ij+48j^2)

// i^2= j^2= ij=-1; (Relative to **Re**)

= (18+39i^2+36ij+52ij+48j^2)+(26+27)i+(24j+36)j

= (18-39-36-52-48) + (26+27)i + (24+36)j

= -157+53i+60j

$(2+3i+4j)(5+6i+7j) + (2+3i+4j)(4+7i+5j)$

$= 2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j) + 2(4+7i+5j)+3i(4+7i+5j)+4j(4+7i+5j)$

$= (10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2) + (8+14i+10j + 12i+21i^2+15ij + 16j+28ij+20j^2)$

// $i^2= j^2= ij=-1$; (Relative to **Re**)

$= (10+18i^2+21ij+24ij+28j^2 + 8+21i^2+15ij+28ij+20j^2)+(12+15+14+12)i+(14+20+10+16)j$

$= (10-18-21-24-28+8-21-15-28-20)+(12+15+14+12)i+(14+20+10+16)j$

$= -157+53i+60j$

Result: pass

a · (b + c) = (a · b) + (a · c)

Application-2:

(2+3i+4j) x [(5+6i+7j) + (4+7i+5j)]

= (2+3i+4j) x [(5+4)+(6+7)i+(7+5)j]

= (2+3i+4j) x (9+13i+12j)

= 2(9+13i+12j)+3i(9+13i+12j)+4j(9+13i+12j)

= $18+26i+24j + 27i+39i^2+36ij + 36j+52ij+48j^2$

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

= $(18+39i^2+48j^2)+(26i+27i+36ij+52ij)+(24j+36j)$

= (18-39-48)+(26+27-36-52)i+(24+36)j

= -69-35i+60j

(2+3i+4j) (5+6i+7j) + (2+3i+4j) (4+7i+5j)

= 2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j) + 2(4+7i+5j)+3i(4+7i+5j)+4j(4+7i+5j)

= $(10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2) + (8+14i+10j + 12i+21i^2+15ij + 16j+28ij+20j^2)$

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

= $(10+18i^2+28j^2+8+21i^2+20j^2)+(12i+15i+21ij+24ij+14i+12i+15ij+28ij)+(14j+20j+10j+16j)$

= (10-18-28+8-21-20)+(12+15-21-24+14+12-15-28)i+(14+20+10+16)j

=-69-36i+60j

Result: pass

a · (b + c) = (a · b) + (a · c)

Application-3:

(2+3i+4j) x [(5+6i+7j) + (4+7i+5j)]

= (2+3i+4j) x [(5+4)+(6+7)i+(7+5)j]

= (2+3i+4j) x (9+13i+12j)

= 2(9+13i+12j)+3i(9+13i+12j)+4j(9+13i+12j)

= $18+26i+24j + 27i+39i^2+36ij + 36j+52ij+48j^2$

// Relative to Im_j, $(-1_{ij}) = -1$ (imaginary number)

= $(18+39i^2+48j^2)+(26i+27i)+(24j+36j+36ij+52ij)$

= (18-39-48)+(26+27)i+(24+36-36-52)j

= -69+53i-28j

(2+3i+4j) (5+6i+7j) + (2+3i+4j) (4+7i+5j)

= 2(5+6i+7j)+3i(5+6i+7j)+4j(5+6i+7j) + 2(4+7i+5j)+3i(4+7i+5j)+4j(4+7i+5j)

= $(10+12i+14j + 15i+18i^2+21ij + 20j+24ij+28j^2) + (8+14i+10j + 12i+21i^2+15ij + 16j+28ij+20j^2)$

// Relative to Im_i, $(-1_{ij}) = -1$ (imaginary number)

= $(10+18i^2+28j^2+8+21i^2+20j^2)+(12i+15i+14i+12i)+(14j+20j+10j+16j+21ij+24ij+15ij+28ij)$

= (10-18-28+8-21-20)+(12+15+14+12)i+(14+20+10+16-21-24-15-28)j

=-69+53i-28j

Result: pass

7.7 Limitations of Argand Diagram

With standard norms, Argand diagram is two dimensional with real axis and imaginary axis. (or x-axis and y-axis). But, Argand diagram shows four dimensions.

I am self-centered, therefore in my view,

(+) I; in the real (x) axis

(-) You; in the real (x) axis

(-)(-) They (your friend) = (+) Imaginary; in imaginary (y) axis

(-)(-)(-) They (your friend's friend) = (-) Imaginary; in imaginary (y) axis

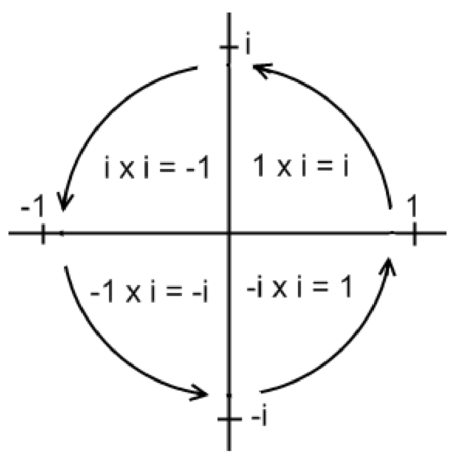
At first glance you will see a **circular pattern** in the Argand diagram.

$0^0 = (+1)$

$90^0 = (+i)$

$180^0 = (-1)$

$270^0 = (+i)$



One of the limitations of Argand diagram is, it can show only **two imaginary relationships**.

There is a **recursive (spiral)** pattern in the Argand diagram.

0^0 = (+1)

90^0 = (+i)

180^0 = (-1)

270^0 = (+i)

360^0 = (+1)

450^0 = (+i)

540^0 = (-1)

630^0 = (+i)

720^0 = (+1)

810^0 = (+i)

900^0 = (-1)

990^0 = (+i)

Argand diagram loses information (imaginary relationship hierarchy) when limiting between 0^0 and 359^0. It suggests a limitation in current mathematics.

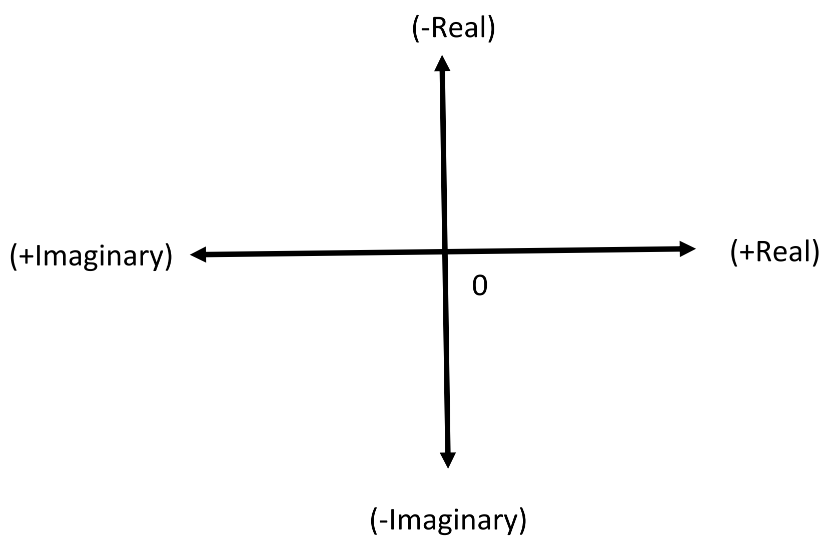
To overcome **recursive limitation** of Argand diagram, author suggests to use **multi-dimensional arrays** to present multiples of imaginary relationships.

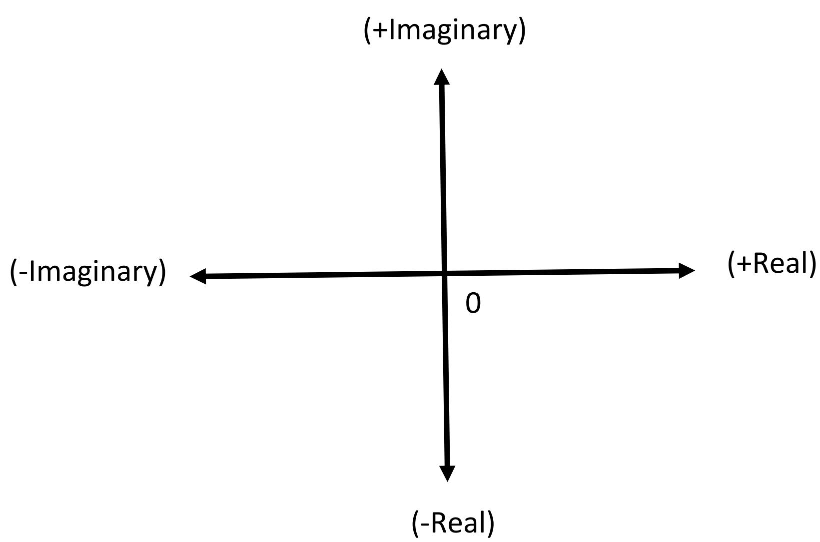
For example:

Array $Imaginary_1[5,3]$ // 5 columns, 3 rows

Another limitation of Argand diagram is, separate imaginary part and real part of complex numbers (a+bi) cannot do direct calculations, because imaginary axis is mutually exclusive (90^0) real axis. In contrary positive dimension and negative dimension of the real axis has direct calculations.

When necessary, author suggests to rearrange the four dimensions of Argand diagram to enable direct calculations between real numbers and imaginary numbers.

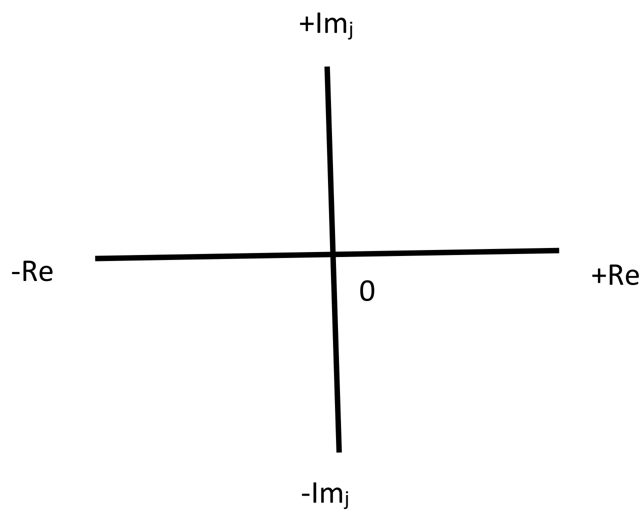




When calculating higher dimensions of Imaginary numbers, 2D Argand diagram can be used to directly interact with real axis.

Example:

In (a+bi+cj) X (x+yi+zj), calculation is at 3rd and 4th dimensions of imaginary numbers are considered.



2nd recursion (360^0 to 719^0 circle) of the 2D Argand diagram is similar to above.

These improvements in mathematics will benefit, by expanding its scope (boundary). Author recommends to conduct more research to confirm the benefits.

7.8 Fruit Basket Example – Imaginary Numbers

Suppose there is a fruit basket with apples, oranges, banana, pineapple, pears, grapes, etc.

If **I am an apple**, in my self-centered view,

I am (+) = **+real**

You, my friend **orange** is (-) = **-real**

Your friend **banana** is (-)(-) = **+imaginary**

Your friend's friend **pineapple** is (-)(-) (-) = **-imaginary**

Pears and **grapes** are friends, **not related** to me

Therefore, they are **unimaginary** to me

Fruits in a fruit basket are physical representations of real numbers, imaginary numbers and unimaginary numbers.

8. Conclusions

With reference to analysis, imaginary number applications in real world (calculation) are available. There is confusion in regard to the nature of imaginary numbers. No philosophical explanation of Imaginary numbers available using real word objects.

This research paper successfully fulfilled the knowledge gap, creating a historical landmark in mathematics. Now, imaginary numbers have sensible meaning. Mathematics will not be a blind belief system.

The physical representations of imaginary numbers are not imaginary, and they do exist in real world like apples, oranges and bananas. Imaginary numbers are referred with ‘imaginary’ term, because of **imaginary relationship**.

9. Recommendations

With the knowledge presented in this paper, new research can find **meaning** to imaginary/complex number **applications**. Research students can introduce **new calculation** methods for multidimensional imaginary numbers. It will help to solve the **unsolved** problems in mathematics like Riemann Hypothesis, which uses imaginary numbers. A complete system of relationships can be formulated and calculated using the fundamental knowledge discussed in this paper.

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