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On the minimum number of pairs that fulfill Goldbach's Conjecture

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Abstract
Based on numerical experiments for all even numbers less than 3.6×10^5, we propose a closed form expression, which is anticipated to estimate the total number of pairs of primes (p, q) that fulfill Goldbach's conjecture: $p + q = 2n$.

1. Introduction

On June 7, 1742, Christian Goldbach in a letter to Leonhard Euler [Gold 42] argued that "*every even natural number > 4 can be written as a sum of two primes*", namely:

$$2n = p + q \quad \text{where } n>2, \text{ and } p, q \text{ are prime numbers.} \tag{1-1}$$

It is reminded that the set of all primes is $\mathbb{P}=\{2,3,5,7,11,\ldots\}$; in other words, it does not include the unit.

Most of the classic works have been included in a collective volume by Wang [Wang00], in which except of those in the original papers, 234 additional citations that refer to the period 1901-2001 are included.

It is noteworthy that, in 2000, Eq(1-1) was verified using computers for even numbers up to 4×10^{16} [Rich00], and the attempt was repeated by T. Oliveira e Silva (http://www.ieeta.pt/~tos/goldbach.html) with the help of distributed computing network to $n \leq 1.609\times10^{18}$ and in selected areas up to 4×10^{18}. However, these checks do not constitute conclusive evidence of validity of Eq(1-1), and the effort continues today [Mitt10].

Interestingly, the object of the Goldbach conjecture has been the subject of statistical approach in the useful paper of Sheldon [Shel03].

Within this framework, the aim of this work is to recast Goldbach's conjecture and try to establish the minimum number of conjecture's validations for a given even number $2n$.

2. Experimental observations

In a previous work we have shown that, based on probabilities [Shel03], an average measure for the number of pairs (p,q) that fulfill Goldbach's conjecture is approximated by [Mark12]:

$$G(2n) \approx \frac{2n}{\left(\log(n)\right)^2} \tag{2-1}$$

In a second work [Prov12], we have shown that, if the true numbers of pairs are represented in the form of points on the [$2n - G(2n)$] plane, the resulting cloud of points is mostly above and less below the line described by (2-1), as clearly shown in **Figure 1**.

The question is to determine the lower bound of the above-mentioned cloud of points, which corresponds to the minimum possible number of pairs for a given even number $2n$.

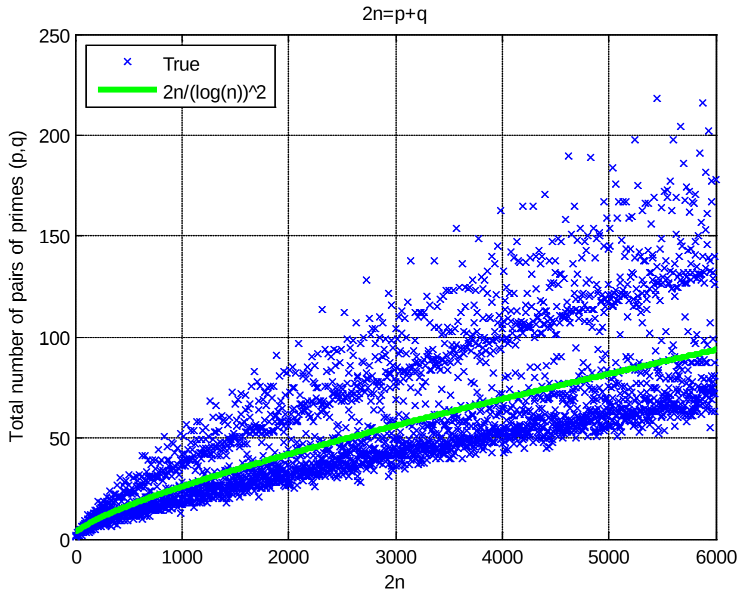

Figure 1: Total number of all pairs of primes (p,q), which correspond to even numbers $2n$ for $6 \leq 2n \leq 6000$.

Computer findings show that a draft estimation for the minimum number of pairs (p,q) is given by $p(2n) \approx 2/3G(2n)$. The aforementioned formula is rarely violated at only 462 even numbers out of the almost ½×285368 ones. Clearly, this violation is gradually vanishing until $2n$ = 285368. For even numbers greater than 285368, no violation has been observed yet. Results are shown in **Figure 2**.

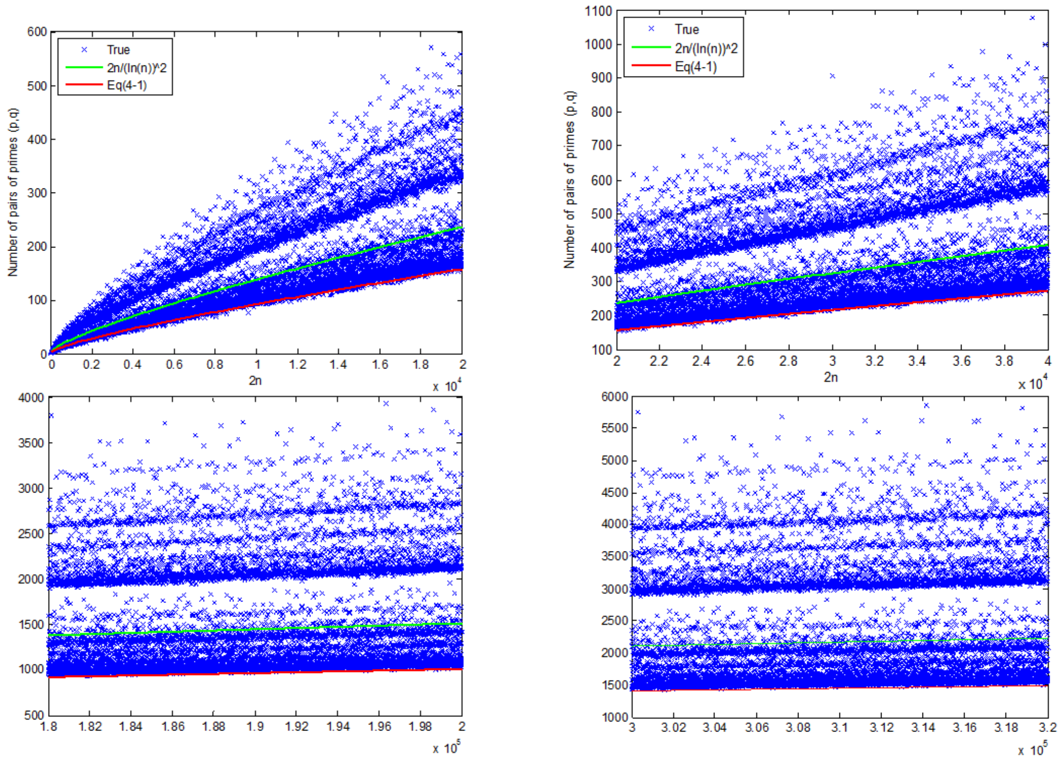

Figure 2: Cloud of points representing the total number of pairs (p,q) that fulfill Goldbach's conjecture compared with the red line representing $p(2n) \approx 2/3G(2n)$: x-axis corresponds to $2n$, while the green line represents Eq(2-1).

It is clarified that the bottom right subplot of **Figure 2** refers to the interval $2n \in [300000, 320000]$ in which no violation of the lower limit occurs.

Let us now elucidate what happens with the cloud of points. Particularly we choose the area in which the last of abovementioned violations occurs (i.e. for for 2*n* = 285368). Using the cells (6λ-2, 6λ, 6λ+2), in **Figure 3** we construct triangles (blue lines) of which the centroids are represented by connected red ones. The green line represents Eq(2-1), whereas the magenta line refers to $p(2n) = 4n/3(\ln(n))^2$. We notice that most of the centroids are above the green line, while the violation occurs for the right end of the corresponding cell: 2*n* = (285364, 285366, 285368).

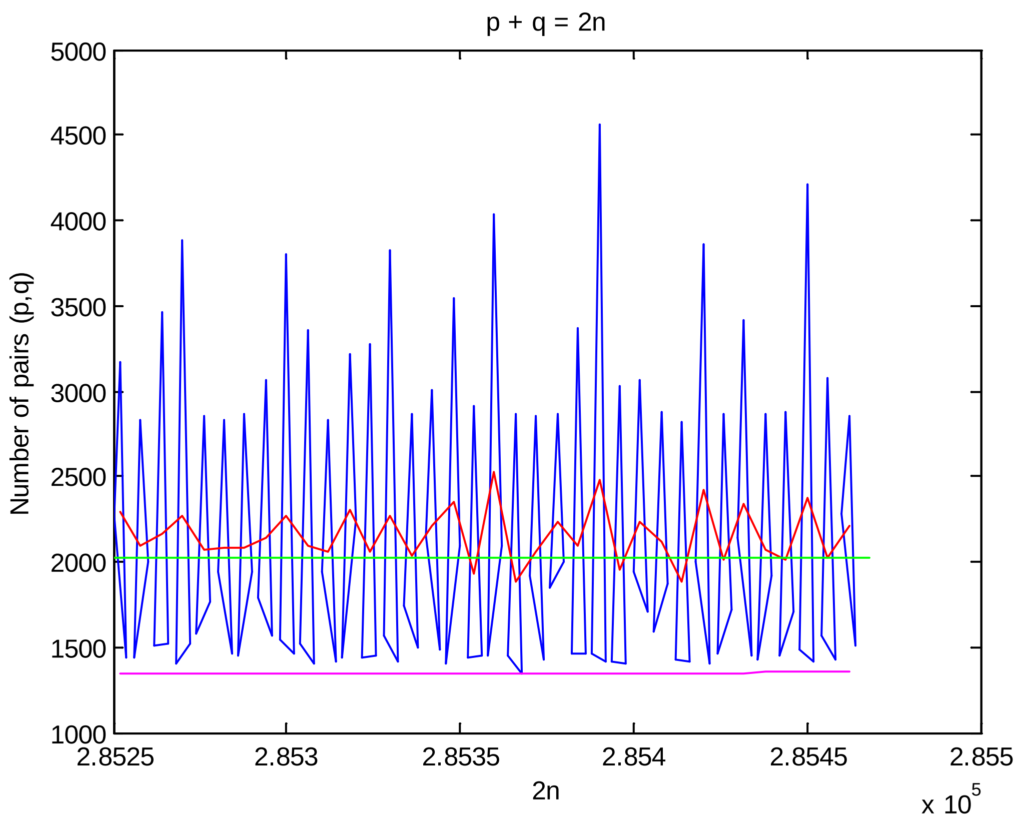

Figure 3: Number of pairs (*p*,*q*) that fulfill Goldbach's conjecture in the neighborhood of latest noticed violation.

The latest findings till today show that no violation occurs in the interval $2n \in [285368, 360000]$. It is anticipated that the proposed formula will be valid for even greater values of 2*n*.

3. Conclusion

Based on numerical experimentation, the main goal of this study was to determine a closed form that expresses the lower limit concerning the total number of pairs (*p*,*q*) of primes that fulfils Goldbach's conjecture, i.e. $2n = p + q$. The analysis was based on the initial finding that the spatial distribution of the pairs (*p*,*q*) is not arbitrary but there are local relationships, at least on the level of a basic cell (6λ-2, 6λ, 6λ+2) of successive even numbers 2*n*. The relationship

$p(2n) = \frac{4n}{3(\ln n)^2}$ has been validated until $2n$ = 360000. An empirical proof is under construction.

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