the general science journal

The Minimum Speed of a Free Massive Particle

Pavle I. Premović

Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, pavleipremovic@yahoo.com, Niš, Serbia

Abstract

The equations for the minimum speed and kinetic energy of a free massive particle are derived within the non-relativistic and special relativistic frameworks. These equations are based on de Broglie's relation between momentum and wavelength of this particle.

Keywords: Particle, de Broglie's wavelength, speed, kinetic energy, relativistic.

Introduction

The particle-wave duality of matter is one of the biggest mysteries in science because a massive particle and a wave are opposite each other in every way. Indeed, this particle is a discrete entity enclosed in a relatively minimal volume of space, while a wave propagates over a large region of space.

In modern physics, it is now widely accepted that light (or in more general terms, electromagnetic radiation) has a dual nature. The wave-like nature of light explains most of its properties: reflection, refraction, diffraction, interference and Doppler effect. On the other hand, the photoelectric effect and Compton effect can only be explained based on the particle-photon nature of light. The wavelength of this photon λ can be expressed as follows

$$\lambda = h/p \quad \dots (1)$$

where h (= 6.62×10^{-34} J sec^{-1}) is Planck's constant and p is the photon's momentum.

Elementary physics shows that the momentum of a free massive particle p is

$$p = m\upsilon$$

where m is the relativistic mass of this particle and υ is its speed relative to an observer at rest. De-Broglie postulated that a free massive particle shows the dualistic wave-particle character just like light and the eqn. (1) can be also applied to this particle

$$\lambda = h/m\upsilon \quad \dots (2).$$

In the first part of this paper, we will deal with a free[1] "non-relativistic" massive particle (from now on "non-relativistic" particle) and in the second part we will deal, in general, with a free massive particle (hereinafter free particle).

In general, the "non-relativistic" particles are those whose speed υ is far less than the speed of light or $\upsilon << c$ ($\approx 3\times10^8$ m sec^{-1}). Physicists usually assume that the massive particles with $\upsilon/c \leq 0.1$ (or $\upsilon \leq 0.1c$) are "non-relativistic". De Broglie's particle-wave duality has also been verified experimentally for the "non-relativistic" (e.g. electron or neutron diffraction) and relativistic (e.g. electron diffraction) massive particles.

Derivations and Discussion. Suppose now that the "non-relativistic" particle is spherical and has a diameter and a mass at rest: D_0 and m_0. De Broglie's wave requires a medium through which to spread and that is of course - matter. From analogy with light, we hypothesize that the maximum de Broglie's wavelength λ_{max} of a "non-relativistic particle" must be equal to its diameter.

Mathematically speaking

$$\lambda_{max} = D_0.$$

Applying the formula (2), we get

$$\lambda_{max} = h/m_0\upsilon_{min}$$

where υ_{min} is the minimum speed of a "non-relativistic" particle. Substituting λ_{max} of this equation with D_0 and after some rearrangement we get

$$\upsilon_{min} = h/m_0D_0 = \beta \quad \ldots (3).$$

According to this equation, for a massive "non-relativistic" particle to have a minimum velocity β equals zero its diameter must be infinitely large, which is, of course, impossible.

The minimum kinetic energy of a "non-relativistic" particle is

$$KE_{min} = 1/2(m_0\beta^2) \quad \ldots (4).$$

In sum, eqn. (3) appears also to set a lower limit for its speed, momentum and kinetic energy.

Examples:

(1) Alpha (α)-particle consists of two protons and two neutrons. The mass of the α-particle is about 6.65×10^{-27} kg and its diameter is about 3.6×10^{-15} m. Employing eqn. (3) we calculate that the minimum speed of this particle is about 2.8×10^7 m sec^{-1} (or about 0.093c). In other words, this is roughly the lowest possible speed of the free α-particle.

[1] It is a free particle in the sense that it is experiencing no net force.

The kinetic energy of the free α-particles emitted by the uranium isotopes (^{226}U - ^{238}U) ranges from about 4.2 MeV (= 6.7×10^{-13} J) up to about 7.6 MeV (= 12.1×10^{-13} J). Using the equation for the "non-relativistic" kinetic energy KE = $1/2(m\upsilon_\alpha^2)$ we estimated that the speed of their α-particles υ_α range is about 1.4×10^7 m sec^{-1} up to about 1.9×10^7 m sec^{-1}. These values agree in order of magnitude with the above rough estimate of the lowest possible speed of the free α-particle.

Alpha particles are relatively big and heavy and are not able to penetrate very far through a medium. As a result of scattering collisions with various nuclei of the medium through which the free α-particle passes, its kinetic energy is reduced and usually becomes a helium atom capturing two electrons from its surroundings.

(2) Macroscopic non-relativistic objects would have a minimum speed extremely low. For the golf ball with a mass of about 0.05 kg and a diameter of 0.05 m we estimate [using eqn. (3)] its minimum speed would be about 2.7×10^{-31} m. The average speed of a golf ball is about 50 m sec^{-1}.

In the next part of this paper, we will consider a relativistic particle and the consequences of that consideration in the case of a "non-relativistic" particle.

Strictly speaking, all free massive particles are relativistic. Even if their speed is much less than the speed of light (or $\upsilon << c$), they are still relativistic. So, there is only a relativistic free massive particle, or better to say a free massive particle, in general. This is why the term non-relativistic is put under quotation marks.

The special theory of relativity sets the light speed c as an upper limit to the speed of a massive particle. According to this theory, the mass of a relativistic particle

$$m = m_0/\sqrt{(1 - \upsilon^2/c^2)} \quad \dots (5).$$

However, Special relativity states that the above spherical "non-relativistic" particle traveling at a relativistic speed would contract in the direction of motion becoming the prolate spheroid-shaped relativistic particle, Fig 1. Its diameter at rest D_0 would be shortened in the direction of its motion, by the factor $(1- \upsilon^2/c^2)^{1/2}$.

In equation form,

$$L = D_0\sqrt{(1 - \upsilon^2/c^2)} \quad \dots (6)$$

where L is the length of a relativistic particle[2] (hereinafter length) along its direction of motion Fig. 1.

Multiplying eqns. (5) and (6) we have

$$mL = m_0D_0.$$

[2] In fact, its equatorial axis.

Eqn. (6) shows that in contrast to the previous non-relativistic particle whose diameter D_0 is assumed to be constant; the length of a relativistic particle L depends on the speed of this particle υ. Its wavelength λ cannot be larger than the length L. In equation form,

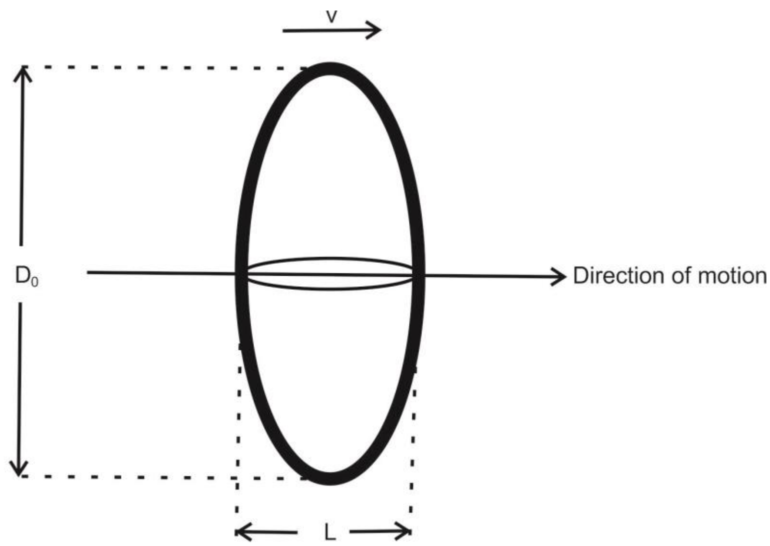

Fig. 1. The shape and dimensions of the free spherical particle at relativistic speed.

$$\lambda \leq L = D_0\sqrt{(1 - \upsilon^2/c^2)}.$$

Combining eqn. (2) and the left end of this expression we have

$$\upsilon \geq h/mL.$$

Substituting into this expression m_0D_0 instead mL we obtain

$$\upsilon \geq h/m_0D_0$$

having a minimum value

$$\upsilon_{min} = h/m_0D_0 = \beta.$$

So, its minimum kinetic energy

$$KE_{min} [= (1/2m\upsilon_{min}^2)] = 1/2(m_0\beta^2)$$

These equations are identical to eqn. (3) and eqn. (4) derived for υ_{min} and KE_{min} of a "non-relativistic" particle. In other words, our approach to the minimum speed and kinetic energy of "a non-relativistic" massive particle in the first part of this work sounds reasonable.

Conclusion. The equations for the minimum speed and kinetic energy of a free massive particle are derived. These equations are based on the non-relativistic and special relativistic formulations using de Broglie's relation between linear momentum and the wavelength of this particle.