Aberration of Electric Field and Radiation from Accelerated Charged Particle

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Abstract: A particle of charge K and mass m, moving at time t with velocity $\mathbf{v}$, in electric field of intensity $\mathbf{E}$ and magnitude E, is subject to aberration of electric field. Accelerating force is $\mathbf{F} = (KE/c)(\mathbf{c} - \mathbf{v}) = m(d\mathbf{v}/dt)$, where $\mathbf{c}$ is velocity of light, of magnitude c. Charge K is accelerated to speed of light c as limit, with constant mass and emission of radiation. Force $\mathbf{F}$ on moving charged particle, is less than force $K\mathbf{E}$ on stationary one, the difference $\mathbf{F} - K\mathbf{E}$ is radiation reaction force and radiation power is scalar product $-\mathbf{v}.(\mathbf{F} - K\mathbf{E})$. Aberration of electric field is missing in physics today, warranting special relativity and quantum theories to fill the vacuum. Speed of light c is obtained as a limit, for charged particles accelerated by an electric field, without recourse to special relativity, and radiation is obtained outside quantum mechanics.
Keywords: Aberration. Acceleration, Electric Charge. Energy. Field. Force. Mass. Special Relativity, Radiation, Velocity.

1. Introduction: A moving observer is subject to aberration of light, a universal phenomenon discovered in 1728 by astronomer, James Bradley. As electrical force is transmitted with velocity of light $\mathbf{c}$, particle of charge K, moving in electric field, is subject to aberration of electric field, a missing link in physics [1, 2]. Relativistic and quantum mechanics do not link the gaps well.

Figure 1 depicts aberration of electric field, for a particle of charge K at P, moving in time t with velocity $\mathbf{v}$, at angle θ to force of repulsion $\mathbf{F}$ of electric field of intensity $\mathbf{E}$, due to stationary particle of positive charge Q at O. The vector $(\mathbf{c} - \mathbf{v})$ is relative velocity between the electrical force, transmitted at velocity of light c and charge K. At $\mathbf{v} = \mathbf{c}$ the force cannot catch up and impact on the charge, making $\mathbf{c}$ the maximum velocity to which a charged particle may accelerated by an electric field.

In Figure 1, electric field appears to reach K with velocity $\mathbf{c}$ such that relative velocity $(\mathbf{c} - \mathbf{v})$ is in OP direction, line of action of force. Aberration angle α is between the vectors $\mathbf{c}$ and $(\mathbf{c} - \mathbf{v})$ as given by equation 1, sine rule, in triangle PRN. Equation (1) is like Bradley's formula for aberration of light. It is independent of separation OP, applicable at astronomical and atomic levels. Similar situation is in Figure 2, for an electron of charge $-e$ at P, moving at velocity $\mathbf{v}$ under attraction of charge Q.

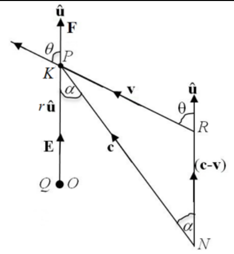

Figure 1: Aberration angle α due to particle of positive charge K and mass m_k moving with velocity $\mathbf{v}$, at angle θ to force of repulsion $\mathbf{F}$ due to electric field $\mathbf{E}$, of stationary charge Q at O.

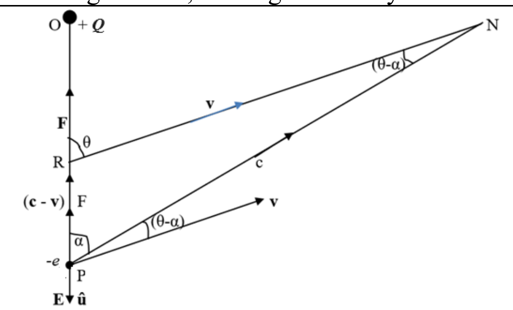

Figure 2: Aberration angle α due to electron of negative charge $-e$ and mass m_e moving with velocity $\mathbf{v}$, at angle θ to force of attraction $\mathbf{F}$ due to positive Q at O.

2. Accelerating Force: In figures 1 and 2, apparent displacement of charge Q, from OP to NP, in angle α, is aberration of electric field, such that relative velocity $(\mathbf{c} - \mathbf{v})$ between electrical force, at velocity of light $\mathbf{c}$, and charge K or $-e$ moving with velocity $\mathbf{v}$, at time t, is in the line joining the charges. Accelerating force $\mathbf{F}_1$ due to force of repulsion on charge K of mass m_k, a constant equal to rest mass m_o, is expressed in equation (2). For electron of negative charge $-e$ and mass m_e, a constant as rest mass, accelerating force $\mathbf{F}_2$, due to attraction, is in equation (3). Speed of light being a limit is implicit in equations (2) and (3).

$$\sin\alpha = (v/c)\sin\theta \quad (1)$$

$$\mathbf{F}_1 = (KE/c)(\mathbf{c} - \mathbf{v}) = m_k(d\mathbf{v}/dt) \quad (2)$$

$$\mathbf{F}_2 = (-eE/c)(\mathbf{c} - \mathbf{v}) = m_e(d\mathbf{v}/dt) \quad (3)$$

Equations (2) and (3), in terms of angles θ and α in Figures (1) and (2), are given in (4) and (5), respectively:

$$(KE/c)\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)}\,\hat{\mathbf{u}} = m_k(d\mathbf{v}/dt) \quad (4)$$

$$(-eE/c)\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)}\,\hat{\mathbf{u}} = m_e(d\mathbf{v}/dt) \quad (5)$$

where $(\theta - \alpha)$ is the angle between the vectors $\mathbf{c}$ and $\mathbf{v}$ in Figures 1 and 2. Equation (5) is also expressed in terms of θ and α, as:

$$\mathbf{F}_2 = (-eE/c)\sqrt{c^2 + v^2 - 2cv(\cos\theta\cos\alpha + \sin\theta\sin\alpha)}\,\hat{\mathbf{u}} = m_e d\mathbf{v}/dt = -m_e(v^2/r)\hat{\mathbf{u}} = -m_o(v^2/r)\hat{\mathbf{u}} \quad (6)$$

Equations (1) and (4) for $\theta = 0$ (acceleration) and $\theta = \pi$ (deceleration) give equations (7) and (8), respectively, as:

$$(KE/c)(c - v)\hat{\mathbf{u}} = m_k(dv/dt)\hat{\mathbf{u}} \quad (7)$$

$$(KE/c)(c + v)\hat{\mathbf{u}} = -m_k(dv/dt)\hat{\mathbf{u}} \quad (8)$$

3. Equations of Motion: Solutions of first order differential equations (7) and (8) are given in equations (9) and (10), in speed v at time t for a particle of charge K accelerated or decelerated, by an electric field E, from initial speed u, where $a = KE/c$:

$$v = c - (c - u)\exp(-at/c) \quad (9)$$

$$v = -c + (c + u)\exp(-at/c) \quad (10)$$

Equations (1) and (6) for $\theta = \pi/2$ (circular motion of radius r, speed v and acceleration $-(v^2/r)\hat{\mathbf{u}}$, at constant mass $m_e = m_o$, gives:

$$\mathbf{F}_2 = (-eE/c)\sqrt{c^2 - v^2}\,\hat{\mathbf{u}} = m_e d\mathbf{v}/dt = -m_e(v^2/r)\hat{\mathbf{u}} = -m_o(v^2/r)\hat{\mathbf{u}} \quad (11)$$

4. Mass-Velocity Formula: Equation (11) gives eE in equation (12) and relativistic mass-velocity formula as in equation (13).

$$eE = m_o(v^2/r)\Big/\sqrt{1 - \frac{v^2}{c^2}} = \gamma m(v^2/r) \quad (12)$$

$$m = m_o\Big/\sqrt{1 - \frac{v^2}{c^2}} = \gamma m_o \quad (13)$$

In equation (12), force is as a constant at eE while mass m varies as in mass-velocity formula (13), where γ is Lorentz factor.

5. Radiation Power: From equation (3), accelerating force on an electron of charge $-e$, moving with velocity $\mathbf{v}$, is more than the force $-e\mathbf{E}$ on a stationary one, the difference is radiation reaction force, something akin to frictional force $\mathbf{R}_f$, thus:

$$\mathbf{R}_f = -(eE/c)(\mathbf{c} - \mathbf{v}) + e\mathbf{E} \quad (14)$$

Radiation power, scalar product $-\mathbf{v}.\mathbf{R}_f$, with reference to angles θ and α in Figure 2, is obtained, outside quantum mechanics, as:

$$-\mathbf{v}.\mathbf{R}_f = -\mathbf{v}.\{(eE/c)(\mathbf{c} - \mathbf{v}) + e\mathbf{E}\} = eEv\{\cos\theta - \cos(\theta - \alpha) + (v/c)\}$$

In rectilinear motion with $\theta = 0$ or π radians, radiation power is eEv^2/c. In circular motion with $\theta = \pi/2$ radians, round a central force of attraction, radiation power is 0. Rutherford's nuclear model of hydrogen atom is without radiation and inherently stable.

6. Conclusion: It is not mass of a particle that increases with speed, in equation 13, but accelerating force which decreases with speed, reducing to zero at speed of light c, (equations 2 & 3), to make c the ultimate limit (equations 9 & 10). Speed c becomes a limit outside special relativity and radiation is obtained, from accelerated charged particles, outside quantum mechanics.

References: [1]. doi.org/10.33140/ATCP.04.04.06 [2]. doi.org/10.33140/ATCP.03.02.01