Did Maxwell Over-Speculate on the Physical Nature of Electromagnetic Waves?

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Abstract. Since James Clerk Maxwell wrote his Treatise in 1873, it has been generally believed that electromagnetic radiation consists of oscillating electric and magnetic fields, perpendicular to each other and in phase, and mutually perpendicular to the direction of propagation. It will now be investigated as to how exactly Maxwell arrived at this perhaps over-simplistic conclusion.

The Electromagnetic Momentum

I. The relevant analysis of the physical nature of electromagnetic waves begins in chapter XX of Maxwell’s 1873 paper “A Treatise on Electricity and Magnetism” [1]. In the Section 790, entitled ‘Plane Waves’, Maxwell begins by proposing the existence of a plane wave moving with a front that is normal to the z axis. He then recalls the equation of magnetic induction, \( \nabla \times \mathbf{A} = \mathbf{B} \), which had its origins in the sea of molecular vortices explained in his earlier 1861 paper “On Physical Lines of Force” [2]. Although Maxwell was no longer overtly promoting the sea of molecular vortices at this stage, the equation of magnetic induction still nevertheless implies a rotation axis, with the magnetic induction vector, \( \mathbf{B} \), being an axial vector representing the vorticity of a circulating electromagnetic momentum, denoted by \( \mathbf{A} \). The precise physical nature of the vector field, \( \mathbf{A} \), is often queried, but it must by all accounts broadly correspond with that of the physical nature of another important vector field known as the electric current density, and usually denoted by the symbol \( \mathbf{J} \). Meanwhile, it should be realized that the magnetic induction equation in itself does not predict any waves, but Maxwell has chosen this equation for the starting point in his analysis.
Magnetic and Electric Disturbances

II. It is not until further down Section 790 in Maxwell’s Treatise that the ingredients necessary to derive a wave equation are introduced. The purpose of initially introducing the magnetic induction equation, $\nabla \times \mathbf{A} = \mathbf{B}$, seems to have been to supply two of the vector fields that are involved in the undulations. Meanwhile Maxwell reduces the magnetic induction equation from a three-dimensional vortex equation, to equation (13), which is a one-dimensional projection, with $\mathbf{A}$ being dependent only on $z$. He does this in anticipation of the wave equation which he will be deriving further down, as he wants to focus on one plane only. After writing equation (13), Maxwell makes the conclusion that the magnetic disturbance is in the plane of the wave. It can be assumed that by the term magnetic disturbance, Maxwell is referring to the magnetic intensity, $\mathbf{H}$, sometimes referred to as the magnetic force, where $\mathbf{B} = \mu \mathbf{H}$. And we know through $\nabla \times \mathbf{A} = \mathbf{B}$, that $\mathbf{A}$ is perpendicular to $\mathbf{H}$.

At equation (14), Maxwell introduces the reciprocal curl equation in the form of Ampère’s Circuituial Law, $\mu \mathbf{J} = \nabla \times \mathbf{B} = -\nabla^2 \mathbf{A}$, similarly reduced to a one-dimensional projection in $z$, upon which he makes his second conclusion, this time that the electric disturbance is in the plane of the wave. By the term electric disturbance, Maxwell is referring to the electric current term, $\mathbf{J}$. So, we now have Maxwell concluding that two mutually perpendicular disturbances, $\mathbf{J}$ and $\mathbf{H}$, are both in the plane of the wave, while further down in Section 797, he seems to be suggesting that we should define the plane of polarization as a plane through the ray perpendicular to the plane of the electric disturbance. This would mean in a plane parallel to the $\mathbf{H}$ field. Maxwell therefore makes a distinction between the plane of the wave, the plane of the field disturbances, and the plane of polarization.

The Wave Equation in the Electromagnetic Momentum

III. Maxwell then goes into more detail about the electric disturbance. At equation (15) he writes $\mathbf{J} = \partial \mathbf{D}/\partial t$, where $\mathbf{D}$ is the electric displacement vector. The electric current, $\mathbf{J}$, in the context is Maxwell’s displacement current. At equation (16), he writes the electric elasticity equation, $\mathbf{D} = \varepsilon \mathbf{E}$, and at equation (17), he writes the Faraday time varying electromagnetic induction equation, $\mathbf{E}_K = -\partial \mathbf{A}/\partial t$.

Then comes an interesting surreptitious substitution (which is retrospectively vindicated). When Maxwell originally invented displacement current in Part III of his 1861 paper, [2], the electric force was based on Coulomb’s law of electrostatics and dielectric polarization. But Maxwell now combines equations (16) and (17) by replacing the electrostatic force, $\mathbf{E}_S$, with the electromagnetic force, $\mathbf{E}_K$. When he combines the result with equation (15),
this leads to equation (18), which becomes a simple harmonic motion equation in electric disturbance, providing that we equate \( \mathbf{A} \) with \( \mathbf{H} \). This suggests that the electromagnetic momentum, \( \mathbf{A} \), and Maxwell’s displacement current, \( \mathbf{J} = \partial \mathbf{D} / \partial t \), are one and the same thing. Maxwell then derives the EM wave equation in the form, \( \nabla^2 \mathbf{A} = \mu \varepsilon \partial^2 \mathbf{A} / \partial t^2 \), or at least the projection of it in \( z \), as is seen at equation (19). The overt identification of \( \mathbf{A} \) with \( \mathbf{J} \) was made in “Displacement Current and the Electrotonic State”[3].

The Sinusoidal Solutions

IV. Maxwell then presents sinusoidal solutions to the wave equation in \( \mathbf{A} \) at equation (20). But so far there has been no input information that might tell us the relative orientation between \( \mathbf{H} \) and \( \mathbf{E}_K \). We do know however from the equation of magnetic induction, \( \nabla \times \mathbf{A} = \mathbf{B} \), where \( \mathbf{B} = \mu \mathbf{H} \), that \( \mathbf{A} \) and \( \mathbf{H} \) are mutually perpendicular and in the same time phase, and likewise from Ampère’s Circuital Law, \( \nabla \times \mathbf{B} = \mu \mathbf{J} \), that \( \mathbf{J} \) is perpendicular to, and in the same time phase as \( \mathbf{H} \). We further know from Maxwell’s analysis that \( \mathbf{J} \) and \( \mathbf{H} \) are identified with the electric disturbance and the magnetic disturbance respectively, while it should now be reasonably clear from Maxwell’s derivation of the EM wave equation in \( \mathbf{A} \), that electric current, \( \mathbf{J} \), and electromagnetic momentum, \( \mathbf{A} \), are one and the same thing. Then we come to Fig. 66. As regards the relative phase and the relative orientation of the two disturbances, the figure would therefore be fine if it represented \( \mathbf{J} \) and \( \mathbf{H} \), or \( \mathbf{A} \) and \( \mathbf{H} \), although this knowledge would be based on Ampère’s Circuital Law and the magnetic induction equation, and nothing to do with any sinusoidal solutions.

The labelling, however, suggests that Fig.66 in fact represents \( \mathbf{D} \) and \( \mathbf{H} \), because in the figure, Maxwell is now using the term electric displacement which would normally apply to the electric displacement vector \( \mathbf{D} \), rather than using the term electric disturbance which would apply to \( \mathbf{A} \) or \( \mathbf{J} \) in the context of his analysis. So, did Maxwell mean \( \mathbf{D} \) or \( \mathbf{J} \) for the electric displacement in Fig. 66? In the script to the right of Fig.66, Maxwell expressly states that the electric force, \( \mathbf{E}_K = -\partial \mathbf{A} / \partial t \), is in a plane perpendicular to the magnetic force \( \mathbf{H} \), and so this would suggest that he really did mean \( \mathbf{D} \) in the figure, although he gives absolutely no account of how he concluded that \( \mathbf{D} \) and \( \mathbf{E}_K \) would be perpendicular to \( \mathbf{H} \), or why he believed that these would be in phase with \( \mathbf{H} \), or indeed how he concluded that \( \mathbf{E}_K \) and \( \mathbf{H} \) would be mutually perpendicular to the direction of propagation. It seems that this has all just been assumed by Maxwell, possibly in part because he has conflated \( \mathbf{J} \) and \( \mathbf{D} \). Furthermore, from the relationship, \( \mathbf{J} = \partial \mathbf{D} / \partial t \), in the context of a sinusoidal wave function in time, it is clear that \( \mathbf{J} \) is at its maximum when \( \mathbf{D} \) is at its minimum, and hence that \( \mathbf{J} \) is out phase with \( \mathbf{D} \) by 90 degrees. Therefore, since \( \mathbf{D} \) and \( \mathbf{E}_K \) are in phase, and
since $\mathbf{J}$ and $\mathbf{H}$ are in phase, then $\mathbf{E}_K$ and $\mathbf{H}$ must be out of phase by 90 degrees. So, if Maxwell really does mean *electric displacement*, $\mathbf{D}$, in Fig. 66, as it seems he does, then he has got it wrong on two counts, and these mistakes have been carried into the textbooks. Maxwell was wrong as regards the phase difference between $\mathbf{E}_K$ and $\mathbf{H}$, and it is also being proposed here that he was wrong as regards the presumed mutual perpendicular orientation as between $\mathbf{E}_K$, $\mathbf{H}$, and the direction of propagation $z$.

And so, based on Maxwell’s sinusoidal solutions to the electromagnetic wave equation, we are still left no wiser as to the relative orientation as between $\mathbf{E}_K$ and $\mathbf{H}$, or indeed the relative orientation as between $\mathbf{A}$ or $\mathbf{J}$ and the propagation direction $z$. We have established from Maxwell’s equations that $\mathbf{J}$ and $\mathbf{H}$ are in phase with each other, and that $\mathbf{E}_K$ is out of phase with both by 90 degrees. While we have established that Maxwell’s equations can be used to derive a wave equation in $\mathbf{A}$, and hence by extension in $\mathbf{E}_K$ and $\mathbf{H}$ too, the sinusoidal solutions nevertheless tell us nothing about the relative orientations of the various electromagnetic field vectors.

**Conclusion**

V. In his 1873 Treatise [1], Maxwell derived an electromagnetic wave equation in the vector field, $\mathbf{A}$, which denotes the *electromagnetic momentum*. However, it seems that he was over ambitious in his subsequent propositions regarding what this wave equation could tell us about the relative orientations of the various electromagnetic field functions involved in the wave. Likewise, he made similar over ambitious conclusions regarding the first electromagnetic wave equation which he derived for the magnetic intensity, $\mathbf{H}$, in his 1864 paper “*A Dynamical Theory of the Electromagnetic Field*” [4].

Based on the sinusoidal solutions, we do not actually know what the relative orientation of the $\mathbf{A}$, $\mathbf{E}_K$, and $\mathbf{H}$ fields are with respect to the propagation direction. It has been suggested in “*Wireless Telegraphy Beyond the Near Magnetic Field*” [5], that these three vectors apply on the picoscopic scale in connection with Maxwell’s molecular vortices, and that these vortices undergo a complete 180 degrees tumble (precession) during the transit of an electromagnetic wave pulse, with the precessional axis of the $\mathbf{H}$ field being perpendicular to the direction of propagation, and the direction of propagation being in the plane of the precessing $\mathbf{H}$ vector. And this will be so no matter what the relative orientation is as between the direction of propagation and the prevailing background magnetic field lines. It is additionally explained how the act of precessing is crucial to the energy flow process. Meanwhile, the orientation of $\mathbf{E}_K$ relative to $\mathbf{H}$ depends on the angle between the direction of propagation and the prevailing background magnetic field lines.
In the 1937 Encyclopaedia Britannica, an article entitled “Ether (in physics)” [6] appeared, written by Sir Oliver Lodge. Lodge states, in relation to the speed of light, that,

“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves—i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”.

It is unclear in the article as to who was promoting this idea, but the idea seems to have been shelved after the second world war. It difficult however to believe that whoever wrote it was not on the right tracks. It would be interesting to find out whose idea this was. Indeed, according to Tesla, mankind knew long ago that space was rendered into tiny whirls. In an unpublished article written in 1907 [7], Tesla states,

“Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance”.

And in the 1910 E.T. Whittaker chronology of aether history [8], it reads,

“All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools.

Returning to Section 790 in Maxwell’s 1873 Treatise and reading on beyond Fig.66, Maxwell talks about a ray of plane polarized light containing both magnetic disturbances and electric disturbances that are mutually perpendicular to each other and he questions whether the plane of polarization is parallel to the magnetic disturbance or to the electric disturbance. Whether by the term electric disturbance, he is referring to J or D, some people have difficulty with the concept of two mutually perpendicular vectors occupying the
same plane. This problem is however swiftly solved once we realize that these vectors refer to mechanical parameters in tiny electric dipolar fluid vortices that fill all of space [9]. The vector fields, \( \mathbf{A} \), \( \mathbf{D} \), \( \mathbf{E}_K \), and \( \mathbf{H} \), in radio waves are not on the scale of the actual radio wave amplitudes or wavelengths. They are on the picoscosopic scale and cannot therefore be detected in the manner of a laboratory-scale magnetic field.

And so, it is concluded that electromagnetic radiation is a fine-grained vortex-gyroscopic process that cannot be accurately described in terms of being exclusively transverse or longitudinal. While \( \mathbf{E}_K \) and \( \mathbf{H} \) are out of phase in time by ninety degrees in a wireless electromagnetic wave, there is no fixed orientation as between them. The situation is different however in a DC transmission line pulse where the relative orientation between the electrostatic \( \mathbf{E}_S \) field and the magnetic \( \mathbf{H} \) field are mutually perpendicular and in phase, as well as being measurable on the laboratory scale [10].

References

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https://www.equipes.lps.u-psud.fr/Montambaux/histoire-physique/Maxwell-2.pdf


https://www.researchgate.net/publication/303112543_Displacement_Current_and_the_Electrotonic_State

The derivation of the electromagnetic wave equation in \( \mathbf{H} \) begins on page 497 in the first link. Then see the note at the top of page 499 in the second link.

https://www.researchgate.net/publication/335169091_Wireless_Radiation_Beyond_the_Near_Magnetic_Field

The quote below is in relation to the speed of light,
“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves—i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”

http://gsjournal.net/Science-Journals/Historical%20Papers/Mechanics%20/20/Electrodynamics/Download/4105

[7] O’Neill, John J., “PRODIGAL GENIUS, Biography of Nikola Tesla”, Long Island, New York, 15th July 1944, quoting Tesla from his 1907 paper “Man’s Greatest Achievement” which was published in 1930 in the Milwaukee Sentinel,

“Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance”.
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“All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools. It will be seen that Bernoulli is a thorough Cartesian in spirit; not only does he reject action at a distance, but he insists that even the elasticity of his aether shall be explicable in terms of matter and motion. This aggregate of small vortices, or "fine-grained turbulent motion," as it came to be called a century and a half later,* is interspersed with solid corpuscles, whose dimensions are small compared with their distances apart. These are pushed about by the whirlpools whenever the aether is disturbed, but never travel far from their original positions. A source of light communicates to its surroundings a disturbance which condenses the nearest whirlpools; these by their condensation displace the contiguous corpuscles from their equilibrium position; and these in turn produce condensations in the whirlpools next beyond them, so that vibrations are propagated in every direction from the luminous point. It is curious that Bernoulli speaks of these vibrations as longitudinal, and actually contrasts them with those of a stretched cord, which, "when it is slightly displaced from its rectilinear form, and then let go, performs transverse vibrations in a direction at right angles to the direction of the cord." When it is remembered that the objection to longitudinal vibrations, on the score of polarization, had already been clearly stated by Newton, and that Bernoulli's aether closely resembles that which Maxwell invented in 1861-2 for the express purpose of securing transversality of vibration, one feels that perhaps no man ever so narrowly missed a great discovery. Bernoulli explained refraction by combining these ideas with those of his father. Within the pores of ponderable bodies
the whirlpools are compressed, so the centrifugal force must vary in intensity from one medium to another. Thus a corpuscle situated in the interface between two media is acted on by a greater elastic force from one medium than from the other; and by applying the triangle of forces to find the conditions of its equilibrium, the law of Snell and Descartes may be obtained. * Cf. Lord Kelvin's vortex-sponge aether, described later in this work.”
