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Reflection of light as a mechanical phenomenon applied to the Michelson interferometer-Rev1

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Abstract. Considering that the speed of light has the same constant speed *c* before and after reflection, Michelson and Morley predicted a fringe shift of $4.00\text{E}-01$ for their experiment. Considering the reflection of light as a mechanical phenomenon, the speed of light is different before and after reflection, and the theoretical fringe shift is $2.00\text{E}-05$. This magnitude of the fringe shift is not experimentally observable. Therefore, the reflection of light as a mechanical phenomenon explains the null result of the Michelson–Morley experiment.

Keywords: geometrical optics; reflection of light; speed of light; interference of light; Michelson interferometer; Michelson–Morley experiment

1. Introduction

This study is a continuation of the article "Reflection of light as a mechanical phenomenon applied to a particular Michelson interferometer" [1].

The derivation of the light paths and fringe shift within the Michelson interferometer is presented here for two hypotheses: the speed of light is the same before and after reflection as was considered in the Michelson–Morley experiment [2], and the reflection of light is viewed from a mechanical perspective according to which the speed of light is different before and after reflection [1].

Sections 2, 3, 4, and 5 present the derivation of the fringe shift for four positions of the interferometer rotated in steps of 90° applicable to both hypotheses. Section 6 offers the numerical calculation of the fringe shift for these hypotheses.

As per the notations used in this study, points marked by a letter without an index are points seen by an observer in an inertial frame. Points marked by a letter with an index are instances of points from the inertial frame seen by an observer in the fixed frame. Points with the same index belong to the same instance, not necessarily in time-sequential order.

2. Derivation of the light paths with the initial position of the interferometer

Figure 1 illustrates the initial position of the interferometer when the velocity of the inertial frame v and the velocity of light from the source c have opposite directions.

The transmitted rays from the source travel through the beam splitter M to mirror M_1 and the reflected rays from the source are directed by beam splitter M to mirror M_2; both rays travel back to M where they interfere.

The beam splitter M makes an angle of $45° - e/2$ with the direction of the velocity v; therefore, the reflected rays make the angle e with the vertical direction. Mirror M_1 makes the angle e with the vertical direction; therefore, the transmitted and reflected rays have the same direction along the interference path to the screen.

From the multitude of the transmitted and reflected rays, there is one pair of rays that continuously intercepts at a point A of M; this is true for all points on M. The pair of the transmitted ray in red and the reflected ray in blue interferes at point A.

The lengths of the interferometer arms AB and AC are equal to L.

The derivation of the light paths starts when the wavefront of light from the source is at line D_1E_1.

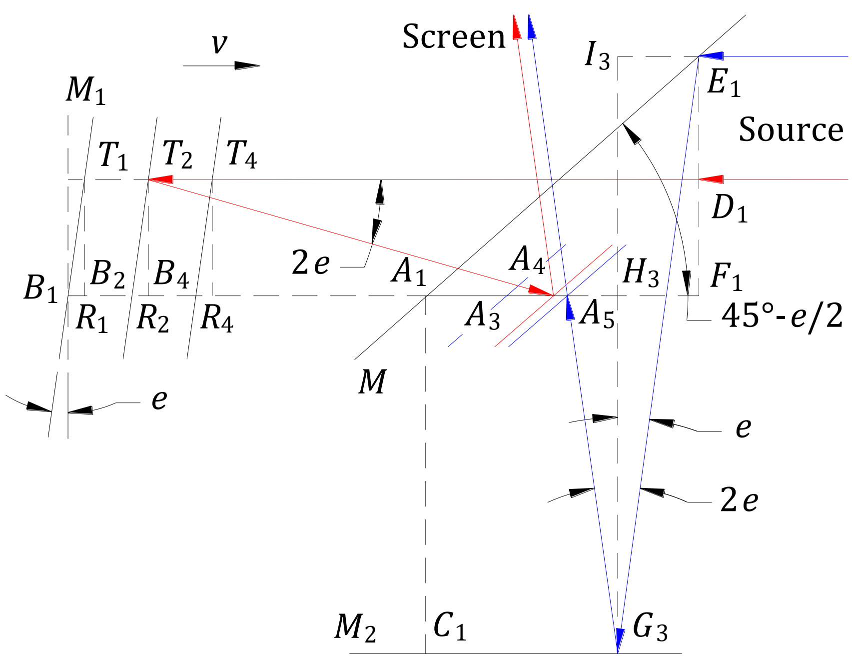

Figure 1. Light path geometry with the initial position of the interferometer.

In time t_{21}, the reflected ray travels from point E_1 to point G_3 with the speed c_{21}, and M travels the distance $A_1A_3 = vt_{21}$.

In triangle $E_1G_3I_3$, $E_1G_3 = c_{21}t_{21}$, $G_3I_3 = c_{21}t_{21}\cos e$, and $E_1I_3 = c_{21}t_{21}\sin e \Rightarrow$

$$t_{21} = \frac{G_3I_3}{c_{21}\cos e} = \frac{G_3H_3 + H_3I_3}{c_{21}\cos e} = \frac{L + E_1F_1}{c_{21}\cos e}.$$

In time t_{22}, the reflected ray travels from point G_3 to point A_5 with the speed c_{22}, and M travels the distance $A_3A_5 = vt_{22}$.

In triangle $A_5G_3H_3$, $A_5G_3 = c_{22}t_{22}$, $G_3H_3 = c_{22}t_{22}\cos e$, and $A_5H_3 = c_{22}t_{22}\sin e$ $\Rightarrow$

$$t_{22} = \frac{G_3H_3}{c_{22}\cos e} = \frac{L}{c_{22}\cos e}.$$

$F_1H_3 = E_1I_3 = c_{21}t_{21}\sin e$.

$A_1F_1 = A_1A_3 + F_1H_3 + A_3A_5 + A_5H_3 = vt_{21} + c_{21}t_{21}\sin e + vt_{22} + c_{22}t_{22}\sin e$.

From triangle $A_1E_1F_1$, $E_1F_1 = A_1F_1\tan(\pi/4 - e/2) = (vt_{21} + c_{21}t_{21}\sin e + vt_{22} + c_{22}t_{22}\sin e)\tan(\pi/4 - e/2)$.

Substituting the formula of E_1F_1 in the formula of time t_{21} $\Rightarrow$

$$t_{21} = \frac{L + E_1F_1}{c_{21}\cos e} = \frac{L + (vt_{21} + c_{21}t_{21}\sin e + vt_{22} + c_{22}t_{22}\sin e)\tan(\pi/4 - e/2)}{c_{21}\cos e} \quad \Rightarrow$$

$$t_{21} = \frac{L + (v + c_{22}\sin e)\,t_{22}\tan(\pi/4 - e/2)}{c_{21}\cos e - (v + c_{21}\sin e)\tan(\pi/4 - e/2)}.$$

Time $t_2 = t_{21} + t_{22}$ can be calculated, and also the distance $A_1F_1 = (v + c_{21}\sin e)t_{21} + (v + c_{22}\sin e)t_{22}$.

In time t_{12}, the transmitted ray travels from point T_2 to point A_4 with the speed c_{12}, and M_1 travels the distance $B_2B_4 = R_2R_4 = T_2T_4 = vt_{12}$.

In triangle $A_4R_2T_2$, $A_4T_2 = c_{12}t_{12}$, $A_4R_2 = c_{12}t_{12}\cos 2e$, and $R_2T_2 = A_4R_2\tan 2e$ $\Rightarrow$

$$t_{12} = \frac{A_4R_2}{c_{12}\cos 2e}.$$

From triangle $B_2R_2T_2$, $R_2T_2 = B_2R_2/\tan e$.

The two equalities of R_2T_2, $A_4R_2\tan 2e = B_2R_2/\tan e$, yield $B_2R_2 = A_4R_2\tan e\tan 2e$.

$A_4R_2 = A_4B_4 + B_2B_4 - B_2R_2 \;\Rightarrow\; c_{12}t_{12}\cos 2e = L + vt_{12} - B_2R_2 \;\Rightarrow$

$$t_{12} = \frac{L - B_2R_2}{c_{12}\cos 2e - v} = \frac{L - A_4R_2\tan e\tan 2e}{c_{12}\cos 2e - v}.$$

The equality of the two formulas for time t_{12} yields the distance A_4R_2.

$$\frac{A_4R_2}{c_{12}\cos 2e} = \frac{L - A_4R_2\tan e\tan 2e}{c_{12}\cos 2e - v} \quad \Rightarrow$$

$$A_4R_2\left(\frac{1}{c_{12}\cos 2e} + \frac{\tan e\tan 2e}{c_{12}\cos 2e - v}\right) = \frac{L}{c_{12}\cos 2e - v}.$$

Distance A_4R_2 can be calculated, and then time t_{12} with one of the two formulas.

In time t_{11}, the transmitted ray travels from point D_1 to point T_2 with the speed c_{11}, and M_1 travels the distance $B_1B_2 = R_1R_2 = T_1T_2 = vt_{11}$.

$D_1T_2 = A_1F_1 + A_1B_1 - B_1B_2 - B_2R_2 \;\Rightarrow\; c_{11}t_{11} = A_1F_1 + L - vt_{11} - B_2R_2 \;\Rightarrow$

$$t_{11} = \frac{A_1F_1 + L - B_2R_2}{c_{11} + v} = \frac{A_1F_1 + L - A_4R_2\tan e\tan 2e}{c_{11} + v}.$$

Distances A_1F_1 and A_4R_2 are known. Thus, time t_{11} followed by time $t_1 = t_{11} + t_{12}$ can be calculated.

The difference of time $\Delta t_1 = t_2 - t_1$, the period of the light is $T = \lambda/c$, and the number of periods or wavelengths comprised in Δt_1 is $N_1 = \Delta t_1/T = c\Delta t_1/\lambda$.

If the speed of light is the same before and after reflection, speeds c_{11}, c_{12}, c_{21}, and c_{22} are equal to the constant speed c.

For the reflection of light as a mechanical phenomenon, speeds c_{11}, c_{12}, c_{21}, and c_{22} are given according to reference [1], by formula $c_{rf} = c_s + v\cos a + v\cos b$, where c_{rf} is the speed of the reflected ray and c_s is the speed of light from a source or a mirror. Both speeds are in the fixed frame. Angle a corresponds to the opposite direction of the incident ray, and angle b to the direction of the reflected ray. The direction of angles a and b are outward in space from the point of collision. Angles a and b are measured counterclockwise from the direction of the velocity vector v with its origin at the point of collision.

Because the transmitted ray comes directly from the light source, the ray enters the interferometer with the speed $c_{11} = c_s = c$.

The transmitted ray from the source is reflected at point T_2. Consider the velocity vector v with its origin at T_2. Angle a, measured counterclockwise from the direction of the velocity vector v to the opposite direction of the incident ray, is 0°. Angle b, measured counterclockwise from the direction of the velocity vector v to the reflected ray direction, is $360° - 2e$. The speed of the reflected ray in the fixed frame $c_{12} = c_s + v\cos a + v\cos b = c_{11} + v\cos 0° + v\cos(360° - 2e) = c + v + v\cos 2e$.

The ray from T_2 is reflected at A_4 along the path of interference with a speed c_{13}. Applying the same reasoning at A_4 as at T_2, the speed of the reflected ray in the fixed frame $c_{13} = c_s + v\cos a + v\cos b = c_{12} + v\cos(180° - 2e) + v\cos(90° + e) = (c + v + v\cos 2e) - v\cos 2e - v\sin e = c + v - v\sin e$.

The reflected ray from source enters the interferometer after it is redirected at E_1. Applying the same reasoning at E_1 as at T_2, the speed of the reflected ray in the fixed frame $c_{21} = c_s + v\cos a + v\cos b = c + v\cos 0° + v\cos(270° - e) = c + v - v\sin e$.

The ray from E_1 is reflected at G_3 towards A_5 and along the path of interference. Applying the same reasoning at G_3 as at T_2, the speed of the reflected ray in the fixed frame $c_{22} = c_s + v\cos a + v\cos b = c_{21} + v\cos(90° - e) + v\cos(90° + e) = (c + v - v\sin e) + v\sin e - v\sin e = c + v - v\sin e = c_{21}$.

Speeds c_{22} and c_{13} are equal along the path of interference and the interference can take place.

The speed and the wavelength of the interfering rays change accordingly such that the period of light is the same constant $T = \lambda/c$, for any angle e, applicable for both hypotheses.

3. Derivation of the light paths with the interferometer rotated 90° from its initial position

Figure 2 depicts the light paths of the interferometer rotated 90° counterclockwise from the initial position shown in Figure 1.

The derivation of the light paths starts when the wavefront of light from the source is at line D_1E_1.

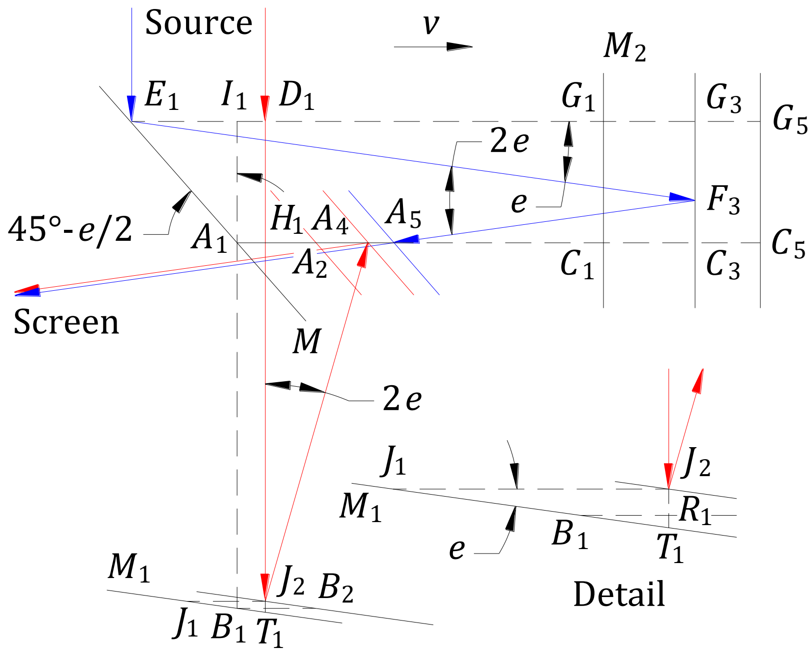

Figure 2. Light path geometry when the interferometer is rotated by 90° from its initial position.

In time t_{21}, the reflected ray travels from point E_1 to point F_3 with the speed c_{21}, and M_2 travels the distance $C_1C_3 = G_1G_3 = vt_{21}$.

In triangle $E_1F_3G_3$, $E_1F_3 = c_{21}t_{21}$, $E_1G_3 = c_{21}t_{21}\cos e$, and $F_3G_3 = c_{21}t_{21}\sin e \quad \Rightarrow$

$$t_{21} = \frac{F_3G_3}{c_{21}\sin e}.$$

$$E_1G_3 = E_1I_1 + I_1G_1 + G_1G_3 \quad \Rightarrow \quad c_{21}t_{21}\cos e = E_1I_1 + L + vt_{21} \quad \Rightarrow$$

$$t_{21} = \frac{L + E_1I_1}{c_{21}\cos e - v}.$$

The equality of the two formulas for time t_{21} yields the distance F_3G_3.

$$\frac{F_3G_3}{c_{21}\sin e} = \frac{L + E_1I_1}{c_{21}\cos e - v} \quad \Rightarrow \quad F_3G_3 = \frac{(L + E_1I_1)c_{21}\sin e}{c_{21}\cos e - v}.$$

In time t_{22}, the reflected ray travels from point F_3 to point A_5 with the speed c_{22}, and M_2 travels the distance $C_3C_5 = G_3G_5 = vt_{22}$.

In triangle $A_5C_3F_3$, $A_5F_3 = c_{22}t_{22}$, $A_5C_3 = c_{22}t_{22}\cos e$, and $C_3F_3 = c_{22}t_{22}\sin e \quad \Rightarrow$

$$t_{22} = \frac{C_3F_3}{c_{22}\sin e}.$$

$$A_5C_5 = A_5C_3 + C_3C_5 \quad \Rightarrow \quad L = c_{22}t_{22}\cos e + vt_{22} \quad \Rightarrow \quad t_{22} = \frac{L}{c_{22}\cos e + v}.$$

The equality of the two formulas for time t_{22} yields the distance C_3F_3.

$$\frac{C_3F_3}{c_{22}\sin e} = \frac{L}{c_{22}\cos e + v} \quad \Rightarrow \quad C_3F_3 = \frac{Lc_{22}\,sin\,e}{c_{22}\cos e + v}.$$

Triangle $E_1A_1I_1$ gives $E_1I_1 = A_1I_1\tan(\pi/4 - e/2) = (F_3G_3 + C_3F_3)\tan(\pi/4 - e/2)$.

Substituting the formula of F_3G_3 and C_3F_3 in the formula of E_1I_1 $\Rightarrow$

$$E_1I_1 = \frac{(L + E_1I_1)c_{21}\sin e\tan(\pi/4 - e/2)}{c_{21}\cos e - v} + \frac{Lc_{22}\sin e\tan(\pi/4 - e/2)}{c_{22}\cos e + v} \quad \Rightarrow$$

$$E_1I_1\left(1 - \frac{c_{21}\sin e\tan(\pi/4 - e/2)}{c_{21}\cos e - v}\right) = \left(\frac{Lc_{21}\sin e}{c_{21}\cos e - v} + \frac{Lc_{22}\sin e}{c_{22}\cos e + v}\right)\tan(\pi/4 - e/2).$$

Distance E_1I_1 can be calculated, then time t_{21} followed by time $t_2 = t_{21} + t_{22}$. Distance $A_1I_1 = E_1I_1/\tan(\pi/4 - e/2)$ can also be calculated.

In time t_{11}, the transmitted ray travels from point D_1 to point J_2 with the speed c_{11}, M travels the distance $A_1A_2 = vt_{11}$, and M_1 travels the distance $J_1J_2 = vt_{11}$.

In time t_{12}, the transmitted ray travels from point J_2 to point A_4 with the speed c_{12}, and M travels the distance $A_2A_4 = vt_{12}$.

In triangle $A_4H_1J_2$, $A_4J_2 = c_{12}t_{12}$, $H_1J_2 = c_{12}t_{12}\cos 2e$, and $A_4H_1 = c_{12}t_{12}\sin 2e$ $\Rightarrow$

$$t_{12} = \frac{H_1J_2}{c_{12}\cos 2e} = \frac{H_1R_1 - J_2R_1}{c_{12}\cos 2e} = \frac{A_1B_1 - (J_2T_1 - R_1T_1)}{c_{12}\cos 2e} = \frac{L - J_2T_1 + R_1T_1}{c_{12}\cos 2e}.$$

In triangle $J_1J_2T_1$, $J_2T_1 = J_1J_2\tan e = vt_{11}\tan e$. In triangle $B_1R_1T_1$, $R_1T_1 = B_1R_1\tan e$.

The distance $B_1R_1 = A_1H_1 = A_1A_2 + A_2A_4 - A_4H_1$, then $R_1T_1 = B_1R_1\tan e = (A_1A_2 + A_2A_4 - A_4H_1)\tan e = (vt_{11} + vt_{12} - c_{12}t_{12}\sin 2e)\tan e$.

Substituting the formula of J_2T_1 and R_1T_1 in the formula of time t_{12} $\Rightarrow$

$$t_{12} = \frac{L - J_2T_1 + R_1T_1}{c_{12}\cos 2e} \quad \Rightarrow \quad t_{12} = \frac{L - vt_{11}\tan e + (vt_{11} + vt_{12} - c_{12}t_{12}\sin 2e)\tan e}{c_{12}\cos 2\mathrm{e}} \quad \Rightarrow$$

$$t_{12} = \frac{L}{c_{12}\cos 2e - (v - c_{12}\sin 2e)\tan e}.$$

$$D_1J_2 = D_1H_1 + H_1J_2 = A_1I_1 + (H_1R_1 - J_2R_1) = A_1I_1 + H_1R_1 - (J_2T_1 - R_1T_1) \quad \Rightarrow$$

$$c_{11}t_{11} = A_1I_1 + L - vt_{11}\tan e + (vt_{11} + vt_{12} - c_{12}t_{12}\sin 2e)\tan e \quad \Rightarrow$$

$$t_{11} = \frac{L + A_1I_1 + (v - c_{12}\sin 2e)t_{12}\tan e}{c_{11}}.$$

Distance A_1I_1 is known. Thus, time t_{11} followed by $t_1 = t_{11} + t_{12}$ can be calculated.

The difference of time $\Delta t_2 = t_2 - t_1$, and the number of wavelengths comprised in Δt_2 is $N_2 = c\Delta t_2/\lambda$.

By rotating the interferometer from the initial position to the 90° position, the fringe shift is $N_{12} = N_2 - N_1$.

If the speed of light is the same before and after reflection, speeds c_{11}, c_{12}, c_{21} and c_{22} are equal to the constant speed c. For the reflection of light as a mechanical phenomenon, these speeds are as follows:

$c_{11} = c$.

$c_{12} = c_{11} + v\cos 90° + v\cos(90° - 2e) = c + v\sin 2e$.

$c_{13} = c_{12} + v\cos(270° - 2e) + v\cos(180° + e) = (c + v\sin 2e) - v\sin 2e - v\cos e = c - v\cos e$.

$c_{21} = c + v\cos 90° + v\cos(360° - e) = c + v\cos e$.

$c_{22} = c_{21} + v\cos(180° - e) + v\cos(180° + e) = (c + v\cos e) - v\cos e - v\cos e = c - v\cos e = c_{13}$.

4. Derivation of the light paths with the interferometer rotated by 180° from its initial position

Figure 3 depicts the light paths of the interferometer rotated 180° counterclockwise from the initial position shown in Figure 1.

The derivation of the light paths starts when the wavefront of light from the source is at line D_1E_1.

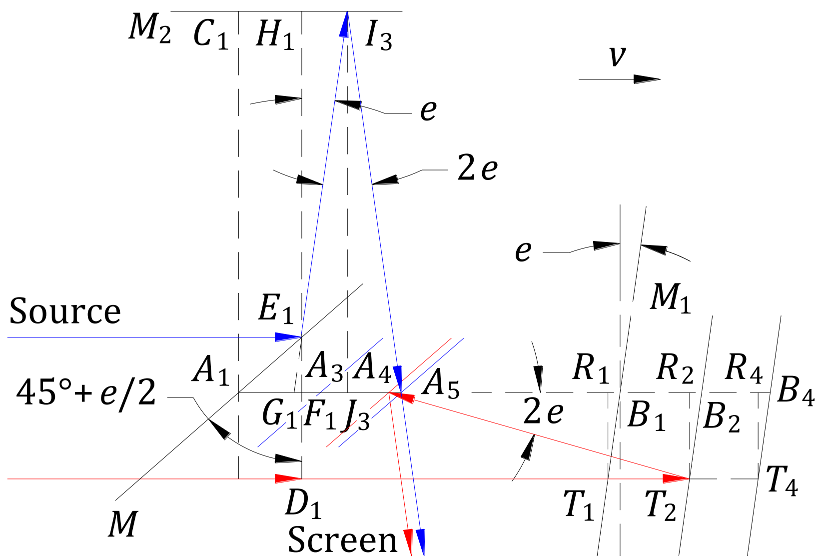

Figure 3. Light path geometry when the interferometer is rotated by 180° from its initial position.

The reflected ray travels from point E_1 to point I_3 with the speed c_{21}, in time t_{21}, and from point I_3 to point A_5 with the speed c_{22}, in time t_{22}.

$E_1H_1 = L - E_1F_1$. In triangle $E_1H_1I_3$, $E_1I_3 = c_{21}t_{21}$, and $E_1H_1 = c_{21}t_{21}\cos e \Rightarrow$

$$t_{21} = \frac{E_1H_1}{c_{21}\cos e} = \frac{L - E_1F_1}{c_{21}\cos e}.$$

In triangle $A_5I_3J_3$, $A_5I_3 = c_{22}t_{22}$, and $I_3J_3 = c_{22}t_{22}\cos e \Rightarrow$

$$t_{22} = \frac{I_3J_3}{c_{22}\cos e} = \frac{L}{c_{22}\cos e}.$$

$$t_2 = t_{21} + t_{21} = \frac{L - E_1F_1}{c_{21}\cos e} + \frac{L}{c_{22}\cos e}.$$

In triangle $A_1E_1F_1$, $A_1F_1 = E_1F_1\tan(\pi/4 + e/2)$. $A_5F_1 = (2L - E_1F_1)\tan e$.

Time t_2 can be calculated with following formula as well:

$$t_2 = \frac{A_1A_5}{v} = \frac{A_1F_1 + A_5F_1}{v} = \frac{E_1F_1\tan(\pi/4 + e/2) + (2L - E_1F_1)\tan e}{v}.$$

The equality of the two formulas for time t_2 yields the distance E_1F_1.

$$\frac{L - E_1F_1}{c_{21}\cos e} + \frac{L}{c_{22}\cos e} = \frac{E_1F_1\tan(\pi/4 + e/2) + (2L - E_1F_1)\tan e}{v} \quad \Rightarrow$$

$$\frac{E_1F_1}{c_{21}\cos e} + \frac{E_1F_1\tan(\pi/4 + e/2) - E_1F_1\tan e}{v} = \frac{L}{c_{21}\cos e} + \frac{L}{c_{22}\cos e} - \frac{2L\tan e}{v} \quad \Rightarrow$$

$$E_1F_1\left(\frac{1}{c_{21}\cos e} + \frac{\tan(\pi/4 + e/2) - \tan e}{v}\right) = L\left(\frac{1}{c_{21}\cos e} + \frac{1}{c_{22}\cos e} - \frac{2\tan e}{v}\right).$$

Distance E_1F_1 can be calculated, then time t_{21} followed by time $t_2 = t_{21} + t_{21}$. Distance $A_1F_1 = E_1F_1\tan(\pi/4 + e/2)$ can also be calculated.

In time t_{12}, the transmitted ray travels from point T_2 to point A_4 with the speed c_{12}, and the mirror M_1 travels the distance $B_2B_4 = R_2R_4 = T_2T_4 = vt_{12}$.

In triangle $A_4R_2T_2$, $A_4T_2 = c_{12}t_{12}$, $A_4R_2 = c_{12}t_{12}\cos 2e$, and $R_2T_2 = c_{12}t_{12}\sin 2e$.

In triangle $B_2R_2T_2$, $B_2R_2 = R_2T_2\tan e = c_{12}t_{12}\sin 2e\tan e$.

$A_4B_4 = A_4R_2 + B_2R_2 + B_2B_4 \;\Rightarrow\; L = c_{12}t_{12}\cos 2e + c_{12}t_{12}\sin 2e\tan e + vt_{12} \;\Rightarrow$

$$t_{12} = \frac{L}{c_{12}\cos 2e + c_{12}\sin 2e\tan e + v}.$$

In time t_{11}, the transmitted ray travels from point D_1 to point T_2 with the speed c_{11}, and M_1 travels the distance $B_1B_2 = R_1R_2 = T_1T_2 = vt_{11}$.

$D_1T_2 = F_1R_2 = A_1R_2 - A_1F_1 = A_1B_1 - B_1R_1 + R_1R_2 - A_1F_1 = A_1B_1 - B_2R_2 + R_1R_2 - A_1F_1 \;\Rightarrow\; c_{11}t_{11} = L - c_{12}t_{12}\sin 2e\tan e + vt_{11} - A_1F_1 \;\Rightarrow$

$$t_{11} = \frac{L - c_{12}t_{12}\sin 2e\tan e - A_1F_1}{c_{11} - v}.$$

Distance A_1F_1 is known. Thus, time t_{11} can be calculated and then time $t_1 = t_{11} + t_{12}$.

The difference of time $\Delta t_3 = t_2 - t_1$, and the number of wavelengths comprised in Δt_3 is $N_3 = c\Delta t_3/\lambda$.

By rotating the interferometer from the 90° position to the 180° position, the fringe shift is $N_{23} = N_3 - N_2$.

If the speed of light is the same before and after reflection, speeds c_{11}, c_{12}, c_{21}, and c_{22} are equal to the constant speed c. For the reflection of light as a mechanical phenomenon, these speeds are as follows:

$c_{11} = c$.

$c_{12} = c_{11} + v\cos 180° + v\cos(180° - 2e) = c - v - v\cos 2e$.

$c_{13} = c_{12} + v\cos(360° - 2e) + v\cos(270° + e) = (c - v - v\cos 2e) + v\cos 2e + v\sin e = c - v + v\sin e$.

$c_{21} = c + v\cos 180° + v\cos(90° - e) = c - v + v\sin e$.

$c_{22} = c_{21} + v\cos(270° - e) + v\cos(270° + e) = (c - v + v\sin e) - v\sin e + v\sin e = c - v + v\sin e = c_{21} = c_{13}$.

5. Derivation of the light paths with the interferometer rotated by 270° from its initial position

Figure 4 depicts the light paths of the interferometer rotated 270° counterclockwise from the initial position of Figure 1.

The derivation of the light paths starts when the wavefront of light from the source is at line D_1E_1.

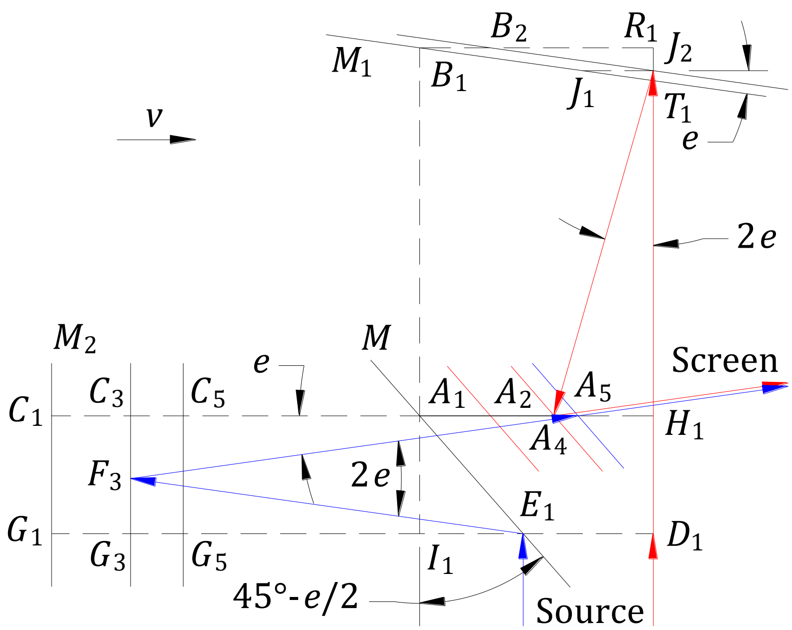

Figure 4. Light path geometry when the interferometer is rotated by 270° from its initial position.

In time t_{21}, the reflected ray travels from point E_1 to point F_3 with the speed c_{21}, and M_2 travels the distance $C_1C_3 = G_1G_3 = vt_{21}$.

In triangle $E_1F_3G_3$, $E_1F_3 = c_{21}t_{21}$, $E_1G_3 = c_{21}t_{21}\cos e$, and $F_3G_3 = c_{21}t_{21}\sin e \;\Rightarrow$

$$t_{21} = \frac{F_3G_3}{c_{21}\sin e}.$$

$$E_1G_3 = E_1I_1 + I_1G_1 - G_1G_3 \;\Rightarrow\; c_{21}t_{21}\cos e = E_1I_1 + L - vt_{21} \;\Rightarrow$$

$$t_{21} = \frac{L + E_1I_1}{c_{21}\cos e + v}.$$

The equality of the two formulas for time t_{21} yields the distance F_3G_3.

$$\frac{F_3G_3}{c_{21}\sin e} = \frac{L + E_1I_1}{c_{21}\cos e + v} \;\Rightarrow F_3G_3 = \frac{(L + E_1I_1)c_{21}\sin e}{c_{21}\cos e + v}.$$

In time t_{22}, the reflected ray travels from point F_3 to point A_5 with the speed c_{22}, and M_2 travels the distance $C_3C_5 = G_3G_5 = vt_{22}$.

In triangle $A_5C_3F_3$, $A_5F_3 = c_{22}t_{22}$, $A_5C_3 = c_{22}t_{22}\cos e$, and $C_3F_3 = c_{22}t_{22}\sin e \;\Rightarrow$

$$t_{22} = \frac{C_3F_3}{c_{22}\sin e}.$$

$$A_5C_3 = A_5C_5 + C_3C_5 \;\Rightarrow\; c_{22}t_{22}\cos e = L + vt_{22} \;\Rightarrow$$

$$t_{22} = \frac{L}{c_{22}\cos e - v}.$$

The equality of the two formulas for time t_{22} yields the distance C_3F_3.

$$\frac{C_3F_3}{c_{22}\sin e}=\frac{L}{c_{22}\cos e-v}\quad\Rightarrow\quad C_3F_3=\frac{Lc_{22}\sin e}{c_{22}\cos e-v}.$$

Triangle $A_1E_1I_1$ gives $E_1I_1=A_1I_1\tan(\pi/4-e/2)=(F_3G_3+C_3F_3)\tan(\pi/4-e/2)$.

Substituting the formula of F_3G_3 and C_3F_3 in the formula of E_1I_1,

$$E_1I_1=\frac{(L+E_1I_1)c_{21}\sin e\tan(\pi/4-e/2)}{c_{21}\cos e+v}+\frac{Lc_{22}\sin e\tan(\pi/4-e/2)}{c_{22}\cos e-v}\quad\Rightarrow$$

$$E_1I_1\left(1-\frac{c_{21}\sin e\tan(\pi/4-e/2)}{c_{21}\cos e+v}\right)=\left(\frac{Lc_{21}\sin e}{c_{21}\cos e+v}+\frac{Lc_{22}\sin e}{c_{22}\cos e-v}\right)\tan(\pi/4-e/2).$$

Distance E_1I_1 can be calculated, then time t_{21} followed by time $t_2=t_{21}+t_{22}$. Distance $A_1I_1=E_1I_1/\tan(\pi/4-e/2)$ can also be calculated.

In time t_{11}, the transmitted ray travels from point D_1 to point J_2 with the speed c_{11}, M travels the distance $A_1A_2=vt_{11}$, and M_1 travels the distance $J_1J_2=B_1B_2=vt_{11}$.

In time t_{12}, the transmitted ray travels from point J_2 to point A_4 with the speed c_{12}, and M travels the distance $A_2A_4=vt_{12}$.

In triangle $A_4H_1J_2$, $A_4J_2=c_{12}t_{12}$, $H_1J_2=c_{12}t_{12}\cos 2e$, and $A_4H_1=c_{12}t_{12}\sin 2e\quad\Rightarrow$

$$t_{12}=\frac{H_1J_2}{c_{12}\cos 2e}=\frac{H_1R_1-J_2R_1}{c_{12}\cos 2e}=\frac{H_1R_1-(R_1T_1-J_2T_1)}{c_{12}\cos 2e}=\frac{L-R_1T_1+J_2T_1}{c_{12}\cos 2e}.$$

In the triangle $B_1R_1T_1$, $R_1T_1=B_1R_1\tan e$. In the triangle $J_1J_2T_1$, $J_2T_1=J_1J_2\tan e=vt_{11}\tan e$.

$B_1R_1=A_1H_1=A_1A_2+A_2A_4+A_4H_1=vt_{11}+vt_{12}+c_{12}t_{12}\sin 2e$, and then $R_1T_1=B_1R_1\tan e=(vt_{11}+vt_{12}+c_{12}t_{12}\sin 2e)\tan e$.

Substituting the formula of R_1T_1 and J_2T_1 in the formula of time $t_{12}\ \Rightarrow$

$$t_{12}=\frac{L-R_1T_1+J_2T_1}{c_{12}\cos 2e}=\frac{L-(vt_{11}+vt_{12}+c_{12}t_{12}\sin 2e)\tan e+vt_{11}\tan e}{c_{12}\cos 2\mathrm{e}}\quad\Rightarrow$$

$$t_{12}=\frac{L}{c_{12}\cos 2e+(v+c_{12}\sin 2e)\tan e}.$$

$D_1J_2=D_1H_1+H_1J_2=D_1H_1+H_1R_1-(R_1T_1-J_2T_1)=A_1I_1+L-R_1T_1+J_2T_1$.

$$t_{11}=\frac{D_1J_2}{c_{11}}=\frac{L+A_1I_1-R_1T_1+J_2T_1}{c_{11}}$$

$$=\frac{L+A_1I_1-(vt_{11}+vt_{12}+c_{12}t_{12}\sin 2e)\tan e+vt_{11}\tan e}{c_{11}}\quad\Rightarrow$$

$$t_{11}=\frac{L+A_1I_1-(v+c_{12}\sin 2e)t_{12}\tan e}{c_{11}}.$$

Distance A_1I_1 is known. Thus, time t_{11} followed by time $t_1=t_{11}+t_{12}$ can be calculated.

The difference of time $\Delta t_4=t_2-t_1$, and the number of wavelengths comprised in Δt_4 is $N_4=c\Delta t_4/\lambda$.

By rotating the interferometer from the 180° position to the 270° position, the fringe shift is $N_{34}=N_4-N_3$. By rotating the interferometer from the 270° position to the initial position, the fringe shift is $N_{41}=N_1-N_4$.

If the speed of light is the same before and after reflection, speeds c_{11}, c_{12}, c_{21}, and c_{22}

are equal to the constant speed c. For the reflection of light as a mechanical phenomenon, these speeds are as follows:

$c_{11} = c.$

$c_{12} = c_{11} + v\cos 270° + v\cos(270° - 2e) = c - \sin 2e.$

$c_{13} = c_{12} + v\cos(90° - 2e) + v\cos e = (c - \sin 2e) + v\sin 2e + v\cos e = c + v\cos e.$

$c_{21} = c + v\cos 270° + v\cos(180° - e) = c - v\cos e.$

$c_{22} = c_{21} + v\cos(360° - e) + v\cos e = (c - v\cos e) + v\cos e + v\cos e = c + v\cos e = c_{13}.$

6. Numerical calculation

6.1. Numerical calculation of the fringe shift

The following tables give an indication of how the fringe shift varies as a function of angle e from aberration angle $e = \sin^{-1} v/c = 1.00\mathrm{E} - 04$ rad to $e = 0$ rad, when the speed of light is considered the same before and after reflection, and when the speed of light is considered different before and after reflection.

Table 1. Difference of the light paths in wavelength and fringe shift for the four positions, for angle e equal to aberration angle, $e = 1.00\mathrm{E} - 04$ rad.

Speed of light is the same before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	2.00E–01	6.00E–01	1.99E–01	6.00E–01
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	–4.00E–01	4.00E–01	–4.00E–01	4.00E–01
Speed of light is different before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	1.99E–01	1.99E–01	2.00E–01	1.99E–01
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	–1.99E–05	1.99E–05	2.00E–05	–2.00E–05

Table 2. Difference of the light paths in wavelength and fringe shift for the four positions, for angle $e = 1.00\mathrm{E} - 05$ rad.

Speed of light is the same before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	2.00E–03	4.02E–01	2.00E–03	4.02E–01
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	–4.00E–01	4.00E–01	–4.00E–01	4.00E–01
Speed of light is different before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	1.99E–03	1.99E–03	2.00E–03	1.99E–03
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	–1.94E–07	1.87E–07	2.09E–07	–2.02E–07

Table 3. Difference of the light paths in wavelength and fringe shift for the four positions, for angle $e = 0$ rad.

Speed of light is the same before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	0	4.00E–01	0	4.00E–01
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	–4.00E–01	4.00E–01	–4.00E–01	4.00E–01
Speed of light is different before and after reflection	**N_1**	**N_2**	**N_3**	**N_4**
	0	0	0	0
	N_{41}	**N_{12}**	**N_{23}**	**N_{34}**
	0	0	0	0

For calculations, the velocity of the inertial frame $v = 3.00\text{E} + 04$ m/s, velocity of light $3.00\text{E} + 08$ m/s, wavelength of light $550\text{E} - 09$ m, and length of the interferometer's arms $L = 11$ m, which is the same data used by Michelson and Morley in their experiment.

The data of Table 3 for the reflection of light as a mechanical phenomenon when the speed of light is different before and after reflection verifies the result of reference [1].

6.2. Discussions

For the hypothesis that the speed of light has the same constant speed c before and after reflection, the theoretical fringe shift is approximately $4.00\text{E} - 01$ for any angle e on the interval of 0 to aberration angle.

For the hypothesis that the reflection of light is a mechanical phenomenon, the fringe shift is approximately $2.00\text{E} - 05$ for angle e equal to the aberration angle and decreases to zero for $e = 0$. For high precision experiments, angle e tends to decrease toward zero along with the corresponding fringe shift. This hypothesis predicts an unobservable fringe shift that explains the negative result of the Michelson–Morley experiment.

References

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