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THREE SIMPLE PROOFS OF A MASS-ENERGY EQUATION AS $E = ½ mc^2$

Musa D. Abdullahi, Department of Electrical and Electronics Engineering
College of Technology, Kaduna Polytechnic, Kaduna, Nigeria. E-mail: musadab@outlook.com

Abstract: A mass-energy equation is derived, in three different ways, as $E = ½ mc^2$, by considering an electrostatic field as a medium having pressure and density and transmitting light at speed c, the kinetic energy of a charged particle moving with velocity **v** and the induction electric field due to a particle of charge Q and mass m moving with acceleration $d\mathbf{v}/dt$. The relativistic mass-energy equation is corrected, and the mass-velocity formula is queried.

1. INTRODUCTION

The mass-energy law, of special relativity, gives the energy content E of a particle of mass m as:

$$E = mc^2 \tag{1}$$

This has become the most famous equation in the world [1, 2]. The mass m in equation (1) is supposed to vary with speed v of the particle in accordance with the relativistic mass-velocity formula:

$$m = m_o\left(1-\frac{v^2}{c^2}\right)^{\frac{-1}{2}} = \gamma m_o \tag{2}$$

where m_o is the rest mass and γ the Lorentz factor. The kinetic energy is supposed to be accounted for in an increase of mass of the particle, which becomes infinitely large at the limiting speed of light $v = c$.

This paper proposes that mass of a moving particle remains constant at m_o and the limiting speed c for a particle of charge Q accelerated by an electric field E_o to velocity **v**, is the result of radiation. The radiation reaction force is $-QE_o\mathbf{v}/c$ and radiation power is QE_ov^2/c, in rectilinear motion. At the speed c, radiation reaction force becomes equal and opposite to the impressed force $Q\mathbf{E}_o$. This makes the accelerating force zero and the particle continues to move with speed c, radiating energy equal to potential energy lost.

The first proof of a mass-energy equivalence law is based on an electrostatic field as a physical medium, having pressure and density and supporting transmission of radiation at the speed of light [3, 4]. The second proof invokes aberration of electric field [5] and the third proof uses Faraday's law of electromagnetic induction. The paper ends with some discussions of the results obtained and a conclusion.

2. ENERGY DENSITY AND PRESSURE AND OF AN ELECTROSTATIC FIELD.

The energy density w of electrostatic field of magnitude E_o, is given by the well-known formula:

$$w = \frac{\varepsilon_o}{2}E_o^2 \tag{3}$$

where ε_o is the permittivity of space. Pressure P in the electrostatic field E_o , of charge intensity ε_oE_o, is:

$$P = \varepsilon_o E_o^2 = 2w \tag{4}$$

The electrostatic field as a medium of mass density ρ, transmits radiation at the speed of light c, so that:

$$c^2 = \frac{P}{\rho} = \frac{2w}{\rho}$$

$$w = \frac{1}{2}\rho c^2 \tag{5}$$

For a volume V containing mass M, equation (5) becomes the first proof of a mass-energy law, as.

$$E = \int_V w(dV) = \frac{1}{2}\int_V \rho c^2(dV) = \frac{1}{2}Mc^2 \tag{6}$$

3. KINETIC ENERGY OF A MOVING CHARGED PARTICLE

Figure 1 shows a particle of charge Q moving at velocity **v**, relative to an observer. The electrostatic field $\mathbf{E}_o$ in the direction of velocity vector **c**, at angle θ to **v**, generating a magnetic field **H**, given by vector:

$$\mathbf{H} = \varepsilon_o \mathbf{v}\times\mathbf{E}_o \tag{7}$$

Relative to the moving particle, $\mathbf{v} = 0$ and **H** should be 0. Due to aberration of electric field, $\mathbf{E}_o$ of magnitude E_o, increasing in the direction of velocity, becomes a dynamic electric field $\mathbf{E}_v$ as expressed in equation (8).

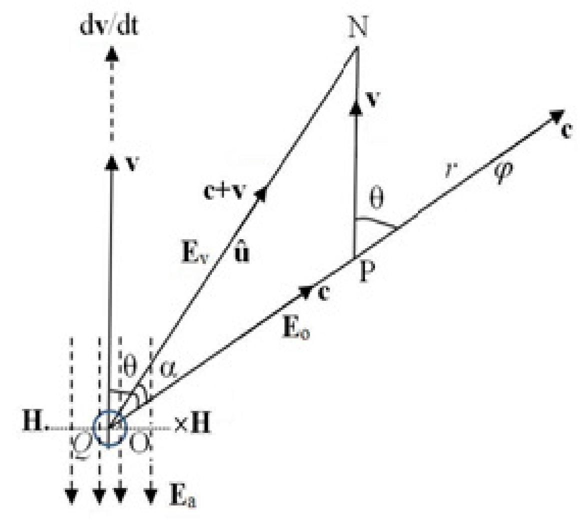

Figure 1: A particle of charge Q moving with its field $\mathbf{E}_o$ at angle θ to velocity $\mathbf{v}$, setting up potential φ, dynamic electric field $\mathbf{E}_v$, magnetic field $\mathbf{H}$ and induction field $\mathbf{E}_a$

$$\mathbf{E}_v = \frac{E_o}{c}(\mathbf{c}+\mathbf{v}) = \frac{E_o}{c}\sqrt{c^2+v^2+2vc\cos\theta}\,\hat{\mathbf{u}} \qquad (8)$$

where c is the speed of light in a space. $\mathbf{E}_v$ accounts for the potential and kinetic energy of the particle. Magnetic field $\mathbf{H}$ does not carry energy. Aberration angle $\propto$ is given by:

$$\sin\alpha = \frac{v}{c}\sin(\theta-\alpha) \qquad (9)$$

Energy of the dynamic electric field $\mathbf{E}_v$ is expressed, with reference to Figure 1 and equation (8), as volume integral:

$$\frac{\varepsilon_o}{2}\int_V E_v^2(dV) = \frac{\varepsilon_o}{2}\int_V E_o^2\left(1+\frac{v^2}{c^2}+\frac{2v}{c}\cos\theta\right)(dV) \qquad (10)$$

The first term, on the right-hand side of equation (10), is the electrostatic energy E of the charge. The second term is the kinetic energy K and the third term integrates to zero.

$$K = \frac{\varepsilon_o}{2}\int_V \frac{E_o^2 v^2}{c^2}(dV) = E\frac{v^2}{c^2} = \frac{1}{2}mv^2 \qquad (11)$$

Equation (11) is the second proof of a mass-energy law.

4. INDUCTION ELECTRIC FIELD DUE TO AN ACCELERATED CHARGED PARTICLE

The magnetic field $\mathbf{H}$, of a particle of charge Q moving with velocity $\mathbf{v}$, given by equation (7), is:

$$\mathbf{H} = \varepsilon_o \mathbf{v}\times\mathbf{E}_o = -\varepsilon_o \mathbf{v}\times\nabla\varphi = \varepsilon_o\nabla\times\varphi\mathbf{v} \qquad (12)$$

where ∇ is a vector operator and φ is the potential at a point, due to the charge. If the particle is accelerated, an induction electric field $\mathbf{E}_a$ is generated, as given by Faraday's law:

$$\nabla\times\mathbf{E}_a = -\mu_o\frac{d\mathbf{H}}{dt} = -\mu_o\varepsilon_o\nabla\times\varphi\frac{d\mathbf{v}}{dt}$$

$$\mathbf{E}_a = -\mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt} \qquad (13)$$

where μ_o is the permeability. $\mathbf{E}_a$ acts on Q, of mass m to produce the inertial force, such that:

$$Q\mathbf{E}_a = -\mu_o\varepsilon_o Q\varphi\frac{d\mathbf{v}}{dt} = -2\mu_o\varepsilon_o E\frac{d\mathbf{v}}{dt} = -m\frac{d\mathbf{v}}{dt} \qquad (14)$$

where the product $Q\varphi = QU$ is equal to twice the electrostatic energy E of the charge Q in its own potential φ developed from 0 to U. With $c^2 = 1/\mu_o\varepsilon_o$, c being the speed of light in free space, equation (14) becomes:

$$E = \frac{m}{2\mu_o\varepsilon_o} = \frac{1}{2}mc^2 \qquad (15)$$

5. RESULTS AND DISCUSSIONS

- Energy E in Equations (6), (11) and (15) is the electrostatic energy of a charged particle or a body.
- Kinetic energy of a charged particle, moving relative to an observer, is in its electrostatic field.
- The induction electric field in equation (13) is responsible for inertia and electromagnetic radiation.

6. CONCLUSION

- The expression $E = ½mc^2$ is the correct mass-energy equivalence equation, in contrast to $E = mc^2$.

REFERENCES

1. M Jammer (1961): *Concept of Mass in Classical and Modern Physics*. Cambridge University Press.
2. M Lange (2001) "The most famous equation", J. of Philosophy 98: 219-238.
3. M Abdullahi (2004): https://www.academia.edu/7692116/On_the_Energy_and_Mass_of_Electric_Charges_in_a_Body
4. M Abdullahi (2019): https://www.academia.edu/39154172/AN_ELECTRIC_FIELD_MODEL_OF_THE_AETHER
5. https://www.academia.edu/39281167/ABERRATION_OF_ELECTRIC_FIELD_AND_ACCELERATED_MOTION_OF_AN_ELECTRON_WITH_CONSTANT_MASS