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A FURTHER EXPLANATION OF MAXWELL'S DISPLACEMENT CURRENT

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Abstract

A magnetic field **H** or electric current intensity **J** in a conductor, changing with time *t*, generates an induction electric field **E**, as given by Faraday's law. An oscillating electric field **E** gives rise to displacement current intensity $\varepsilon_o(d\mathbf{E}/dt)$, where ε_o is the permittivity. The addition of $\varepsilon_o(d\mathbf{E}/dt)$ to the conduction current intensity **J**, by Maxwell, was crucial in deriving the electromagnetic wave equation. The field **E** accounts for inertia where the accelerating force $m(d\mathbf{v}/dt)$ on a particle of charge Q and mass m, moving with velocity **v**, is equal and opposite to the inertial force expressed as $Q\mathbf{E}$.

Keywords: Displacement Current, Electric Field, Magnetic Field, Magnetic Vector Potential, Vector, Wave Equation.

1. INTRODUCTION

A charge Q and its electric field $\mathbf{E}_o$, moving at velocity **v**, create magnetic field **H**, as vector product:

$$\mu_o\mathbf{H} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o = -\mu_o\varepsilon_o\mathbf{v}\times\nabla\varphi = \mu_o\varepsilon_o\nabla\times\varphi\mathbf{v} = \nabla\times\mathbf{A} \quad (1) \qquad \mathbf{A} = \mu_o\varepsilon_o\varphi\mathbf{v} \quad (2)$$

where μ_o is the permeability and ε_o the permittivity of space, φ the potential at a point and **A** the magnetic vector potential. A charged particle moving with velocity **v** or a current of intensity **J** in a conductor, generates a magnetic field **H** around the current, as in Figure 1. A magnetic field **H** changing with time *t*, creates induction field **E**, given by Faraday's law of 1831 and equation (1), in terms of curl of a vector, thus:

$$\nabla\times\mathbf{E} = -\mu_o\frac{d\mathbf{H}}{dt} = -\nabla\times\frac{d\mathbf{A}}{dt} \quad (3) \qquad \mathbf{E} = -\frac{d\mathbf{A}}{dt} = -\mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt} \quad (4)$$

A changing field **E** creates displacement current intensity $\varepsilon_o(d\mathbf{E}/dt)$, as introduced by Maxwell in 1865.

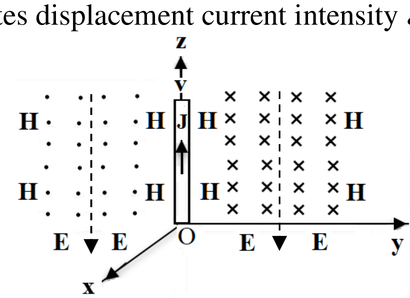

Figure 1: Current intensity **J** in the **z**-direction, due to oscillating charges creating magnetic field **H** and induction electric field **E**.

2. INERTIA AND MASS-ENERGY EQUIVALENCE LAW

It is proposed that the induction electric field **E**, given by equation (4), acts on the same charge Q producing it, to create the inertial force $Q\mathbf{E}$, equal and opposite to the accelerating force on mass m, thus:

$$Q\mathbf{E} = -\mu_o\varepsilon_o\varphi Q\frac{d\mathbf{v}}{dt} = -m\frac{d\mathbf{v}}{dt} \qquad \frac{\varphi Q}{2} = E_n = \frac{m}{2\mu_o\varepsilon_o} = \frac{mc^2}{2} \quad (5)$$

where $\varphi Q/2 = E_n$ is the intrinsic energy of charge Q in its own potential and $1/\sqrt{(\mu_o\varepsilon_o)} = c$ the speed of light.

3. ELECTROMAGNETIC WAVE EQUATION

Maxwell added displacement current to conduction current intensities to gives Ampere's law as:

$$\nabla\times\mathbf{H} = \mathbf{J} + \varepsilon_o\frac{d\mathbf{E}}{dt} \quad (6)$$

Taking the curl of equation (3), in free space, with no charge distribution and $\mathbf{J} = 0$, gives the wave equation:

$$\nabla\times(\nabla\times\mathbf{E}) = \mu_o\frac{d}{dt}(\nabla\times\mathbf{H}) = \nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E} = -\mu_o\frac{d}{dt}\left(\mathbf{J} + \varepsilon_o\frac{d\mathbf{E}}{dt}\right) \quad (7) \qquad \nabla^2\mathbf{E} = \mu_o\varepsilon_o\frac{d^2\mathbf{E}}{dt^2} = \frac{1}{c^2}\frac{d^2\mathbf{E}}{dt^2} \quad (8)$$

4. CONCLUSIONS

1. Induction electric field **E** is needed to give inertia of a mass m and a mass-energy law as $E_n = ½mc^2$.
2. Maxwell's displacement current, $\varepsilon_o d\mathbf{E}/dt$, is crucial in deriving the electromagnetic wave equation.