the general science journal

The Significance of the Poynting Vector

Frederick David Tombe,
Belfast, Northern Ireland,
United Kingdom,
sirius184@hotmail.com
27th January 2020

Abstract. **We can multiply an electric field by a magnetic field to obtain the Poynting vector, S = E×H, and the product applies to the energy flow in electromagnetic radiation, where E is specifically an electric field that has been induced by time-varying electromagnetic induction. This is because the form of the Poynting vector follows from Ampère's circuital law and Faraday's law of time-varying electromagnetic induction. Although there is no known theoretical basis that would justify swapping the electromagnetic E field in S with an electrostatic E field, the dimensions of S would nevertheless remain unchanged if we did do so. As such, it has been asked whether or not the product E×H is predictive of any energy flow when an electrostatic field is superimposed upon a steady state magnetic field, or would it just amount to multiplying apples and bananas?**

Historical Background

I. A telegrapher's equation linking the speed of light to electric signals propagating along a conducting wire was first derived by German physicist Gustav Kirchhoff in 1857 **[1]**. In Kirchhoff's theory, it was assumed that the energy travelled inside the conducting wires. Some years later however, in 1883, English physicist John Henry Poynting made a proposal regarding the transfer of energy in electric circuits. Poynting proposed that at least some of the energy is actually transferred through the space outside the conducting wires **[2]**. This idea was also taken up around about the same time by English electrical engineer Oliver Heaviside **[3]**.

It was already known since the time of Faraday and Henry that electrical energy can be transferred through the space between two electric circuits in the case of electromagnetic induction, but Poynting and Heaviside were now suggesting that in the case of electrical energy that is applied directly to a circuit, that some of the energy travels through the space in the immediate the vicinity of the conducting wires.

Poynting's Theorem

II. The derivation of Poynting's Theorem in this section begins by considering the equation of continuity as applied to the sum of two energy density fields in space. One of these is the electromagnetic energy density field, $\frac{1}{2}[\varepsilon_o\mathbf{E_K}\cdot\mathbf{E_K} + \mu_o\mathbf{H}\cdot\mathbf{H}]$, which is sourced in a dynamic magnetic field where $\mathbf{E_K} = -\partial\mathbf{A}/\partial t$, and where the magnetic vector potential, $\mathbf{A}$, satisfies $\nabla\times\mathbf{A} = \mu_o\mathbf{H}$. The other is the electrostatic energy density field, $\frac{1}{2}\varepsilon_o\mathbf{E_S}\cdot\mathbf{E_S}$, which is sourced in an identifiable electric charge and where $\mathbf{E_S} = -\nabla\varphi$. The two $\mathbf{E}\cdot\mathbf{E}$ terms represent the potential energy associated with stress in the all-pervading elastic solid which acts as the medium for the propagation of light. This mathematical form is in the likeness of the form used for the potential energy, $\frac{1}{2}kx^2$, that is stored in a stretched mechanical spring, where $\mathbf{E}$ corresponds to k. See equation **(8)** in section **IV** below, and also Part III in Maxwell's 1861 paper ***"On Physical Lines of Force"*** **[4]**. The electric permittivity, ε_o, is the inverse of the elastic constant. Meanwhile, the $\mathbf{H}\cdot\mathbf{H}$ term represents fine-grained kinetic energy in the magnetic field in like manner to the familiar mechanical term, $\frac{1}{2}mv^2$, where $\mathbf{H}$ corresponds to the speed, v. See Parts I and II in Maxwell's 1861 paper. The magnetic permeability, μ_o, is the mass density term. The total energy density, *w*, is therefore,

$$w = \tfrac{1}{2}\varepsilon_o\mathbf{E_K}^2 + \tfrac{1}{2}\mu_o\mathbf{H}^2 + \tfrac{1}{2}\varepsilon_o\mathbf{E_S}^2 \qquad \textbf{(1)}$$

Taking the partial time derivative of **(1)** leads to,

$$\partial w/\partial t = \varepsilon_o\mathbf{E_K}\cdot\partial\mathbf{E_K}/\partial t + \mu_o\mathbf{H}\cdot\partial\mathbf{H}/\partial t + \varepsilon_o\mathbf{E_S}\cdot\partial\mathbf{E_S}/\partial t \qquad \textbf{(2)}$$

The first term on the right-hand side of equation **(2)** contains Maxwell's displacement current, $\varepsilon_o\partial\mathbf{E_K}/\partial t$, as is used in electromagnetic radiation, and we know from Ampère's Circuital Law, as applied in space, that,

$$\nabla\times\mathbf{H} = \varepsilon_o\partial\mathbf{E_K}/\partial t \qquad \textbf{(3)}$$

Regarding the second term on the right-hand side of equation **(2)**, we know from Faraday's Law that,

$$\nabla\times\mathbf{E_K} = -\mu_o\partial\mathbf{H}/\partial t \qquad \textbf{(4)}$$

Substituting equations **(3)** and **(4)** into equation **(2)** we get,

$$\partial w/\partial t = \mathbf{E_K}\cdot\nabla\times\mathbf{H} - \mathbf{H}\cdot\nabla\times\mathbf{E_K} + \varepsilon_o\mathbf{E_S}\cdot\partial\mathbf{E_S}/\partial t \qquad \textbf{(5)}$$

Hence, using the vector identity,

$$\partial w/\partial t = -\nabla\cdot(\mathbf{E_K}\times\mathbf{H}) + \varepsilon_o\mathbf{E_S}\cdot\partial\mathbf{E_S}/\partial t \qquad \textbf{(6)}$$

the Poynting vector, **S**, is then defined as,

$$\mathbf{S} = \mathbf{E_K}\times\mathbf{H} \qquad \textbf{(7)}$$

The Significance of the Poynting Vector

III. What amazes most people about the Poynting vector is its apparent predictive power in relation to energy flow. The prediction is of course rooted in Faraday's Law which is implicit in the Poynting vector, as seen from the derivation in Section **II** above. We know from experiments that Faraday's Law, in relation to self-inductance, describes an impedance, and that as such, it possesses a strong correspondence to Newton's Second Law of Motion, and indeed through Lenz's Law, it also encroaches into the territory of Newton's Third Law. The more interesting aspect of Faraday's Law is however the fact that the changing magnetic field in a primary circuit can cause an electric field in a secondary circuit. This is the inertial aspect which connects Faraday's Law to Newton's First Law of Motion and hence to conservation of momentum. We also observe this inertial effect when the power is disconnected from an inductance circuit. The magnetic field collapses and gives the current a final forward kick.

Ampère's Circuital Law has no corresponding reciprocal cause and effect implications beyond that an electric current has an associated magnetic field, and indeed we can't even combine these two Maxwell curl equations in the context of a single electric circuit since the **E** terms in each equation don't correspond to each other. But when we do combine the two equations in space, we obtain electromagnetic wave equations in $\mathbf{E_K}$ and **H**, as well as the Poynting vector, $\mathbf{S} = \mathbf{E_K}\times\mathbf{H}$. The Poynting vector, **S**, plays the same role within the equation of continuity of electromagnetic energy that conduction current, **J**, plays within the equation of continuity of charge. Electromagnetic radiation would therefore appear to be a complex electric current that flows through space, and when it strikes a conducting wire, the component that strikes the wire at right angles, channels into a simple conduction current, **J**, which then flows along the wire. This is like the case of convectively induced electromagnetic induction where a current is induced in a conducting wire that moves at right angles to a magnetic field. The right-angle deflection betrays the action of a Coriolis force, $\mathbf{F} = 2m\mathbf{v}\times\boldsymbol{\omega}$. The Coriolis force is strongly associated with vortices, usually atmospheric cyclones, and it possesses a mathematical form very similar to that of the convective electromagnetic force, $\mathbf{E_C} = \mu_o\mathbf{v}\times\mathbf{H}$.

This could all be explained if space were to be densely packed with tiny electric circuits, and so it is proposed that the Poynting vector represents a complex electric current undergoing a fine-grained vortex flow of electric fluid through a dense sea of rotating electron-positron dipoles [5], [6], [7], [8]. These dipolar vortices will have a vorticity, **H**, equivalent to the magnetic intensity, where $\mathbf{H} = 2\boldsymbol{\omega}$. The inertial implications contained within Faraday's Law translate into the travelling energy associated with what is both an electric current and a fine-grained sinusoidal wave. The current is continually flowing between neighbouring rotating electron-positron dipolar vortices, each which constitutes a tiny electric circuit in its own right. This complex electric current is ***Maxwell's Displacement Current***. In comparison with waves in general, $\mathbf{E_K}$ corresponds to the potential energy component while **H** corresponds to the kinetic energy component. From conservation of energy, these two components will therefore be out of phase with each other by ninety degrees as is confirmed from the two curl equations. See **Appendix I**.

The Poynting vector therefore applies to wireless radiation providing that we can isolate **H** from that of the already existing background magnetic field [9]. Hence, the applicability of the Poynting vector would be most significant in AC transformers. In the case of transformers, the Poynting vector would represent the energy flow that leaves the primary circuit, flows through space, and enters the secondary circuit.

Ironically, the Poynting vector does not apply in the context that J.H. Poynting himself and Oliver Heaviside originally intended. The trolley waves that troll the conducting wires of a transmission line constitute a travelling magnetic field and the situation is not covered by the time-varying aspect of Faraday's Law. It may become relevant to convectively induced electromagnetic induction, where $\mathbf{E_C} = \mu_o\mathbf{v}\times\mathbf{H}$, but without the time-varying aspect of Faraday's Law being involved, we can't connect these trolley waves to the Poynting vector. Electromagnetic waves radiate outwards from an electric circuit at right angles to the electric current in the conducting wire. They do not travel alongside the current.

The Electrostatic Component

IV. An AC transmission line involves an important electrostatic component, $\mathbf{E_S}$. In the case of high tension overhead cross-country power transmission lines, it is the most dangerous component, and since it carries the lossless potential energy, which is greater than the kinetic energy of the electric current and the magnetic field, it is perhaps the most important component. The electrostatic component is a capacitative effect associated with dielectric linear polarization. It crosses the gap between the two wires in a transmission line, or the gap to Earth, and it moves along at the speed of light* with the magnetic pulse which it is superimposed upon, and which it gets absorbed into.

Although the electrostatic component, $\mathbf{E_S}$, is not included in the Poynting vector, it was nevertheless smuggled into it at the very outset in J.H. Poynting's original derivation **[2]**. Poynting's own derivation begins as if he has ignored $\mathbf{E_K}$ altogether. He starts by splitting the energy terms at equation (1) *(in his own paper)* into electrostatic and electromagnetic energy components. But in the electromagnetic energy component, he uses only **H** and he omits $\mathbf{E_K}$. That's his first mistake. Then at equation (5) *(in his own paper)*, he produces Maxwell's electromotive force equation. This was originally equation (77) in Maxwell's 1861 paper **[4]**, and later equation (D) in the original listing of eight Maxwell's equations in his 1865 paper ***"A Dynamical Theory of the Electromagnetic Field"*** **[10]**. J.H. Poynting then substitutes all three of Maxwell's EMF terms into what had been exclusively an electrostatic energy component. This meant that he had added the electrostatic term into itself, along with the time-varying electromagnetic term $\mathbf{E_K}$, and also along with the convective electromagnetic term, $\mu_o\mathbf{v}\times\mathbf{H}$, which ultimately wasn't involved in the Poynting vector. Next, he segregates both of the non-convective terms, $\mathbf{E_S}$ and $\mathbf{E_K}$, from the convective term, and from then on, he treats the two as a single bundled entity.

By comparison with the less cumbersome derivation in Section **II** above, it would be like as if J.H. Poynting had used a single potential energy term in equation **(1)** in the form $\frac{1}{2}\,\varepsilon_o(\mathbf{E_S} + \mathbf{E_K})^2$. This would only be legitimate if $\mathbf{E_S}$ and $\mathbf{E_K}$ were two mutually orthogonal vectors, but in general they are not. The electrostatic field, $\mathbf{E_S}$, can be superimposed at any angle to the electromagnetic field $\mathbf{E_K}$. In a transmission line, the two are orthogonal, but this still doesn't legitimize the presence of $\mathbf{E}_S$ in the Poynting vector, because $\mathbf{E_S}$ is not involved in the constituent equations **(3)** and **(4)**.

So, what about the electrostatic term, $\varepsilon_o\mathbf{E_S}\cdot\partial\mathbf{E_S}/\partial t$, which is the last term on the right-hand side of equation **(2)**? It relates to a time-varying electrostatic field, such as we would find in the vicinity of an AC circuit, or in particular, in the vicinity of a charging capacitor. The term $\varepsilon_o \mathbf{E_S} \cdot \partial \mathbf{E_S} / \partial t$ relates to linear polarization in a dielectric, where Maxwell's fifth equation, the electric elasticity equation, applies as in,

$$\mathbf{D} = -\varepsilon_o \mathbf{E_S} \qquad \textbf{(8)}$$

and hence,

$$\varepsilon_o \mathbf{E_S} \cdot \partial \mathbf{E_S} / \partial t = -\mathbf{E_S} . \mathbf{J_D} \qquad \textbf{(9)}$$

where **D** is the electric displacement vector and where,

$$\mathbf{J_D} = \partial \mathbf{D} / \partial t \qquad \textbf{(10)}$$

is the original displacement current as derived by Maxwell in Part III of his 1861 paper **[4], [11]**. Although Maxwell originally used dielectric polarization in order to derive displacement current in 1861, when he came to apply it to the derivation of the electromagnetic wave equation in his 1865 paper, **[10]**, he was now using $\mathbf{E_K}$ in the displacement current, **[12]**.

Meanwhile, equation **(6)** now becomes,

$$\nabla \cdot \mathbf{S} + \mathbf{J_D} . \mathbf{E_S} = -\partial w / \partial t \qquad \textbf{(11)}$$

* See **Appendix II** – The Speed of Light

Conclusion - Multiplying Apples and Bananas?

V. As mentioned in the above sections, the electrostatic term, $\mathbf{E_S}$, has no place in the Poynting vector, yet if we were to form the product $\mathbf{E_S} \times \mathbf{H}$ in connection with a static electrostatic field superimposed upon the magnetic field of a static bar magnet, the product would nevertheless indicate a non-zero value for energy flow even though there is no known basis for assuming that any energy flow should be occurring. It has often been debated as to whether or not the product $\mathbf{E_S} \times \mathbf{H}$ is predictive of some hidden energy flow that has been hithertofore undetected, or if it's simply a meaningless product equivalent to multiplying apples and bananas.

While the answer to this question has never been agreed upon, it is not unreasonable to assume that an electrostatic field superimposed upon a magnetic field might induce some kind of physical interaction with the physical fabric of the magnetic field. The double helix theory of the magnetic field, **[7]**, **[8]**, **[9]**, predicts that space is densely packed with tiny rotating electron-positron dipoles on the pico-scale. An externally applied electrostatic field would therefore cause a torque to act on these tiny dipoles such as to cause them to precess.

It's within the context of one of these tiny vortices where the hidden physical linkage between $\mathbf{E_S}$ and $\mathbf{E_K}$ is revealed. $\mathbf{E_S}$ ($= -\nabla\varphi$) is the radial force field due to tension or pressure in the aether, while the magnetic vector potential, $\mathbf{A}$, is the transverse component of the momentum of the aether flow. $\mathbf{E_K}$ ($= -\partial\mathbf{A}/\partial t$) is the transverse force field in which the curl of $\mathbf{E_K}$ results in Faraday's Law, $\nabla\times\mathbf{E_K} = -\mu_o\partial\mathbf{H}/\partial t$. For further details, see ***"The Positronium Orbit in the Electron-Positron Sea"*** **[9]**.

On the scale of the tiny dipoles, the externally applied electrostatic force field, $\mathbf{E_S}$, will become $\mathbf{E_K}$ since it will be acting transversely to their polar origins. The induced precession will involve a fine-grained circular energy flow, like a *Larmor precession* within the fabric of the magnetic field, on top of the already existing fine-grained rotational kinetic energy.

But what if the electrostatic field is exactly perpendicular to the magnetic field? It would then be more difficult to see how precession might be induced, and this is where magnetization and polarization overlap. Precession of the rotating dipoles, which is actually destructive to the magnetic field, (See the last paragraph of Section **IV** in ***"Maxwell's Displacement Current in Capacitors"*** regarding the collapsing of a magnetic field as the capacitor is charging up **[11]**), is something that best occurs when the electric and magnetic fields are parallel and hence would be of no interest in relation to the Poynting vector. In the perpendicular case however, we might get a better idea of what could be occurring by considering how a test charge would act in the combined field. The electrostatic field would accelerate the test charge, hence inducing the convective force, $\mathbf{F} = q\mu_o\mathbf{v}\times\mathbf{H}$, which would deflect the test charge at right angles to the direction of both of the two fields, and by extension, this should also apply to the electromagnetic momentum field, otherwise known as the magnetic vector potential, $\mathbf{A}$, which is associated with every force field and which constitutes electric current in its purest form. Hence, if we were to place a cylindrical capacitor in a magnetic field with its axis of symmetry parallel to the $\mathbf{H}$ field, such that the electrostatic field, $\mathbf{E_S}$, is radially centred on the axis of cylindrical symmetry, then we might expect the aethereal current, $\mathbf{A}$, associated with the radial electrostatic field within the region enclosed between the two concentric capacitor plates, to be absorbed into the fine-grained rotation of the magnetic field. In other words, an electrostatic field superimposed perpendicularly on a magnetic field serves to enhance the magnetic field.

We can extend this principle into the dynamic state in the case of AC current and apply it to the strong electrostatic field that exists in the vicinity of high voltage overhead power cables where the field lines radiate perpendicularly from the cables. The electrostatic field lines emanate from the cycles of non-zero electric charge density that come with the electric current in the cables. A magnetic field surrounds these cyclical electric current surges, and close to the cables, the travelling electrostatic field intersects the magnetic field at right angles. The radial electrostatic field should therefore be absorbed into the magnetic field, and the enhanced field should travel parallel to the current in the cables.

The Poynting vector, $\mathbf{S} = \mathbf{E_K}\times\mathbf{H}$, is associated with time-varying electromagnetic induction such as would radiate outwards into space from an electric circuit, and such as we find in wireless radiation, but the magnetic trolley waves that troll power cables arise from the actual motion of the source of the magnetic field itself. It's reasonable therefore to deduce that we cannot use the Poynting vector at all in connection with trolley waves, as these are simply travelling magnetic fields. The conclusion is that the Poynting vector only exists where Faraday's Law of time-varying electromagnetic induction applies. The momentum behind the energy transfer associated with the Poynting vector comes from the time-varying induction. In the case of a trolley wave however, although this also has momentum, this momentum comes from the source electric current which plays no role in the derivation of the Poynting vector. The product $\mathbf{E_S}\times\mathbf{H}$ is therefore a case of multiplying apples and bananas.

Appendix I
(Electromagnetic Waves)

While the two Maxwell curl equations, as in Faraday's Law and Ampère's Circuital Law, imply that $\mathbf{E_K}$ and $\mathbf{H}$ should be out of phase with each other by ninety degrees in an electromagnetic wave, Maxwell himself said the complete opposite in his 1873 publication ***"A Treatise on Electricity and Magnetism"*** **[13]**. The curl equations imply that $\mathbf{E_K}$ and $\mathbf{H}$ should be out of phase by ninety degrees because the curl terms will be zero when the time-varying terms are zero. The argument can easily be illustrated in the case of Faraday's Law where $\mathbf{H}$ will be at its maximum magnitude when $\partial\mathbf{H}/\partial t$ is zero, and hence when $\nabla\times\mathbf{E_K}$ is zero, and hence when $\mathbf{E_K}$ is at its minimum magnitude, which will also be zero. In the case of Ampère's Circuital Law, $\mathbf{E_K}$ will be at its maximum magnitude when the displacement current $\varepsilon_o\partial\mathbf{E_K}/\partial t$ is zero, and hence when $\nabla\times\mathbf{H}$ is zero, and hence when $\mathbf{H}$ is at its minimum magnitude.

However, on page 438 of the Treatise, Maxwell introduces an electric wave, $\mathbf{E_K}$, that propagates at right angles to the magnetic wave, $\mathbf{H}$, for which he originally derived the EM wave equation. Moving on to page 439 we can see at equations (19) and (20) how he has isolated two mutually perpendicular components of **H** that would naturally be in phase with each other. Note that Maxwell splits **H** into its three Cartesian components, F, G, and H, whereas the corresponding components of $\mathbf{E_K}$ are P, Q, and R. Then for reasons unexplained, Maxwell presents a diagram (Fig. 67) in which he depicts an electromagnetic wave with $\mathbf{E_K}$ and **H** shown in phase. It is clear that this must be a mistake and that Maxwell has got the two components of **H** in equations (19) and (20) mixed up with $\mathbf{E_K}$ and **H**. This mistake seems to have been carried through into the modern textbooks.

Appendix II
(The Speed of Light)

The correspondence between the speed of an electrical signal along a wire on the one hand, and the speed of light on the other hand, is based largely on aether hydrodynamics, on the principle that electric current is primarily an aethereal fluid that flows between positive particles (sources) and negative particles (sinks), and at an average speed in the order of the speed of light. This however leads to confusion as regards reconciling Ohm’s Law with Bernoulli’s Principle.

Ohm’s Law implies that for a given resistance, assumed to be constant, then the greater the applied EMF, the greater will be the ensuing electric current. This is fine under certain conditions, but it hardly applies to a DC transmission line pulse which travels along on its own momentum, and which has spread out into the circuit to its maximum extent. In this case, the aether pressure will be at a natural equilibrium with the surrounding dielectric space, and it is in this context where the greatest correspondence is observed between the speed of an electrical signal in a wire speed and the speed of light **[14]**.

The situation is quite different however in a capacitor circuit and in high-voltage cross-country power cables. In both of these cases, the speed of flow (current) reduces as the aether pressure increases. This is Bernoulli’s Principle in operation.

If we consider the overhead section of the electricity grid that crosses the country, this section, enclosed at each end by the high winding of an AC transformer, is in fact like a capacitor. It is actually a mixture between an inductor and a capacitor, but most of the energy involved will be in the form of electrostatic potential energy due to the capacitance. At the step-up end of the line, the current flowing across the gap from the primary winding, in the form of EM radiation, is absorbed into the secondary winding where, due to the high impedance, the velocity reduces while the pressure goes up. It’s as though the primary circuit is feeding a reservoir. When they talk about the high voltage in the overhead power cables, this is not a voltage in the sense of the voltage that drives a current as in Ohm's Law. This voltage, mainly due to capacitance, is acting as an impedance to the flow of the conduction current entering the circuit at the step-up end, although, due to inductance, it can also act as an ejecting EMF at the step-down end. The fact that the overhead power cables are acting like a high-pressure reservoir means that neither the electric current in the cables, nor the electromagnetic and electrostatic over-spills that troll the cables, will travel as fast as the speed of light.

References

[1] Kirchhoff, G.R., ***"On the Motion of Electricity in Wires"***, Philosophical Magazine, Volume XIII, Fourth Series, pages 393-412 (1857)
Pages 280-282 in this link,
https://www.ifi.unicamp.br/~assis/Weber-Kohlrausch(2003).pdf

[2] Poynting, J.H., ***"On the Transfer of Energy in the Electromagnetic Field"***, Philos. Trans. Roy. Soc. London 175, pp. 343-361(1884)
https://en.wikisource.org/wiki/On_the_Transfer_of_Energy_in_the_Electromagnetic_Field

[3] Heaviside, O., ***"The Induction of Currents in Cores"***, The Electrician, Volume 13, pp. 133-4, 21st June 1884

[4] Maxwell, J.C., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)
http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[5] Lodge, Sir Oliver, ***"Ether (in physics)"***, Encyclopaedia Britannica, Fourteenth Edition, Volume 8, Pages 751-755, (1937)
http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105
In relation to the speed of light, *"The most probable surmise or guess at present is that* ***the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion,*** *circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed"*

[6] Tombe, F.D., ***"The Double Helix Theory of the Magnetic Field"*** (2006) Galilean Electrodynamics, Volume 24, Number 2, p.34, (March/April 2013)
https://www.researchgate.net/publication/295010637_The_Double_Helix_Theory_of_the_Magnetic_Field

[7] Tombe, F.D., ***"The Double Helix and the Electron-Positron Aether"*** (2017)
https://www.researchgate.net/publication/319914395_The_Double_Helix_and_the_Electron-Positron_Aether

[8] Tombe, F.D., ***"The Positronium Orbit in the Electron-Positron Sea"*** (2020)
https://www.researchgate.net/publication/338816847_The_Positronium_Orbit_in_the_Electron-Positron_Sea

[9] Tombe, F.D., ***"Wireless Radiation Beyond the Near Magnetic Field"*** (2019)
https://www.researchgate.net/publication/335169091_Wireless_Radiation_Beyond_the_Near_Magnetic_Field

[10] Maxwell, J.C., ***"A Dynamical Theory of the Electromagnetic Field"***, Philos. Trans. Roy. Soc. London 155, pp. 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531--536 (1864). Maxwell presents his eight original equations in Part III under the heading *'General Equations of the Electromagnetic Field'* which begins on page 480. Maxwell's derivation of the electromagnetic wave equation is found in Part VI entitled *'Electromagnetic Theory of Light'* which begins on page 497.
http://www.zpenergy.com/downloads/Maxwell_1864_3.pdf

[11] Tombe, F.D., ***"Maxwell's Displacement Current and Capacitors"*** (2020)
https://www.researchgate.net/publication/338669407_Maxwell's_Displacement_Current_and_Capacitors

[12] Tombe, F.D., ***"Maxwell's Displacement Current"*** (2019)
https://www.researchgate.net/publication/335276838_Maxwell's_Displacement_Current

[13] Maxwell, J.C., ***"A Treatise on Electricity and Magnetism"*** Volume II, pp. 438-439 (1873)
https://archive.org/details/ATreatiseOnElectricityMagnetism-Volume2/page/n461/mode/2up

[14] Tombe, F.D., ***"Cable Telegraphy and Poynting's Theorem"*** (2019) Section **IV**
https://www.researchgate.net/publication/334654102_Cable_Telegraphy_and_Poynting's_Theorem