Cherenkov radiation

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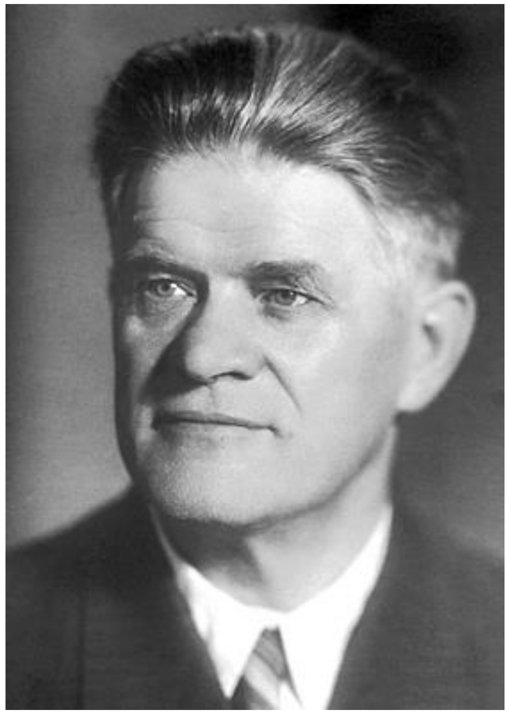
Abstract

Cherenkov radiation is the electromagnetic radiation emitted when a charged particle (such as an electron) passes through an insulator at a constant speed greater than the speed of light in that medium. In this article, we provide a simple, concise discussion about "**Cherenkov radiation**" which demonstrates the characteristic blue glow of an underwater nuclear reactor.



Pavel Alekseyevich Cherenkov was a Soviet physicist who shared the Nobel Prize in physics in 1958 with **Ilya Frank** and **Igor Tamm** for the discovery of Cherenkov radiation, made in 1934.

In some situations, photon behaves like a wave, while in others, it behaves like a particle. The photons can be thought of as both waves and particles. In 1924 a French physicist **Louis de Broglie** developed a formula to relate this dual wave and particle behavior:

$$E = h\upsilon, \qquad c = \lambda\upsilon, \qquad E = \frac{hc}{\lambda} = mc^2,$$

where E and m are the energy and mass of the photon, υ and λ are the frequency and wavelength of the photon, h is the Planck constant, c is the speed of light. From this we obtain the definition of the photon wavelength through the **Planck constant** and the momentum of the photon:

$$\lambda = \frac{h}{mc}$$

This equation is used to describe the wave properties of matter, specifically, the wave nature of the electron:

$$\lambda_e = \frac{h}{m_e v}$$

where λ_e is wavelength, h is Planck's constant, $m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is the relativistic mass of the electron, moving at a velocity v.

$$p_e = \frac{h}{\lambda_e}$$

From this it follows that,

$$\frac{dp_e}{dt} = -\frac{d\lambda_e}{dt} \times \frac{h}{\lambda_e^2}$$

$$\frac{dp_e}{dt} = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}$$

Sir Isaac Newton first presented his three laws of motion in the "**Principia Mathematica Philosophiae Naturalis**" in 1686. His second law defines a force exerted on the electron to be equal to the rate of change in momentum of the electron:

$$F = \frac{dp_e}{dt}$$

$$\boxed{F = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}}$$

According to the law that nothing may travel faster than the speed of light – i.e., according to the **Albert Einstein's law of variation of mass with velocity** (the most famous formula in the world. In the minds of hundreds of millions of people it is firmly associated with the menace of atomic weapons. Millions perceive it as a symbol of **relativity theory**):

$$m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

That the electron's mass $\mathbf{m}_e$ in motion at speed $\mathbf{v}$ is the mass $\mathbf{m_0}$ at rest divided by the factor $\sqrt{1-\frac{v^2}{c^2}}$ implies: the mass of the electron is not constant; it varies with changes in its velocity.

Differentiating the above equation, we get:

$$m_e v\, dv + v^2 dm_e = c^2 dm_e$$

$$dm_e\,(c^2 - v^2) = m_e v\, dv$$

In relativistic mechanics (the arguably most famous cult of modern physics, which has a highly interesting history which dates back mainly to Albert Einstein and may be a little earlier to **H. Poincaré**), we define the energy $\mathbf{m}_e\mathbf{c}^2$ which a moving electron possess to be = $m_0c^2 + E_k$.

$$\mathbf{m}_e\mathbf{c}^2 = m_0c^2 + E_k$$

$$\frac{dm_e c^2}{dt} = \frac{dE_k}{dt} = Fv$$

$$F = \frac{dp_e}{dt} = \frac{dm_e}{dt}v + \frac{dv}{dt}\mathbf{m}_e$$

$$F = F\frac{v^2}{c^2} + \mathbf{m}_e\, a$$

$$F = \frac{m_e a}{1-\frac{v^2}{c^2}}$$

So as v approaches c, the bottom term approaches zero and therefore the force approaches infinity. It requires an infinite amount of force to accelerate the electron to the speed of light.

Because:

$$m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Therefore:

$$F = \frac{m_e^3 a}{m_0^2}$$

For non-relativistic case ($v << c$), the above equation reduces to $F = m_0a$.

$$\frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt} = \frac{m_e^3 a}{m_0^2}$$

From this it follows that,

$$a = \frac{m_0^2 v^2}{h m_e} \times -\frac{d\lambda_e}{dt}$$

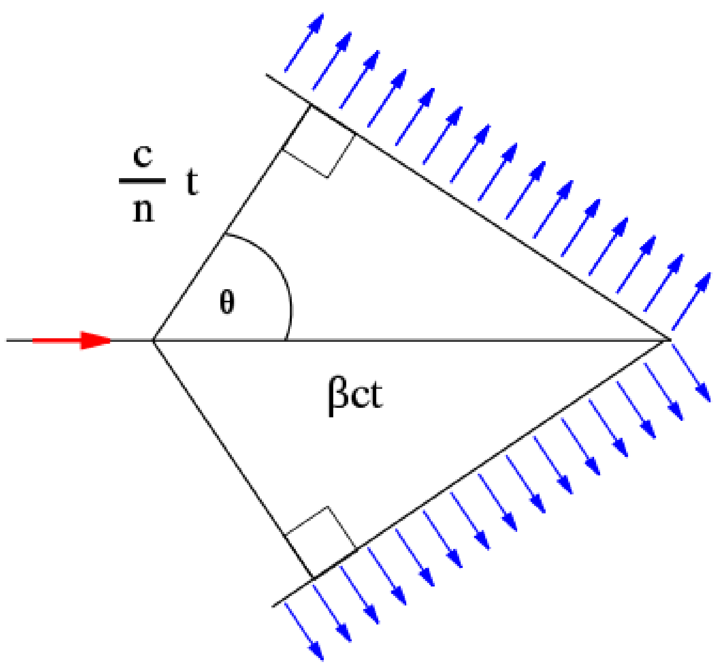
Thus, we have the formula for the calculation of the acceleration of the electron.

Cherenkov radiation is the electromagnetic radiation emitted when a charged particle (such as an electron) travels in a medium with speed **v** such that:

$$\frac{c}{n} < v < c$$

where c is speed of light in vacuum, and n is the refractive index of the medium. We define the ratio between the speed of the particle and the speed of light as:

$$\beta = \frac{v}{c}$$



Using simple trigonometric relation one can determine the Cherenkov angle:

$$\cos\theta = \frac{1}{n\beta}$$

$$\cos\theta = \frac{c}{nv}$$

$$\frac{v^2}{c^2} = \frac{1}{n^2 \cos^2\theta}$$

Since the charged particle is relativistic, we can use the relation:

$$a = \frac{m_0^2 v^2}{h m_e} \times -\frac{d\lambda_e}{dt}$$

$$a = \frac{v^2\sqrt{c^2 - v^2}}{c^2 \lambda_C} \times -\frac{d\lambda_e}{dt}$$

The heavier the charged particle, the higher kinetic energy it must possess to be able to emit Cherenkov radiation.

where $\lambda_C = \frac{h}{m_0 c}$ is the Compton wavelength of the electron.

$$a = \frac{\upsilon_C \sqrt{1-\beta^2}}{n^2 \cos^2\theta} \times -\frac{d\lambda_e}{dt}$$

where υ_C is the Compton frequency of the electron.

$\cos\theta = \frac{c}{nv}$ — The emission of **Cherenkov radiation** depends on the refractive index n of the medium or the velocity v of the particle in that medium.

$$E_e^2 - E_0^2 = (m_e c^2 + m_0 c^2)\,(m_e c^2 - m_0 c^2)$$

Since:

$$(m_e c^2 - m_0 c^2) = E_k$$

$$E_e = \sqrt{E_0^2 + p_e^2 c^2}$$

Therefore:

$$E_k = \frac{p_e^2}{(m_e + m_0)} = \frac{m_e^2 v^2}{(m_e + m_0)}$$

$$E_k = \frac{m_e^2 c^2}{n^2 \cos^2\theta\ (m_e + m_0)}$$

The mass m_e of an electron moving with a velocity v is given by $m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where: m_0 = rest mass of electron and c = speed of light.

$$\sqrt{c^2 - v^2} = \frac{m_0 c}{m_e}$$

$$\frac{\sqrt{c^2 - v^2}}{v} = \frac{m_0 c}{m_e v}$$

$$\sqrt{\frac{c^2}{v^2} - 1} = \frac{\lambda_e}{\lambda_C}$$

$$\lambda_e = \frac{\lambda_C}{\sqrt{\frac{c^2}{v^2} - 1}} = \frac{\lambda_C}{\sqrt{n^2 \cos^2\theta - 1}}$$

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