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Maxwell's Displacement Current in the Two Gauges

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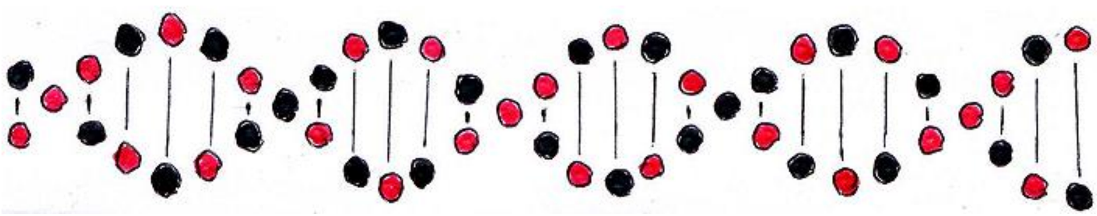

Abstract. ***Displacement current was originally conceived by James Clerk Maxwell in 1861 in connection with linear polarization in a dielectric solid which he believed to pervade all of space. Modern textbooks however adopt a different approach. The official teaching today is that displacement current is a consequence of extending the original solenoidal Ampère's Circuital Law to embrace the conservation of electric charge. Yet, unless either of these two methods leads to a displacement current that is related to Faraday's Law of Induction, then it cannot serve its main purpose, which is to provide a bridge between Ampère's Circuital Law and Faraday's Law, hence enabling the derivation of the electromagnetic wave equations. This matter will be investigated in both the Coulomb gauge and the Lorenz gauge.***

Ampère's Circuital Law

I. The original derivation of *Ampère's Circuital Law* assumes that an electric circuit will be closed, and that there will be no accumulation of electric charge at any point along the circuit. Since this condition does not hold in the case of a charging or discharging capacitor, then Ampère's Circuital Law, in its original solenoidal form, cannot hold in that context. However, when we take into consideration the equation of continuity of electric charge, we can expand Ampère's circuital law into the extended form,

$$\nabla\times\mathbf{B} = \mu(\mathbf{J} + \varepsilon\partial\mathbf{E}_S/\partial t) \qquad \textbf{(1)}$$

where the electrostatic term, $\mathbf{E}_S$, satisfies Gauss's Law,

$$\nabla \cdot \mathbf{E}_S = \rho/\varepsilon \qquad \textbf{(2)}$$

The extra $\varepsilon\partial\mathbf{E}_S/\partial t$ term is known as *Maxwell's Displacement Current*, and it will ensure that the divergence of the right-hand-side of the equation remains zero. It is important to note though, that equation **(1)** is not derived in the textbooks from first principles, but rather by force-fitting with the benefit of hindsight, and it will be realized later, that just as the *Lorenz gauge* is also a force-fit, the displacement current term, $\varepsilon\partial\mathbf{E}_S/\partial t$, is in fact itself a result of extending Ampère's circuital law to incorporate the Lorenz gauge, where it had previously only incorporated the *Coulomb gauge*. It is therefore ironic that Maxwell, the architect of the displacement current, was opposed to the Lorenz gauge, which was named after Danish physicist Ludvig Lorenz, who proposed it in 1867, **[1]**. It will be seen later though, that the displacement current that is involved in wireless electromagnetic radiation, will actually emerge from the original solenoidal version of Ampère's circuital law.

Meanwhile, if we are looking for a derivation of equation **(1)** from first principles, rather than simply a retrospective justification, we can look to a paper written by Dr. Zhong-Cheng Liang of the Nanjing University of Posts and Telecommunications entitled ***"Dark matter and real-particle field theory"***, **[2]**. The subject matter of this paper is actually more fundamental than electromagnetism. In Dr. Liang's paper, he fills all of space with what he refers to as elastic electrons, and from two fundamental field equations, he derives the parent equation *(seen as equations (36) and (B17) in his paper)* that underlies Ampère's circuital law in its full form. It's important to note however, that in Dr. Liang's paper, the speed of light has not yet entered the proceedings. The speed of light is something that will follow later from the elasticity and the density of the particle field.

The Speed of Light

II. Although the primary justification for the displacement current term in equation **(1)** lies in the conservation of charge, this was not the basis upon which James Clerk Maxwell first conceived of the idea. Maxwell first conceived of displacement current in Part III of his 1861 paper ***"On Physical Lines of Force"***, **[3]**, in conjunction with an all-pervading elastic dielectric solid. In 1855, Wilhelm Eduard Weber and Rudolf Kohlrausch, by discharging a Leyden Jar (a capacitor), demonstrated that the ratio of the electrostatic and electrodynamic units of charge is equal to $c\sqrt{2}$, where

c is the directly measured speed of light, [4]. On converting from electrodynamic units into electromagnetic units, Maxwell exposed the speed of light directly, and by comparing the Weber-Kohlrausch ratio with the ratio of the transverse elasticity to the density of his dielectric solid, Maxwell concluded that his dielectric solid is the all-pervading *luminiferous medium* that is responsible for electric, magnetic, and optical phenomena. Meanwhile, back in Part I of the same paper, Maxwell had already derived Ampère's circuital law hydrodynamically in its original solenoidal form. Then, by employing the concept of linear polarization in his dielectric solid, Maxwell presented displacement current in the form $\varepsilon\partial\mathbf{E}_S/\partial t$, where the electric displacement, $\mathbf{D}$, is equal to $\varepsilon\mathbf{E}_S$, and where the displacement current, $\mathbf{J}_D$, is equal to $\partial\mathbf{D}/\partial t$. The electric permittivity, ε, is inversely related to the dielectric constant, which is in turn a measure of the transverse elasticity. Maxwell had therefore assembled equation **(1)** above from two separate parts. And since linear polarization and charge separation in a capacitor are closely related topics, Maxwell was probably dealing with a phenomenon that involves the conservation of charge at a deeper level.

Wireless Electromagnetic Radiation

III. In his 1865 paper, ***"A Dynamical Theory of the Electromagnetic Field"***, [5], Maxwell's displacement current, which had originally been tied up with linear polarization and the electrostatic force, $\mathbf{E}_S$, instead became associated with the time-varying electromagnetic induction force, $\mathbf{E}_K$. It's a major omission on Maxwell's part that he made no attempt to physically justify this transfer of association. Nevertheless, the mathematical justification alone is sufficient indication that Maxwell was on the right tracks, further indicating that displacement current comes in two distinct varieties, and that for the purposes of deriving the electromagnetic wave equations, we are not interested in an electrostatic-based displacement current, but rather in one that is based on time-varying electromagnetic induction. This requires that the dielectric nature of the luminiferous medium is no longer sufficient on its own to explain the elasticity that is associated with a magnetization-based displacement current. We need to refer back to the all-pervading sea of tiny molecular vortices, [3], [6], [7], that Maxwell used in Part II of his 1861 paper in order to explain electromagnetic induction.

We will identify the vector field, $\mathbf{A}_C$, with the circumferential momentum circulating around the edge of these fine-grained vortices. As such, the divergence of $\mathbf{A}_C$ will be zero, and this is the essence of the Coulomb gauge. If we define $\mathbf{A}$ in general as,

$$\mathbf{A} = \mu/4\pi\int_V (\mathbf{J}dV)/r \quad \textbf{(3)}$$

then the Coulomb gauge is the transverse component of **A** within the context of a single vortex. Since the electric field in the displacement current needs to be interchangeable with the electric field in *Faraday's Time Varying Law of Induction*, if it is to be used to derive the electromagnetic wave equations, this means that it should take the mathematical form, $\varepsilon\partial\mathbf{E}_K/\partial t$, such that,

$$\mathbf{E}_K = -\partial\mathbf{A}_C/\partial t \quad \textbf{(4)}$$

where **B** is the vorticity of this circulating current, $\mathbf{A}_C$, as in,

$$\nabla\times\mathbf{A}_C = \mathbf{B} \quad \textbf{(5)}$$

Then further taking the curl of **B**, this expands to,

$$\nabla\times\nabla\times\mathbf{A}_C = \nabla(\nabla\cdot\mathbf{A}_C) - \nabla^2\mathbf{A}_C \quad \textbf{(6)}$$

In Dr. Liang's paper, **[2]**, if we equate α_s with magnetic permeability, μ, while equating c with the speed of light, then Dr. Liang's equation (B17) becomes equivalent to equation **(1)**. Hence, equation **(6)** becomes the special case of Dr. Liang's equation (B16), *in the Coulomb gauge*, and since,

$$\nabla(\nabla\cdot\mathbf{A}) = \varepsilon\partial\mathbf{E}_S/\partial t = 0 \quad \textbf{(7)}$$

being in the Coulomb gauge, equation **(1)** then reduces to,

$$\nabla\times\mathbf{B} = \mu\mathbf{J} \quad \textbf{(8)}$$

which is the original solenoidal form. In the solenoidal context of the perimeter momentum of one of Maxwell's tiny molecular vortices, this results in Ampère's circuital law adopting the mathematical form,

$$\nabla\times\mathbf{B} = \mu\varepsilon\partial\mathbf{E}_K/\partial t \quad \textbf{(9)}$$

with the Coulomb gauge guaranteeing that both sides of the equation will have zero divergence.

If we consider Maxwell's vortices to be dipolar, each comprising of an aether sink (electron) and an aether source (positron), then the induction-based displacement current (in the Coulomb gauge) will be an oscillatory phenomenon tangential to these tiny rotating electron-positron dipoles that fill all of space, and such that pure electric fluid (aether) swirls across from the positron of one dipole into the electron of its neighbour, with this repeating again indefinitely with respect to the next neighbour along the line until the wave is absorbed by a target, **[8]**, **[9]**. We know from equation **(4)** that displacement current in this context is equal to $-\varepsilon\partial^2\mathbf{A}_C/\partial t^2$, and from the oscillatory nature we know that,

$$\mathbf{A}_C = -\varepsilon\partial^2\mathbf{A}_C/\partial t^2 \qquad \textbf{(10)}$$

which means that displacement current is one and the same thing as the circumferential momentum, **[10]**. Maxwell referred to the circumferential momentum as the *electromagnetic momentum* and he identified it with Faraday's *electrotonic state,* yet he never identified it with his displacement current, as he should have done. In modern textbooks, $\mathbf{A}_C$ is referred to as the *magnetic vector potential*.

In the preamble to Part III of Maxwell's 1861 paper, where his sea of molecular vortices gradually gives way to a dielectric solid, he says, ***"I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another."***

Conclusion

IV. The Coulomb gauge and the Lorenz gauge are mutually perpendicular aspects of a single phenomenon. This can be explained within the context of one of the tiny molecular vortices that James Clerk Maxwell presumed to fill all of space. The Coulomb gauge pertains to the transverse aether flow, whereas the Lorenz gauge pertains to the radial flow. It's therefore ironic that the Coulomb gauge does not relate to the radial electrostatic Coulomb force, $\mathbf{E}_S$, but rather to the transverse electromagnetic force, $\mathbf{E}_K$, that is involved when these tiny vortices are angularly accelerating (or precessing). The transverse force is the force that is associated with time-varying electromagnetic induction and with wireless electromagnetic radiation. The radial electrostatic Coulomb force on the other hand is associated with the Lorenz gauge. In the dynamic state when radiation is passing through, these vortices are undergoing an oscillatory angular acceleration, and the *electric fluid* (aether) of which the dipolar vortices are comprised, is being swirled from vortex to vortex, **[9], [11].**

When Maxwell first conceived of the concept of displacement current in his1861 paper, **[3]**, he did so in the context of dielectric polarization and the electrostatic Coulomb force, hence he was working inadvertently in the Lorenz gauge. Yet, when he came to deriving the electromagnetic wave equation in the magnetic disturbance, **H**, in his 1865 paper, **[5]**, he switched to the Coulomb gauge by eliminating the electrostatic Coulomb force in the derivation. Hence displacement current as it is used in the derivation of the electromagnetic wave equations is an induction effect, not directly measurable by experiment. It is an action in its own right, capable of self-propagation in a wave mechanism, and it is not the displacement current originally derived by Maxwell, and neither is it the displacement current that is derived in the textbooks in connection with capacitors. The textbooks therefore teach the wrong displacement current for the purposes of deriving the electromagnetic wave equations. The Lorenz gauge-based displacement current which is taught in the textbooks is not an action in its own right, but rather the reaction to an externally applied electric field, and so it could not be involved in the mechanism of a self-propagating wave. Maxwell believed that Lorenz had missed the point entirely and that we should be using the Coulomb gauge.

Both gauges are of course valid, depending on the context. The Coulomb gauge is the relevant gauge when it comes to the wireless electromagnetic wave propagation mechanism, whereas, in DC transmission line pulses, we would be operating in the Lorenz gauge.

References

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[5] Maxwell, J.C., ***"A Dynamical Theory of the Electromagnetic Field"***, Philos. Trans. Roy. Soc. London **155**, pp 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531-536 (1864).
The derivation of the electromagnetic wave equation in **H** begins on page 497 in the first link below. Note how the electrostatic component, Ψ, is eliminated after equation (68), hence leaving the elastic displacement mechanism in the wave as an effect that is connected exclusively with time-varying electromagnetic induction.
http://www.zpenergy.com/downloads/Maxwell_1864_4.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_5.pdf

[6] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity"***, Chapter 4, pages 100-102, (1910)
*"All space, according to the younger Bernoulli, is permeated by **a fluid aether, containing an immense number of excessively small whirlpools.** The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; **for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools**."*

[7] O'Neill, John J., ***"PRODIGAL GENIUS, Biography of Nikola Tesla"***, Long Island, New York, 15th July 1944, Fourth Part, paragraph 23, quoting Tesla from his 1907 paper ***"Man's Greatest Achievement"*** which was published in 1930 in the Milwaukee Sentinel,
"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".
http://www.rastko.rs/istorija/tesla/oniell-tesla.html
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[8] Tombe, F.D., ***"The Double Helix Theory of the Magnetic Field"*** (2006)
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http://gsjournal.net/Science-Journals/Research%20Papers-Mathematical%20Physics/Download/6371
See also ***"The Double Helix and the Electron-Positron Aether"*** (2017)
http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7057

[9] The 1937 Encyclopaedia Britannica article on *'Ether'* discusses its structure in relation to the cause of the speed of light. It says, *"POSSIBLE STRUCTURE. __ **The question arises as to what that velocity can be due to. The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves _i.e., periodic disturbances across the line of propagation_ and would transmit them at a rate of the order of magnitude as the vortex or circulation speed - - - -"***
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Appendix I
(The Biot-Savart Law in the Coulomb Gauge)

"The Double Helix Theory of the Magnetic Field" **[8]**, is essentially Maxwell's sea of aethereal vortices but with the vortices replaced by rotating electron-positron dipoles. Within the context of a single rotating electron-positron dipole, the angular momentum can be written as $\mathbf{H} = \mathbf{D}\times\mathbf{v}$, where $\mathbf{D}$ is the displacement from the centre of the dipole and $\mathbf{v}$ is the circumferential velocity. When elastically bonded to other dipoles within the wider electron-positron sea, the displacement $\mathbf{D}$ will be related to the transverse elasticity through Maxwell's fifth equation, $\mathbf{D} = \varepsilon\mathbf{E}$. A full analysis can be seen in the articles ***"Radiation Pressure and $E = mc^2$"*** **[12]**, and ***"The 1855 Weber-Kohlrausch Experiment"*** **[4]**. If we substitute $\mathbf{D} = \varepsilon\mathbf{E}$ into the equation $\mathbf{H} = \mathbf{D}\times\mathbf{v}$, this leads to,

$$\mathbf{H} = -\varepsilon\mathbf{v}\times\mathbf{E}_C \qquad \textbf{(11)}$$

See **Appendix II** regarding why the magnitude of $\mathbf{v}$ should necessarily be equal to the speed of light. Equation **(11)** would appear to be equivalent to the Biot-Savart Law if $\mathbf{E}_C$ were to correspond to the Coulomb electrostatic force. However, in the context, $\mathbf{E}_C$ will be the centrifugal force, $\mathbf{E}_C = \mu\mathbf{v}\times\mathbf{H}$, and not the Coulomb force. If we take the curl of equation **(11)** we get,

$$\nabla\times\mathbf{H} = -\varepsilon[\mathbf{v}(\nabla\cdot\mathbf{E}_C) - \mathbf{E}_C(\nabla\cdot\mathbf{v}) + (\mathbf{E}_C\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{E}_C] \qquad \textbf{(12)}$$

Since $\mathbf{v}$ is an arbitrary particle velocity and not a vector field, this reduces to,

$$\nabla\times\mathbf{H} = -\varepsilon[\mathbf{v}(\nabla\cdot\mathbf{E}_C) - (\mathbf{v}\cdot\nabla)\mathbf{E}_C] \qquad \textbf{(13)}$$

Since $\mathbf{v}$ and $\mathbf{E}_C$ are perpendicular, the second term on the right-hand side of equation **(13)** vanishes. In a rotating dipole, the aethereal flow from positron to electron will be cut due to the vorticity, the separate flows surrounding the electron and the positron will be passing each other in opposite directions, and so the Coulomb force of attraction will be disengaged. Hence, the two particles will press against each other with centrifugal force while striving to dilate, since the aether can't pass laterally through itself, and meanwhile the two vortex flows will be diverted up and down into the axial direction of the double helix, [8]. Despite the absence of the Coulomb force in the equatorial plane, $\mathbf{E}_C$ is still nevertheless radial, and like the

Coulomb force, as explained in **Appendix III**, it still satisfies Gauss's Law, this time with a negative sign in the form,

$$\nabla\cdot\mathbf{E}_C = -\rho/\varepsilon \qquad \textbf{(14)}$$

Substituting into equation **(13)** leaves us with,

$$\nabla\times\mathbf{H} = \rho\mathbf{v} = \mathbf{J} = \mathbf{A}_C \qquad \textbf{(15)}$$

and hence since $\mathbf{B} = \mu\mathbf{H}$ then,

$$\nabla\times\mathbf{B} = \mu\mathbf{J} = \mu\mathbf{A}_C \qquad \textbf{(16)}$$

which is Ampère's Circuital Law in the Coulomb gauge as per equation **(9)**.

Appendix II
(The Speed of Light)

Starting with the Biot-Savart law in the Coulomb gauge, $\mathbf{H} = -\varepsilon\mathbf{v}\times\mathbf{E}_C$, where $\mathbf{E}_C = \mu\mathbf{v}\times\mathbf{H}$, means that we can then write $\mathbf{H} = -\varepsilon\mu\mathbf{v}\times(\mathbf{v}\times\mathbf{H})$. It follows therefore that the modulus $|\mathbf{H}|$ is equal to $\varepsilon\mu v^2$H since $\mathbf{v}$, $\mathbf{E}_C$, and $\mathbf{H}$ are mutually perpendicular within a rotating electron-positron dipole. Hence, from the ratio $\varepsilon\mu = 1/c^2$, it follows that the circumferential speed v must be equal to c within such a rotating dipole. In other words, the ratio $\varepsilon\mu = 1/c^2$ hinges on the fact that the circumferential speed in Maxwell's molecular vortices is equal to the speed of light.

Appendix III
(Gauss's Law for Centrifugal Force)

Taking the divergence of the centrifugal force, $\mathbf{E}_C = \mu\mathbf{v}\times\mathbf{H}$, we expand as follows,

$$\nabla\cdot(\mu\mathbf{v}\times\mathbf{H}) = \mu[\mathbf{H}\cdot(\nabla\times\mathbf{v}) - \mathbf{v}\cdot(\nabla\times\mathbf{H})] \qquad \textbf{(17)}$$

Since $\mathbf{v}$ refers to a point particle that is in arbitrary motion, and not to a vector field, then $\nabla\times\mathbf{v} = 0$, and since $\nabla\times\mathbf{H} = \mathbf{J} = \rho\mathbf{v}$, it follows that,

$$\nabla\cdot(\mu\mathbf{v}\times\mathbf{H}) = -\mu\rho\mathbf{v}\cdot\mathbf{v} \qquad \textbf{(18)}$$

then substituting $v = c$ as per **Appendix II**,

$$\nabla\cdot(\mu\mathbf{v}\times\mathbf{H}) = -\mu\rho c^2 \qquad \textbf{(19)}$$

and substituting $c^2 = 1/\mu\varepsilon$, this leaves us with,

$$\nabla\cdot(\mu\mathbf{v}\times\mathbf{H}) = -\rho/\varepsilon \qquad \textbf{(20)}$$

which is a negative version of Gauss's law for centrifugal force.

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