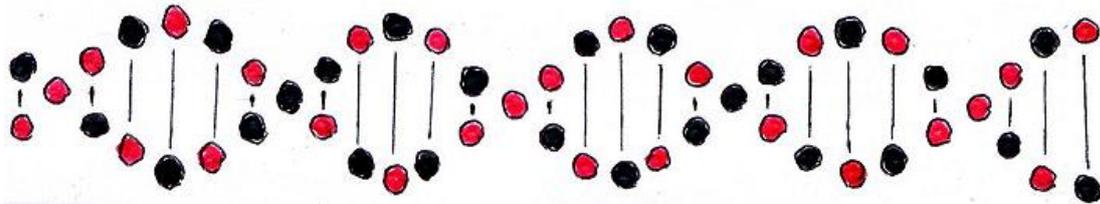


## Ampère's Circuital Law and Displacement Current

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**Abstract.** *Ampère's Circuital Law is the most controversial of Maxwell's equations due to its association with displacement current. The controversy centres around the fact that Maxwell's entire physical basis for introducing the concept of displacement current in the first place, was the existence of a dense sea of molecular vortices pervading all of space. The modern-day physical parameter known as the electric permittivity,  $\epsilon$ , being reciprocally related to the dielectric constant, is historically rooted in the elasticity of this medium. Indeed, the dielectric constant served as the vehicle through which the speed of light was imported into the analysis from the 1855 Weber-Kohlrausch experiment, yet the medium itself has since been totally eliminated from the textbooks. In order to understand how the omission of Maxwell's vortex sea has impacted upon electromagnetic theory, this article will take a close examination of both the Biot-Savart Law and Ampère's Circuital Law.*

### Ampère's Force Law

**I.** *Ampère's Force Law* should not be confused with *Ampère's Circuital Law*. The force law,

$$\mathbf{F} = \mu/4\pi[I_1I_2\oint\oint dl_1 \times (dl_2 \times \hat{r})/r^2] \quad (1)$$

is the magnetic force which acts between two closed electric circuits. ( $\mu$  is magnetic permeability,  $I$  is electric current,  $dl$  is an element of conducting wire, and  $r$  is the distance between elements in opposite circuits) If we define the vector  $\mathbf{B}$  as,

$$\mathbf{B} = \mu/4\pi[I_2\oint(dl_2\times\hat{\mathbf{r}})/r^2] \quad (2)$$

then equation (1) becomes,

$$\mathbf{F} = I_1\oint dl_1\times\mathbf{B} \quad (3)$$

which is identical in principle to the convective term in Maxwell's original electromotive force equation, see Section VIII below. This equation is nowadays known by the misnomer "*The Lorentz Force*", and written as,

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B} \quad (4)$$

where  $q$  is the electric charge of a moving particle and  $\mathbf{v}$  is its velocity relative to Maxwell's sea of molecular vortices. See Section III below. Equation (2) is the integral form of the *Biot-Savart Law*.

### Ampère's Circuital Law

II. *Ampère's Circuital Law* is not a force law. It is a law describing the relationship between an electric current and the surrounding magnetic field. In differential form is written as,

$$\nabla\times\mathbf{B} = \mu\mathbf{J} \quad (5)$$

where  $\mathbf{J}$  is the electric current density. The controversy begins with the assumption that in a region of space that is free of current density, equation (5) becomes,

$$\nabla\times\mathbf{B} = 0 \quad (6)$$

While equation (6) serves to indicate a conservative force, it ignores any physical details relating to the magnetic field in space. This is because we are working in the Lorenz gauge. In order to make equation (5) more relevant to wireless EM radiation in space, we need to derive a form of Ampère's circuital law in the Coulomb gauge. This is attempted in Section VII below.

## Maxwell's Displacement Current

III. It was Maxwell's intention when first proposing the existence of displacement current in the preamble to Part III of his 1861 paper "*On Physical Lines of Force*" [1], that displacement current was a very real electric current in an all-pervading sea of tiny aethereal vortices. Owing to the presence of this sea of molecular vortices, there will be a circulating electric current at every point in space which we will identify with the magnetic vector potential  $\mathbf{A}$ , and also with displacement current, [2]. Maxwell himself identified the vector  $\mathbf{A}$  with Faraday's *electrotonic state* and he called it the *electromagnetic momentum*, but ironically, he never linked it with his very own displacement current. In a steady state magnetic field, we should expect displacement current  $\mathbf{A}$  to be a localized fine-grained circulation of *electric fluid* such that,

$$\nabla \times \mathbf{A} = \mathbf{B} \quad (7)$$

This is Maxwell's second equation in his original listing of eight, [3], and it places  $\mathbf{B}$  into the realm of being a vorticity/angular momentum density. It is now proposed that Ampère's Circuital Law, when applied in deep space, should take the general form,

$$\nabla \times \mathbf{B} = \mu \mathbf{A} \quad (8)$$

while noting that as compared with equation (2), the polar origin has now been shifted from the source electric current elements into the fine-grained structure of the magnetic field itself.

In the dynamic state where time varying electromagnetic induction is occurring, then from Faraday's Law we have,

$$\mathbf{E} = -\partial \mathbf{A} / \partial t \quad (9)$$

and in the oscillating state, the harmonic relationship,

$$\mathbf{A} = -\varepsilon \partial^2 \mathbf{A} / \partial t^2 \quad (10)$$

will hold. The electric permittivity,  $\varepsilon$ , is the elastic constant, inversely related to the dielectric constant in Maxwell's original papers. Hence equation (8) takes on the familiar form,

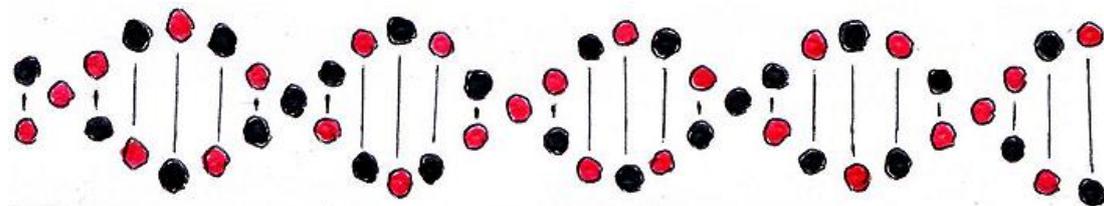
$$\nabla \times \mathbf{B} = \mu \varepsilon \partial \mathbf{E} / \partial t \quad (11)$$

The dynamic state equation (11) is therefore a special case of the general equation (8) relating to wireless radiation, where it is proposed that the displacement current  $\mathbf{A}$  involves a net flow of electric fluid from vortex to vortex while transferring mass and energy through space [4], [5]. This would explain the ballistic nature of light. We have therefore established a continuity in Ampère's Circuital Law as between the steady state and the dynamic state, which would be impossible in a pure vacuum.

## Magnetic Attraction as a Conservative Force

IV. The format of Ampère's Force Law at equation (1) with its inverse square law in distance, along with the product of electric currents in the numerator, has got all the outward characteristics of a conservative *Gauss force* in the likeness of *Coulomb's law of Electrostatics* and *Newton's Law of Gravity*. But due to electromagnetic induction, the source electric currents change during the operation of Ampère's force and this makes it difficult to definitively formulate a theory of conservation of energy in the manner that this can be done in the other two cases. Lenz's Law would suggest that conservation of energy is not breached, but it would be helpful if we could reduce the problem to a purely electrostatic problem, and this is indeed possible in the particular case of magnetic attraction.

We saw in *"The Double Helix Theory of the Magnetic Field"* [6], how rotating electron-positron dipoles could be arranged in a double helix fashion so as to account for the magnetic field. A rotating electron-positron dipole consists of an electron and a positron undergoing a mutual central force orbit such that the rotation axis is perpendicular to a line joining the electron to the positron. If we stack these dipoles on top of each other along their axes of rotation with the electrons placed approximately above the positrons and angularly synchronized in a twisted rope ladder fashion, we will effectively have a helical spring. These helical springs are magnetic lines of force by virtue of the fact that they channel an electrostatic Coulomb tension along a double helix alignment within a neutral electron-positron sea. See Fig.1 below,



**Fig. 1. A close-up view of a single magnetic line of force. A net electrostatic force is channelled along the double helix of electrons and positrons in an electrically**

**neutral electron-positron sea. The circumferential speed of the electrons and positrons is what determines the local speed of light.**

## **Centrifugal Force and Magnetic Repulsion**

**V.** The electric fluid (or aether) is the stuff that all matter is made from. Positive particles are therefore aether sources while negative particles are aether sinks. Consider two electron-positron dipoles sitting side by side while rotating in the same plane and in the same direction. When the electron of one dipole passes the positron of the other dipole in the opposite direction at closest approach, the electrostatic field lines will connect directly between the two. According to Coulomb's law there should be a force of attraction acting between them as in the case of any two particles of opposite charge. However, if the electrostatic  $\mathbf{E}$  field is tied up with an aetherreal fluid momentum density field  $\mathbf{A}$ , then due to the enormous mutual transverse speed as between the two particles, the flow lines between the two dipoles will split, and if this happens, the electrostatic attraction will be converted into a centrifugal repulsion. The  $\mathbf{E}$  field lines will remain irrotational, but their physical cause will have changed. It will no longer be due to a tension in the fluid but instead it will be due to side pressure from the flow lines, and so we will now be dealing with centrifugal force as opposed to the Coulomb force.

The rotating dipoles in one magnetic line of force will be aligned in their mutual equatorial planes with the rotating dipoles in an adjacent line of force, and the mutual transverse velocities existing between them will cause a centrifugal repulsion that will push them apart. Since the magnetic lines of force between two like poles do not join directly together, but instead spread outwards and contact each other laterally, magnetic repulsion then occurs due to centrifugal force.

## **The Significance of Ampère's Circuital Law**

**VI.** When rotating electron-positron dipoles bond together along their rotation axes to form a double helical toroid with nothing in the toroidal hole in the middle, the Coulomb attraction along the double helix would tend to make the helix collapse. If the circumferential speed of each rotating dipole is  $\mathbf{v}$ , then  $\nabla \times \mathbf{v} = \mathbf{H}$  where  $\mathbf{H}$  is the vorticity, and hence  $\nabla \cdot \mathbf{H} = 0$  meaning that  $\mathbf{H}$  is solenoidal. The speed  $\mathbf{v}$  represents the vortex flow of the electric fluid which constitutes an electric current. At the hole in the middle of the toroid there will be a concentration of electric fluid flowing in one direction, and the current density will be  $\rho \mathbf{v} = \mathbf{J}$  where  $\rho$  is

the fluid density in the hole. The concentration of electric current through the hole in the toroid prevents the toroid from collapsing into the hole. Unlike in the case of fluid pouring down a sink, a toroid involves only solenoidal flow and so the fluid circulates around indefinitely. The fluid cannot pass sideways through itself inside the toroidal hole and so the toroid cannot collapse. The double helix toroid is therefore the fundamental basis for stability and the default alignment in the electron-positron sea. It corresponds to a magnetic line of force and  $\mathbf{H}$  corresponds to magnetic intensity. And since  $\mathbf{H}$  forms a circle around the inside of the double helix, it follows therefore that  $\nabla \times \mathbf{H} = \mathbf{J}$ . This is Ampère's Circuital Law which is Maxwell's third equation. See section VIII below.

Within the context that  $\mathbf{v}$  corresponds to the circumferential current within the individual rotating dipoles that fill all of space, the vector  $\mathbf{A}$  will be defined as  $\mu \mathbf{v}$ , where  $\mu$ , known as the magnetic permeability, is a measure of the flux density of magnetic lines of force in the vicinity. In a steady state magnetic field,  $\mathbf{A}$  is a fine-grained *circulating current density* (or momentum density) in the electron-positron sea. The product  $\mu \mathbf{H}$  is known as the magnetic flux density and written as  $\mathbf{B}$ . Hence  $\nabla \times \mathbf{A} = \mathbf{B}$  and Ampère's Circuital Law becomes  $\nabla \times \mathbf{B} = \mu \mathbf{J}$ . The equation  $\nabla \times \mathbf{A} = \mathbf{B}$  is Maxwell's second equation from which  $\nabla \cdot \mathbf{B} = 0$  can be derived. Maxwell referred to the magnetic vector potential  $\mathbf{A}$  as the *electromagnetic momentum*, and although he invented displacement current, he failed to identify it with  $\mathbf{A}$ .

Ampère's Circuital Law means that when a current or a particle, neutral or otherwise, moves through the electron-positron sea, it causes the electron-positron dipoles to align with their rotation axes forming solenoidal rings around the direction of motion. This provides a circular energy flow mechanism which replaces friction with the inertial forces and magnetic repulsion. It's similar in principle to the creation of smoke rings. Maxwell explains Ampère's Circuital Law at equation (9) in Part I of his 1861 paper "*On Physical Lines of Force*" [1].

## The Biot-Savart Law

VII. Returning to the question raised in Section II above, as regards taking the curl of the Biot-Savart Law, it was pointed out that in order for the result to have any useful meaning in space, in terms of establishing the deeper physical nature of electromagnetic radiation, then we need to begin with a version of the Biot-Savart law which is compatible with the Coulomb gauge.

Within the context of a single rotating electron-positron dipole, the angular momentum can be written as  $\mathbf{H} = \mathbf{D} \times \mathbf{v}$ , where  $\mathbf{D}$  is the

displacement from the centre of the dipole and  $\mathbf{v}$  is the circumferential velocity. When elastically bonded to other dipoles within the wider electron-positron sea, the displacement  $\mathbf{D}$  will be related to the transverse elasticity through Maxwell's fifth equation,  $\mathbf{D} = \epsilon\mathbf{E}$ . A full analysis can be seen in the articles "*Radiation Pressure and  $E = mc^2$* " [7], and "*The 1855 Weber-Kohlrausch Experiment*" [8]. If we substitute  $\mathbf{D} = \epsilon\mathbf{E}$  into the equation  $\mathbf{H} = \mathbf{D} \times \mathbf{v}$ , this leads to,

$$\mathbf{H} = -\epsilon\mathbf{v} \times \mathbf{E} \quad (12)$$

See the **Appendix I** regarding why the magnitude of  $\mathbf{v}$  should be equal to the speed of light. Equation (12) would appear to be equivalent to the Biot-Savart Law if  $\mathbf{E}$  were to correspond to the Coulomb force. However, in the context,  $\mathbf{E}$  will be the centrifugal force,  $\mathbf{E}_C = \mu\mathbf{v} \times \mathbf{H}$ , and not the Coulomb force. If we take the curl of equation (12) we get,

$$\nabla \times \mathbf{H} = -\epsilon[\mathbf{v}(\nabla \cdot \mathbf{E}_C) - \mathbf{E}_C(\nabla \cdot \mathbf{v}) + (\mathbf{E}_C \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{E}_C] \quad (13)$$

Since  $\mathbf{v}$  is an arbitrary particle velocity and not a vector field, this reduces to,

$$\nabla \times \mathbf{H} = -\epsilon[\mathbf{v}(\nabla \cdot \mathbf{E}_C) - (\mathbf{v} \cdot \nabla)\mathbf{E}_C] \quad (14)$$

Since  $\mathbf{v}$  and  $\mathbf{E}_C$  are perpendicular, the second term on the right-hand side of equation (14) vanishes. In a rotating dipole, the aethereal flow from positron to electron will be cut due to the vorticity, the separate flows surrounding the electron and the positron will be passing each other in opposite directions, and so the Coulomb force of attraction will be disengaged. Hence, the two particles will press against each other with centrifugal force while striving to dilate, since the aether can't pass laterally through itself, and meanwhile the two vortex flows will be diverted up and down into the axial direction of the double helix, [6]. Despite the absence of the Coulomb force in the equatorial plane,  $\mathbf{E}_C$  is still nevertheless radial, and like the Coulomb force, it still satisfies Gauss's Law, this time with a negative sign in the form,

$$\nabla \cdot \mathbf{E}_C = -\rho/\epsilon \quad (15)$$

See Appendix **II** for the derivation. Substituting into equation (14) this leaves us with,

$$\nabla \times \mathbf{H} = \rho\mathbf{v} = \mathbf{J} = \mathbf{A} \quad (16)$$

and hence since  $\mathbf{B} = \mu\mathbf{H}$  then,

$$\nabla \times \mathbf{B} = \mu \mathbf{J} = \mu \mathbf{A} \quad (17)$$

which is Ampère's Circuital Law as per equation (8).

## Maxwell's Equations and Summary

**VIII.** Maxwell's eight original equations as listed in his 1865 paper entitled "*A Dynamical Theory of the Electromagnetic Field*" [3], in a section entitled '*General Equations of the Electromagnetic Field*', appear below in modern format. It is often claimed that there were twenty equations in the original list, however the first six of the equations below were written out three times each for each of the three Cartesian components.

The modern textbook listings are however simply, (i) The divergence of equation (B) below, as in  $\nabla \cdot \mathbf{B} = 0$ , which also appeared in Maxwell's 1861 paper, (ii) *Gauss's Law* exactly as shown at equation (G) below, (iii) Faraday's Law of Induction (time-varying only), as in the curl of the second term of the right-hand side of equation (D) below, and (iv) Ampère's Circuital Law with Maxwell's displacement current added, as in a combination of equations (A) and (C) below, which also appeared as equation (112) in Maxwell's 1861 paper. The modern textbooks also omit equation (D) below while supplementing the list with an equivalent equation under the misnomer of *The Lorentz Force*. The four equations in the textbooks therefore actually hide things rather than add anything, but the biggest problem with the textbooks is that they have dropped the physical context within which Maxwell originally derived the equations. That context was the sea of molecular vortices, and it makes a mockery of displacement current to present it as a virtual concept in the context of a pure vacuum.

### Maxwell's Original Eight Equations 1864

(A) The Equation of Total Currents including "*Displacement Current*"

$$\mathbf{J}_{total} = \mathbf{J}_{conduction} + \partial \mathbf{D} / \partial t$$

(B) The Equation of Magnetic Force

$$\nabla \times \mathbf{A} = \mu \mathbf{H}$$

(C) The Equation of Electric Current (Ampère's Circuital Law)

$$\nabla \times \mathbf{H} = \mathbf{J}_{total}$$

(D) The Equation of Electromotive Force (The Lorentz Force)

$$\mathbf{E} = \mu \mathbf{v} \times \mathbf{H} - \partial \mathbf{A} / \partial t - \text{grad} \psi$$

(E) The Equation of Electric Elasticity (Hooke's Law)

$$\mathbf{D} = \epsilon \mathbf{E}$$

(F) The Equation of Electric Resistance (Ohm's Law)

$$\mathbf{E} = R \mathbf{J}_{conduction}$$

(G) Gauss's Law for Free Electricity

$$\nabla \cdot \mathbf{D} = \rho$$

(H) The Equation of Continuity

$$\nabla \cdot \mathbf{J} + \partial \rho / \partial t = 0$$

Maxwell's second equation,  $\nabla \times \mathbf{A} = \mu \mathbf{H}$ , was derived on the basis that  $\mathbf{A}$  is a momentum density from which we can then derive the solenoidal equation  $\nabla \cdot \mu \mathbf{H} = 0$  ( $\nabla \cdot \mathbf{B} = 0$ ) that is familiar in the modern listings of four Maxwell's equations. Modern textbooks however reverse the reasoning and work backwards to  $\mathbf{A}$ , giving no physical meaning to  $\mathbf{A}$ .

One might argue that in the steady state, the momentum density will be zero since the circulating electrons and positrons in each of the tiny dipoles cancel each other due to their opposite charges. This might be so if electric current were to be primarily based on the flow of charged particles. There is however a deeper flow of electric fluid which is the primary essence of electric current. In the case of conduction currents, charged particles merely get accelerated along by the electric fluid, with negative particles eating their way in the opposite direction to that of

positive particles, and they never catch up with the aethereal flow due to electrical resistance in the conducting channels.

Nevertheless, due to the fine-grained circulation of  $\mathbf{A}$ , there will be no net displacement current in the steady state in terms of radiation propagating through the luminiferous medium. The term *displacement current* might more accurately be reserved for the dynamic state, but it nevertheless exists too in the steady state, undergoing fine-grained circulation.

Not even Maxwell himself realized that the magnetic vector potential  $\mathbf{A}$ , which he termed the *electromagnetic momentum*, in fact corresponds to his very own displacement current which he used in the derivation of the electromagnetic wave equation.

Maxwell's fourth equation ( $\mathbf{D}$ ) is easily recognizable as the Lorentz force. The three components on the right-hand side of equation ( $\mathbf{D}$ ) appeared in equations (5) and (77) in Maxwell's 1861 paper [1]. If we take the curl of equation ( $\mathbf{D}$ ) we end up with,

$$\nabla \times \mathbf{E} = -(\mathbf{v} \cdot \nabla) \mathbf{B} - \partial \mathbf{B} / \partial t = -d\mathbf{B} / dt \quad (18)$$

See **Appendix B** in "*The Double Helix Theory of the Magnetic Field*" [6] for the full analysis. Equation (18) with the convective term  $(\mathbf{v} \cdot \nabla) \mathbf{B}$  removed, hence leaving only the partial time derivative  $-\partial \mathbf{B} / \partial t$  term, is one of the equations in the modern listings of four Maxwell's equations and it is known as Faraday's Law. It deals only with time varying electromagnetic induction although Faraday's original flux law also takes into account the convective effect  $\mathbf{E}_C = \mathbf{v} \times \mathbf{B}$ .

Maxwell's fifth equation is closely related to Hooke's law and it is most relevant in relation to the elasticity of the electron-positron sea.

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[1] Clerk-Maxwell, J., "*On Physical Lines of Force*", Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)  
[http://vacuum-physics.com/Maxwell/maxwell\\_oplf.pdf](http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf)

[2] Tombe, F.D., "*Displacement Current and the Electrotonic State*" (2008)  
<http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/228>

[3] Clerk-Maxwell, J., "*A Dynamical Theory of the Electromagnetic Field*", Philos. Trans. Roy. Soc. London **155**, pp 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531-536 (1864). The original eight Maxwell's equations are found in the link below in Part III entitled '*General Equations of the Electromagnetic Field*' which begins on page 480,

[http://www.zpenergy.com/downloads/Maxwell\\_1864\\_3.pdf](http://www.zpenergy.com/downloads/Maxwell_1864_3.pdf)

Maxwell's derivation of the electromagnetic wave equation is found in the link below in Part

VI entitled '*Electromagnetic Theory of Light*' which begins on page 497,  
[http://www.zpenergy.com/downloads/Maxwell\\_1864\\_4.pdf](http://www.zpenergy.com/downloads/Maxwell_1864_4.pdf)

[4] Tombe, F.D., "*Wireless Radiation Beyond the Near Magnetic Field*" (2019)  
[https://www.researchgate.net/publication/335169091\\_Wireless\\_Radiation\\_Beyond\\_the\\_Near\\_Magnetic\\_Field](https://www.researchgate.net/publication/335169091_Wireless_Radiation_Beyond_the_Near_Magnetic_Field)

[5] The 1937 Encyclopaedia Britannica article on '*Ether*' discusses its structure in relation to the cause of the speed of light. It says, "*POSSIBLE STRUCTURE. — The question arises as to what that velocity can be due to. The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves — i.e., periodic disturbances across the line of propagation — and would transmit them at a rate of the order of magnitude as the vortex or circulation speed — — —*"  
<http://gsjournal.net/Science-Journals/Historical%20Papers-%20Mechanics%20/%20Electrodynamics/Download/4105>

[6] Tombe, F.D., "*The Double Helix Theory of the Magnetic Field*" (2006)  
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## Appendix I (The Speed of Light)

Starting with the Biot-Savart law in the Coulomb gauge,  $\mathbf{H} = -\varepsilon\mathbf{v}\times\mathbf{E}_C$ , where  $\mathbf{E}_C = \mu\mathbf{v}\times\mathbf{H}$ , means that we can then write  $\mathbf{H} = -\varepsilon\mu\mathbf{v}\times(\mathbf{v}\times\mathbf{H})$ . It follows therefore that the modulus  $|\mathbf{H}|$  is equal to  $\varepsilon\mu v^2 H$  since  $\mathbf{v}$ ,  $\mathbf{E}_C$ , and  $\mathbf{H}$  are mutually perpendicular within a rotating electron-positron dipole. Hence, from the ratio  $\varepsilon\mu = 1/c^2$ , it follows that the circumferential speed  $v$  must be equal to  $c$  within such a rotating dipole. In other words, the ratio  $\varepsilon\mu = 1/c^2$  hinges on the fact that the circumferential speed in Maxwell's molecular vortices is equal to the speed of light.

## Appendix II

(Gauss's Law for Centrifugal Force)

Taking the divergence of the centrifugal force,  $\mathbf{E}_C = \mu \mathbf{v} \times \mathbf{H}$ , we expand as follows,

$$\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = \mu [\mathbf{H} \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{H})] \quad (19)$$

Since  $\mathbf{v}$  refers to a point particle in arbitrary motion, and not to a vector field, then  $\nabla \times \mathbf{v} = 0$ , and since  $\nabla \times \mathbf{H} = \mathbf{J} = \rho \mathbf{v}$ , it follows that,

$$\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\mu \rho \mathbf{v} \cdot \mathbf{v} \quad (20)$$

then substituting  $v = c$  as per **Appendix I**,

$$\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\mu \rho c^2 \quad (21)$$

and substituting  $c^2 = 1/\mu\epsilon$ , this leaves us with,

$$\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\rho/\epsilon \quad (22)$$

which is a negative version of Gauss's law for centrifugal force.