

Conclusions from the Fizeau experiment

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Abstract: I approach the Fizeau experiment and its result with the purpose to know what it unveils to us when the experiment occurs in a fixed frame and Earth's inertial frame. I complete the derivations of the fringe shift of the Fizeau experiment when the experiment takes place in a fixed frame and Earth's inertial frame. The conclusion is that the observation of the experiment in an inertial frame is like the experiment carried out in a fixed frame.

Keywords: Fizeau experiment, Fresnel drag coefficient.

1 Introduction

This article calculates the fringe shift when the experimental device is in a fixed frame and Earth's inertial frame for stationary and moving water in the tubes of the interferometer.

The derivations of the fringe shift employ the Fresnel drag coefficient formula [1-2], and the formula for the velocity of light through a moving medium derived by F Dambi [3].

2 Fizeau experiment

Figure 1 illustrates the schematic of the Fizeau experiment [1, 2] in the initial position. The water in tubes 1 and 2 flows with the velocity u in the directions as marked on the drawing. A ray of light from the source is split into two rays by the beam splitter M_1 . From M_1 , the transmitted ray travels to mirrors M_2 , M_3 , M_4 , back to M_1 , and then to the screen. From M_1 , the reflected ray travels to mirrors M_4 , M_3 , M_2 , back to M_1 , and then to the screen.

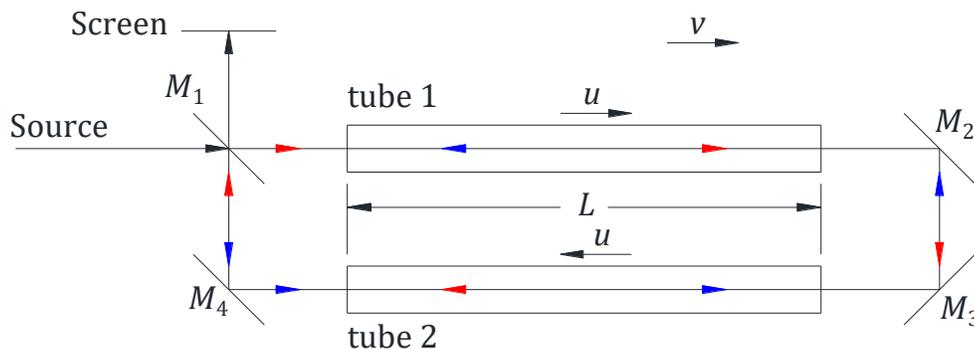


Figure 1. Schematic drawing of the Fizeau experiment.

The two split rays interfere on the screen and yield an image of fringes when water is stationary in the tubes. When the water travels with the velocity u , the image of fringes shifts in one direction. If the water change directions in tubes, the image of fringes shifts in the opposite direction.

The derivations of the fringe shift ignore the distances traveled by light through air because these dimensions cancel one another in the calculation.

In the Fizeau experiment, the velocity of water in the tubes $u = 7.059 \text{ m/s}$, length of the tubes $L = 1.487 \text{ m}$, wavelength of light $\lambda = 0.000526 \text{ m}$, and index of refraction of the water $n = 1.33$. The speed of the Earth's inertial frame $v = 3 \times 10^4 \text{ m/s}$, the speed of light $c = 3 \times 10^8 \text{ m/s}$, and speed of light through a moving medium $c_1 = c/n$.

3 Derivations with the Fresnel drag coefficient formula

The formula of the velocity of light through a moving medium with the Fresnel drag coefficient $k = 1 - 1/n^2$ is $c_2 = c_1 \pm vk = c_1 \pm v(1 - 1/n^2)$.

3.1 Stationary water in the fixed frame

In this case, the velocities u and v are zero, and the velocity of light in the moving medium $c_2 = c_1$, in any direction.

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = \frac{L}{c_2} = \frac{L}{c_1}, \quad t_{12} = \frac{L}{c_2} = \frac{L}{c_1} \Rightarrow t_1 = t_{11} + t_{12}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = \frac{L}{c_2} = \frac{L}{c_1}, \quad t_{21} = \frac{L}{c_2} = \frac{L}{c_1} \Rightarrow t_2 = t_{22} + t_{21}.$$

The difference Δt_1 is $\Delta t_1 = t_2 - t_1 = 0$.

Therefore, no fringes should appear. The path length of the transmitted and reflected rays are not quite equal, and the value of Δt_1 is not zero. Thus an image of fringes is displayed.

3.2 Moving water in the fixed frame

In this case, the velocity $v = 0$ and the velocity of light in the moving medium $c_2 = c_1 \pm uk$, for the transmitted and reflected ray, respectively.

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = \frac{L}{c_1 + uk}, \quad t_{12} = \frac{L}{c_1 + uk} \Rightarrow t_1 = t_{11} + t_{12} = \frac{2L}{c_1 + uk}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = \frac{L}{c_1 - uk}, \quad t_{21} = \frac{L}{c_1 - uk} \Rightarrow t_2 = t_{22} + t_{21} = \frac{2L}{c_1 - uk}.$$

The difference Δt_2 is $\Delta t_2 = t_2 - t_1 = 3.5870835494127800E - 16 \text{ s}$.

The difference between Δt_2 and Δt_1 is $\Delta t_{12} = \Delta t_2 - \Delta t_1 = \Delta t_2$. The fringe shift $N_{12} = c\Delta t_{12}/\lambda = c\Delta t_2/\lambda = 2.0458651422506400E - 01$ fringes. The mean of the observation in the Fizeau experiment was 0.23016 fringes.

3.3 Stationary water in Earth's inertial frame

In this case, the velocity u is zero, and the velocity of light in the moving medium is $c_2 = c_1 \pm vk$. The sign \pm depends on the direction of v and u in each tube.

The times t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = \frac{L}{c_1 + vk - v}, \quad t_{12} = \frac{L}{c_1 - vk + v} \Rightarrow t_1 = t_{11} + t_{12}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = \frac{L}{c_1 + vk - v}, \quad t_{21} = \frac{L}{c_1 - vk + v} \Rightarrow t_2 = t_{22} + t_{21}.$$

The difference Δt_3 is $\Delta t_3 = t_2 - t_1 = 0$.

For stationary water in the fixed frame as well as in the Earth's inertial frame, the differences Δt_1 and Δt_3 are equal to zero.

If the interferometer is rotated 180° from the initial position, then the times t_{11} and t_{12} switch formulas between them and also times t_{21} and t_{22} . The times t_1 and t_2 have the same formulas for the initial position and the 180° position. Thus, rotating the interferometer 360° , no fringe shift is generated.

3.4 Moving water in Earth's inertial frame

In this case, the velocity of light in the moving medium in $c_2 = c_1 \pm (v \pm u)k$ for the transmitted and reflected ray. The sign \pm depends on the direction of v and u in each tube.

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = \frac{L}{c_2 - v} = \frac{L}{c_1 + (v + u)k - v}, \quad t_{12} = \frac{L}{c_2 + v} = \frac{L}{c_1 - (v - u)k + v} \Rightarrow t_1 = t_{11} + t_{12}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = \frac{L}{c_2 - v} = \frac{L}{c_1 + (v - u)k - v}, \quad t_{21} = \frac{L}{c_2 + v} = \frac{L}{c_1 - (v + u)k + v} \Rightarrow t_2 = t_{22} + t_{21}.$$

The difference Δt_4 is $\Delta t_4 = t_2 - t_1 = 3.5870836238590400E - 16$ s.

The difference between Δt_4 and Δt_3 is $\Delta t_{34} = \Delta t_4 - \Delta t_3 = \Delta t_4$. The fringe shift $N_{34} = c\Delta t_{34}/\lambda = c\Delta t_4/\lambda = 2.0458651847104800E - 01$ fringes. The fringe shift is approximately equal to the result for the fixed frame. Considering the incapability of the experiment to make a distinction between the fringe shift of this section and the fringe shift of section 3.2, can be concluded that the observation of the experiment in the inertial frame is like the experiment carried out in the fixed frame.

If in the difference of Δt_4 the speed v is zero, then

$$\Delta t_4 = \frac{2L}{c_1 - uk} - \frac{2L}{c_1 + uk} = \Delta t_2.$$

For the same reasoning as in section 3.3, if the interferometer is rotated 360° , no fringe shift is generated.

4 Derivations with the formula of the velocity of light

The formula of the velocity of light through a moving medium derived by Filip Dambi [3] is $c_2 = \sqrt{c_1^2 + (v \cos a / 2)^2} + v \cos a / 2$, where angle a in figure 1 can be 0° or 180° . Thus, the formula for the speed c_2 becomes $c_2 = \sqrt{c_1^2 + (v/2)^2} \pm v/2$ for $a = 0^\circ$ and $a = 180^\circ$.

4.1 Stationary water in the fixed frame

In this case, the velocities u and v are zero, and the velocity of light in the moving medium $c_2 = c_1$, in any direction. The result for Δt_1 is the same as in section 3.1, $\Delta t_1 = 0$.

4.2 Moving water in the fixed frame

In this case, the velocity $v = 0$ and the velocity of light in the moving medium is $c_2 = \sqrt{c_1^2 + (u/2)^2} \pm u/2$, for the transmitted and reflected ray.

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = t_{12} \Rightarrow t_1 = t_{11} + t_{12} = 2L / \left(\sqrt{c_1^2 + (u/2)^2} + u/2 \right).$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = t_{21} \Rightarrow t_2 = t_{22} + t_{21} = 2L / \left(\sqrt{c_1^2 + (u/2)^2} - u/2 \right).$$

The difference: Δt_2 is $\Delta t_2 = t_2 - t_1 = 4.1261490978744200E - 16$ s.

The difference between Δt_2 and Δt_1 is $\Delta t_{12} = \Delta t_2 - \Delta t_1 = \Delta t_2$. The fringe shift $N_{12} = c\Delta t_{12}/\lambda = c\Delta t_2/\lambda = 2.3533169759740100E - 01$ fringes. The mean of the observation in the Fizeau experiment was 0.23016 fringes.

4.3 Stationary water in Earth's inertial frame

In this case, the velocity u is zero, and the velocity of light in the moving medium is $c_2 = \sqrt{c_1^2 + (v/2)^2} \pm v/2$. The sign \pm depends on the direction of v .

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = L / \left(\sqrt{c_1^2 + (v/2)^2} + v/2 - v \right), \quad t_{12} = L / \left(\sqrt{c_1^2 + (v/2)^2} - v/2 + v \right) \Rightarrow t_1 = t_{11} + t_{12}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = L / \left(\sqrt{c_1^2 + (v/2)^2} + v/2 - v \right), \quad t_{21} = L / \left(\sqrt{c_1^2 + (v/2)^2} - v/2 + v \right) \Rightarrow t_2 = t_{22} + t_{21}.$$

The difference Δt_3 is $\Delta t_3 = t_2 - t_1 = 0$.

For stationary water in the fixed frame as well as in the Earth's inertial frame, the differences Δt_1 and Δt_3 are equal to zero.

4.4 Moving water in Earth's inertial frame

In this case, the velocity of light in the moving medium is $c_2 = \sqrt{c_1^2 + ((v \pm u)/2)^2} \pm (v \pm u)/2$. The sign \pm depends on the directions of v and u .

The time t_{11} and t_{12} in which the transmitted ray travels the distance L in tubes 1 and 2, respectively, are:

$$t_{11} = L / \left(\sqrt{c_1^2 + ((v + u)/2)^2} + (v + u)/2 - v \right), \quad t_{12} = L / \left(\sqrt{c_1^2 + ((v - u)/2)^2} - (v - u)/2 + v \right) \Rightarrow t_1 = t_{11} + t_{12}.$$

The time t_{22} and t_{21} in which the reflected ray travels the distance L in tubes 2 and 1, respectively, are:

$$t_{22} = L / \left(\sqrt{c_1^2 + ((v - u)/2)^2} + (v - u)/2 - v \right), \quad t_{21} = L / \left(\sqrt{c_1^2 + ((v + u)/2)^2} - (v + u)/2 + v \right)$$

$$2 + v) \Rightarrow t_2 = t_{22} + t_{21}.$$

The difference Δt_4 is $\Delta t_4 = t_2 - t_1 = 4.1261491888642900E - 16$ s.

The difference between Δt_4 and Δt_3 is $\Delta t_{34} = \Delta t_4 - \Delta t_3 = \Delta t_4$. The fringe shift $N_{34} = c\Delta t_{34}/\lambda = c\Delta t_4/\lambda = 2.3533170278693700E - 01$ fringes. The fringe shift is approximately equal to the result for the fixed frame. Considering the incapability of the experiment to make a distinction between the fringe shift of this section and the fringe shift of section 4.2, can be concluded that the observation of the experiment in the inertial frame is like the experiment carried out in the fixed frame.

If in the difference of Δt_4 the speed v is zero, then

$$\Delta t_4 = 2L/\left(\sqrt{c_1^2 + (u/2)^2} - u/2\right) - 2L/\left(\sqrt{c_1^2 + (u/2)^2} + u/2\right) = \Delta t_2.$$

5 Conclusions

The results employing the Fresnel drag coefficient and the formula of the velocity of light through a moving medium are different, but the values of them are comparable.

When the water in tubes is stationary in the fixed frame and the inertial frame, the fringe shift is zero. Thus, no difference if the experiment is performed in a fixed frame or Earth's inertial frame.

When the water in tubes is in motion in the fixed frame and the inertial frame, the fringe shift is visually detectable for each of them. However, the fringe shift for moving water in the fixed frame and Earth's inertial frame is visually indistinguishable from one another. Thus we cannot conclude if the experiment occurs in a fixed frame or an inertial frame.

From the perspective of detectability, it can be concluded that the observation of the experiment in the inertial frame is like the experiment performed in the fixed frame.

Different from other interferometers performed in air, or partially in air, the Fizeau interferometer does not predict a fringe shift by rotating the interferometer 360° .

References:

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Note: The paper was revised on 25 February 2019.

Complete derivations for section 4 and a few comments were added.