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# Velocity of light through a moving medium

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**Abstract:** A formula for the velocity of light through a moving medium for any direction of light propagation in that medium was derived. The core of this derivation is based on the fact that light in a moving medium travels through densities that differ from the density of that medium at rest. Applying this formula to the Fizeau experiment, a predicted fringe shift of 0.2353 fringes was obtained, which is different from 0.2022 fringes when using the Fresnel drag coefficient. The result of the Fizeau experiment was 0.23016 fringes, which confirms both theoretical results.

**Keywords:** Fresnel drag coefficient, Fizeau experiment, velocity of light.

## 1 Introduction

Sound waves only travel through a medium, not a vacuum, which led physicists to believe that light also requires a medium for propagation. This hypothetical medium was called aether or ether, which was later abandoned by physicists. The Fresnel drag coefficient [1, 2] based on one of the ether theories predicts that in moving bodies, the ether partially drags the light, which is different from the theory of complete ether dragging.

Fresnel applied the formula of the speed of sound from elasticity theory [1-3] to the propagation of light through the hypothetical ether. This study employs the formula of the speed of waves from elasticity theory to the velocity of light through moving mediums.

This study considers that light in a moving medium travels through different densities depending on the direction of the light's propagation in that medium.

Section 2 presents the formula for the propagation of light waves through a medium at rest, a moving medium when medium and light travel in the same direction, a moving medium when medium and light travel in the opposite direction, and a moving medium when medium travels at an angle to the direction of the moving medium. Section 3 presents the application of the Fresnel drag coefficient and the formula for the velocity of light through a moving medium obtained in this study to the derivation of the Fizeau experiment.

## 2 Velocity of light through a moving medium

### 2.1 Light traveling through a medium at rest

A beam of light travels through a fixed medium of density $\rho_1$ with speed $c_1$ through an imaginary cylinder of an infinitesimal circular cross-section of area $S$ and length $L_1$, as shown in figure 1. The wavefront of light from section $S$, located at point $A$, travels to section $S$, located at point $B$, distance $L_1$, through volume $V_1 = SL_1$. The mass of the medium through which the light travels is $m = \rho_1 V_1 = \rho_1 SL_1$.

The formula of the speed of light in a medium at rest based on electromagnetic theory [3, 4] is $c_1 = 1/\sqrt{\varepsilon_1 \mu_1}$, where $\varepsilon_1$ is the permittivity and $\mu_0$ is the permeability of that medium at rest. The formula of the speed of transverse sound waves through a solid medium from elasticity theory is $v_1 = \sqrt{M_1/\rho_1}$, where $M_1$ is the shear modulus, and $\rho_1$ is the density of the medium through which sound travels. Applying this formula to light waves that are also transverse waves, the speed of light in a medium at rest is $c_1 = \sqrt{M_1/\rho_1}$. Both formulas for $c_1$ have to be equal; therefore $c_1 = \sqrt{M_1/\rho_1} = 1/\sqrt{\varepsilon_1 \mu_1} \Rightarrow M_1 = \rho_1/(\varepsilon_1 \mu_1)$. For light waves, modulus $M_1$ is not a shear modulus anymore. It is a constant depending on the density and electromagnetic properties of the medium at rest through which light travels.

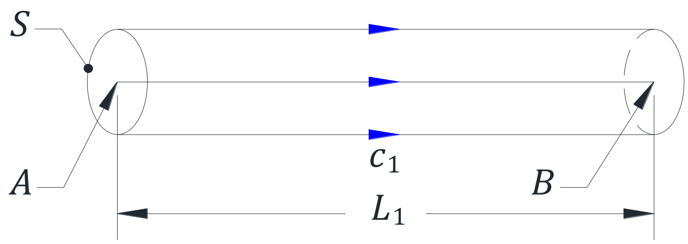


Figure 1. Light traveling through a medium at rest.

Fresnel assumed in his studies that $M_2$ for a moving medium has the same value as that of $M_1$ at rest, and only densities vary, an assumption which this study also employs

## 2.2 Medium and light traveling in the same direction

A medium moves with uniform rectilinear velocity $v$ and a beam of light travels through it with speed $c_2$ in the same direction as velocity $v$, as shown in figure 2. An observer $O$ is attached to a fixed point $A$, and an observer $O'$ is attached to the moving medium. Initially, observers $O$ and $O'$, and the wavefront of light are on section $S$, located at point $A$.

While the wavefront of the light travels the distance $L_2 = AB'$ in time $t$, section $S$ from point $A$ travels the distance $AA'$ and section $S$ from point $B$ travels the distance $BB' = AA' = vt$. The wavefront travels the distance $L_2$ with the speed $c_2$; therefore can be written that $L_1 + vt = c_2 t \Rightarrow t = L_1/(c_2 - v)$. If the wavefront travels in opposite direction, as in section 2.3, $t = L/(c_2 + v)$.

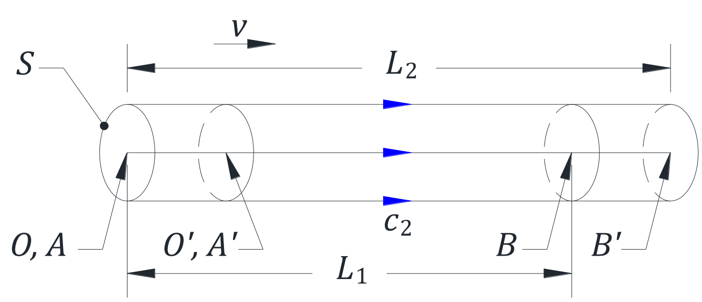


Figure 2. Medium and light traveling in the same direction.

In time $t$, the wavefront of light travels through the same mass of medium $m$, equal to when the medium is at rest, but through a larger volume. Therefore, the moving medium through which the light travels has a density $\rho_2$, which is lower than the density $\rho_1$ of the medium at rest.

Two formulas can be written for the same mass of the medium through which the wavefront of light travels; one for the medium at rest and one for the moving medium:

$$m = \rho_1 S L_1 \text{ and } m = \rho_2 (S L_1 + S v t) = \rho_2 \left( S L_1 + S v \frac{L_1}{c_2 - v} \right) \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{c_2}{c_2 - v}.$$

The speed of light in the moving medium is $c_2 = \sqrt{M_2/\rho_2}$. The ratio between this formula and the equivalent formula for the medium at rest yields

$$\frac{c_2}{c_1} = \sqrt{\frac{M_2}{M_1}} \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{c_2}{c_2 - v}} \quad \Rightarrow$$

$$c_2^2 - v c_2 - c_1^2 = 0. \qquad (1)$$

The second-degree equation (1) with the unknown $c_2$ yields the convenient solution $c_2 = \sqrt{c_1^2 + (v/2)^2} + v/2$. The observer $O$ at fixed point $A$ sees speed $c_2$. The observer $O'$, who is moving with the medium, perceives speed $c_2' = c_2 - v = \sqrt{c_1^2 + (v/2)^2} + v/2 - v = \sqrt{c_1^2 + (v/2)^2} - v/2$.

## 2.3 Medium and light traveling in opposite directions

A medium moves with uniform rectilinear velocity $v$ and a beam of light travels through it with speed $c_2$ in the opposite direction to velocity $v$, as shown in figure 3. Observer $O$ is attached to a fixed point $A$, and observer $O'$ is attached to the moving medium. Initially, observers $O$ and $O'$, and the wavefront of light are on section $S$, located at point $A$. While the wavefront of light travels the distance $L_2 = AB'$ in time $t$, section $S$ from point $A$ travels the distance $AA'$ and section $S$ from point $B$ travels the distance $BB' = AA'$.

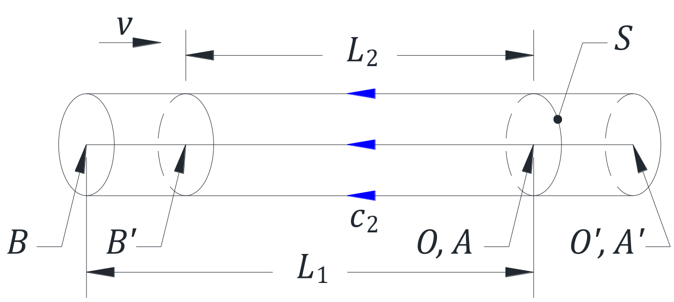


Figure 3. Medium and light traveling in opposite directions.

In time $t$, the wavefront of light travels through the same mass of medium $m$, equal to when the medium is at rest, but through a smaller volume. Therefore, the moving medium through which the light travels has density $\rho_2$, which is higher than the density $\rho_1$ of the medium at rest.

Two formulas can be written for the same mass of the medium through which the wavefront of light travels; one for the medium at rest and one for the moving medium:

$$m = \rho_1 S L_1 \quad \text{and} \quad m = \rho_2 (SL_1 - Svt) = \rho_2 \left( SL_1 - Sv \frac{L_1}{c_2 + v} \right) \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{c_2}{c_2 + v}.$$

The speed of light in the moving medium is $c_2 = \sqrt{M_2/\rho_2}$. The ratio between this formula and the equivalent formula for the medium at rest yields

$$\frac{c_2}{c_1} = \sqrt{\frac{M_2}{M_1}} \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{c_2}{c_2 + v}} \quad \Rightarrow$$

$$c_2^2 + vc_2 - c_1^2 = 0. \qquad (2)$$

The second-degree equation (2) with the unknown $c_2$ yields the convenient solution $c_2 = \sqrt{c_1^2 + (v/2)^2} - v/2$. The observer $O$ at fixed point $A$ sees speed $c_2$. The observer $O'$, who is moving with the medium, perceives speed $c_2' = c_2 + v = \sqrt{c_1^2 + (v/2)^2} - v/2 + v = \sqrt{c_1^2 + (v/2)^2} + v/2$.

## 2.4 Medium and light traveling at an angle *a*

A medium moves with uniform rectilinear velocity $v$ and a beam of light travels through it with speed $c_2$ at an angle $a$ to the direction of the velocity $v$, as shown in figure 4. Observer $O$ is attached to a fixed point $A$, and observer $O'$ is attached to the moving medium. Initially, observers $O$ and $O'$, and the wavefront of light are on section $S$, located at point $A$. While the wavefront of light travels the distance $L_2 = AB$ in time $t$, section $S$ from point $A$ travels the distance $AA'$. Infinitesimal cross-section of area $S'$ has an oval shape. Sections $S'$are perpendicular to the distance $L'$ and their areas $S' = Scos(a' - a)$.

The wavefront of light of section $S$ travels the distance $L_2$ through volume $V_2$. The medium through which the wavefront traveled the distance $L_2$ is in volume $V'$ of the cylinder limited by the sections $S$ at points $A'$ and $B$, which are at the distance $L'$ apart from each other. The volume that is limited by section $S'$ at points $A'$ and $B$, which are at the distance $L'$ apart from each other, is equal to the same volume $V'$. Therefore, the moving medium through which light travels from point $A$ to point $B$ with volume $V_2$ has a density $\rho_2$, which is different from the density $\rho_1$ of the medium comprising volume $V'$. Note that figure 4 does not show section $S'$ at point $B$, and it shows that $\rho_2$ is lower than $\rho_1$.

Two formulas can be written for the same mass of the medium through which the wavefront of light propagates; one for the medium at rest of density $\rho_1$ comprised in volume $V'$ and the other for the moving medium of density $\rho_2$ comprised in volume $V_2$:

$$m = \rho_2 S L_2 \quad \text{and} \quad m = \rho_1 S' L' = \rho_1 L' S cos(a' - a) \quad \Rightarrow$$

$$\frac{\rho_2}{\rho_1} = \frac{L'}{L_2} cos(a' - a) = \frac{L'}{L_2} (cos\, a' \,cos\, a + sin\, a' \,sin\, a).$$

From figure 4:

$sin\, a' = Y/L' \quad \Rightarrow \quad Y = L' \,sin\, a'$ and $sin\, a = Y/L_2 \quad \Rightarrow \quad Y = L_2 \,sin\, a$. Thus:

$Y = L' \,sin\, a' = L_2 \,sin\, a \quad \Rightarrow \quad sin\, a' = (L_2/L')\, sin\, a$.

$cos\, a = X/L_2 \quad \Rightarrow \quad X = L_2 \,cos\, a$ and

$$cos\,a' = \frac{X - vt}{L'} = \frac{X - v\frac{L_2}{c_2}}{L'} = \frac{1}{L'}\left(X - L_2\frac{v}{c_2}\right) \quad \Rightarrow \quad X = L'\,cos\,a' + L_2\frac{v}{c_2}.$$ Thus:

$$X = L_2\,cos\,a = L'\,cos\,a' + L_2\frac{v}{c_2} \quad \Rightarrow \quad cos\,a' = \frac{L_2}{L'}\left(cos\,a - \frac{v}{c_2}\right).$$

Introducing the formulas of $sin\,a'$ and $cos\,a'$ in the ratio $\rho_2/\rho_1$,

$$\frac{\rho_2}{\rho_1} = \frac{L'}{L_2}(cos\,a'\,cos\,a + sin\,a'\,sin\,a) = \frac{L'}{L_2}\left[\frac{L_2}{L'}\left(cos\,a - \frac{v}{c_2}\right)cos\,a + \frac{L_2}{L'}\,sin\,a\,sin\,a\right] \Rightarrow$$

$$\frac{\rho_1}{\rho_2} = \frac{c_2}{c_2 - v\,cos\,a}.$$

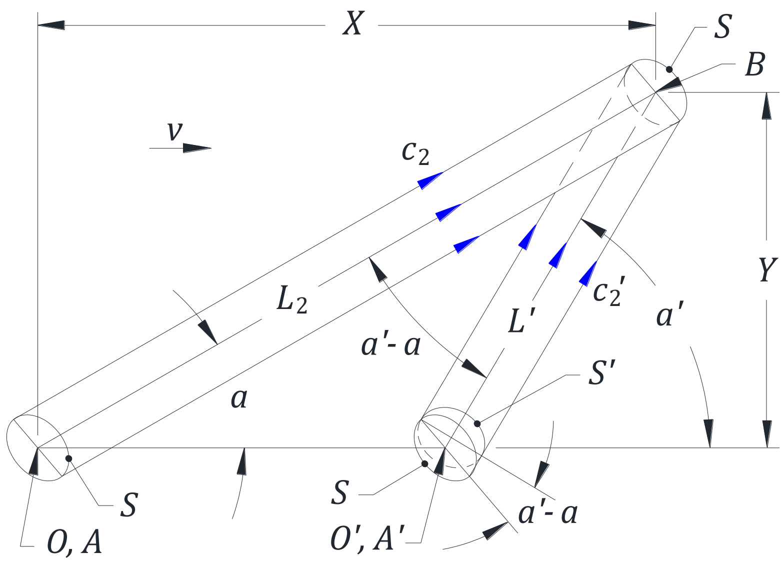


Figure 4. Medium and light traveling at an angle $a$.

The speed of light in the moving medium is $c_2 = \sqrt{M_2/\rho_2}$. The ratio between this formula and the equivalent formula for the medium at rest yields

$$\frac{c_2}{c_1} = \sqrt{\frac{M_2}{M_1}}\sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{c_2}{c_2 - v\,cos\,a}} \quad \Rightarrow$$

$$c_2^2 - (v\,cos\,a)c_2 - c_1^2 = 0. \tag{3}$$

The second-degree equation (3) with the unknown $c_2$ yields the convenient solution

$$c_2 = \sqrt{c_1^2 + \left(\frac{v}{2}cos\,a\right)^2} + \frac{v}{2}cos\,a.$$

For $a = 0°$ and 180°, the formula for $c_2$ of this section yields the formula for $c_2$ of sections 2.2 and 2.3, respectively. For $a = 90°$, the formula for $c_2$ is the speed of the medium at rest, $c_2 = c_1$. For the medium at rest, $v = 0$ and $c_2 = c_1 = c/n_1$, where $n_1$ is the index of refraction of water. In a vacuum there is nothing to move, thus $v = 0$, and with its index of refraction equal to unity, the velocity of light in a vacuum is $c_2 = c_1 = c/n_1 = c$.

Observer $O$ at fixed point $A$ sees speed $c_2$. Observer $O'$, who is moving with the medium, perceives speed $c_2'$, which is calculable in the same manner as $c_2$.

From the formula $sin\, a' = L_2/L' \ sin\, a \quad \Rightarrow \quad sin\, a = (L'/L_2)\, sin\, a'$. From the formula $cos\, a' = \frac{L_2}{L'}\left(cos\, a - \frac{v}{c_2}\right) \quad \Rightarrow \quad cos\, a = \frac{L'}{L_2} cos\, a' + \frac{v}{c_2}$.

Introducing the formulas of $sin\, a$ and $cos\, a$ in the ratio $\rho_2/\rho_1$ yields

$$\frac{\rho_2}{\rho_1} = \frac{L'}{L_2}(cos\, a' \, cos\, a + sin\, a' \, sin\, a) = \frac{L'}{L_2}\left[cos\, a' \left(\frac{L'}{L_2} cos\, a' + \frac{v}{c_2}\right) + sin\, a' \frac{L'}{L_2}\, sin\, a'\right] \Rightarrow$$

$$\frac{\rho_2}{\rho_1} = \frac{L'}{L_2}\left(\frac{L'}{L_2} + \frac{v}{c_2} cos\, a'\right) \quad \Rightarrow \quad \frac{\rho_2}{\rho_1} = \frac{c_2'}{c_2}\left(\frac{c_2'}{c_2} + \frac{v}{c_2} cos\, a'\right) \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{c_2^2}{c_2'(c_2' + v\, cos\, a')}.$$

Introducing the formula of $\rho_1/\rho_2$ in formula $c_2/c_1 = \sqrt{\rho_1/\rho_2}$ gives

$$\frac{c_2}{c_1} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{c_2^2}{c_2'(c_2' + v\, cos\, a')}} \quad \Rightarrow$$

$$c_2'^2 + (v\, cos\, a')c_2' - c_1^2 = 0. \tag{4}$$

The second-degree equation (4) with the unknown $c_2'$ yields the convenient solution

$$c_2' = \sqrt{c_1^2 + \left(\frac{v}{2} cos\, a'\right)^2} - \frac{v}{2} cos\, a'.$$

For $a' = 0°$ and $180°$, the formula for $c_2'$ in this section yields the formula for $c_2'$ of sections 2.2 and 2.3, respectively.

# 3 Fizeau experiment

## 3.1 Experimental device

Figure 5 illustrates the schematic of the Fizeau experiment [1, 2]. The water in tubes 1 and 2 flows with speed $u$ in the directions shown in figure 5.

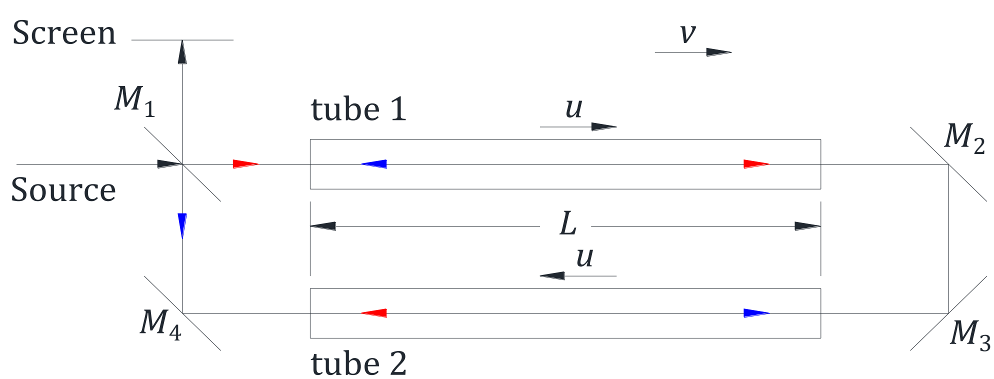


Figure 5. Schematic of the Fizeau experiment.

A ray of light from the source is split into two rays by the beam splitter $M_1$. The transmitted ray travels to opaque mirrors $M_2$, $M_3$, and $M_4$, back to beam splitter $M_1$, and then to a screen. The reflected ray travels to mirrors $M_4$, $M_3$, and $M_2$, back to beam splitter $M_1$, and then to the screen. The two split rays interfere on the screen and yield an interference fringe when water is stationary in the tubes. When the water travels with speed $u$, the fringe image shifts.

The derivation of the two light paths ignores the distances the light travels through air because these dimensions cancel one another in the calculation.

In the Fizeau experiment, the speed of water in the tubes $u = 7.059$ m/s, the length of each tube $L = 1.487$ m, the wavelength of light in a vacuum $\lambda = 526 \times 10^{-9}$ m, and the index of refraction of water $n_1 = 1.33$. The speed of the Earth inertial frame of reference $v = 3 \times 10^4 m/s$, the speed of light in a vacuum $c = 3 \times 10^8$ m/s, and that in a medium at rest $c_1 = c/n_1$.

## 3.2 Application of Fresnel drag coefficient to Fizeau experiment

Applying the Fresnel drag coefficient $k = 1 - 1/n_1^2$ to the Fizeau experiment, the speed of light through the moving water in tubes 1 and 2 is $c_2 = c_1 \pm (v \pm u)k$. The sign ± depends on the direction of $c, v$ and $u$.

The times $t_{11}$ and $t_{12}$ in which the transmitted ray travels in tubes 1 and 2, respectively, are

$$t_{11} = \frac{L}{c_2 - v} = \frac{L}{c_1 + (v+u)k - v}, \quad t_{12} = \frac{L}{c_2 + v} = \frac{L}{c_1 - (v-u)k + v} \Rightarrow$$

$t_1 = t_{11} + t_{12}$.

The times $t_{22}$ and $t_{21}$ in which the reflected ray travels in tubes 2 and 1, respectively, are

$$t_{22} = \frac{L}{c_2 - v} = \frac{L}{c_1 + (v-u)k - v}, \quad t_{21} = \frac{L}{c_2 + v} = \frac{L}{c_1 - (v+u)k + v} \Rightarrow$$

$t_2 = t_{21} + t_{22}$.

The difference of times $t_2$ and $t_1$ is $\Delta t = t_2 - t_1$. The difference $\Delta t$ yields the fringe shift $N = c\Delta t/\lambda = 0.2045$ fringes, which is slightly higher than the shift of 0.2022 fringes obtained by Fizeau derivation. The mean of the observation in the Fizeau experiment was 0.23016 fringes.

## 3.3 Application of the velocity of light through a moving medium to Fizeau experiment

Applying the formula for the velocity of light through a moving medium, derived in this study, to the Fizeau experiment, the speed of the transmitted ray $c_{11}$ in tube 1 and $c_{12}$ in tube 2, and the corresponding times $t_{11}$ and $t_{12}$ the light travels in tubes 1 and 2, respectively, are

$$c_{11} = \sqrt{c_1^2 + \left(\frac{v+u}{2}\right)^2} + \frac{v+u}{2}, \qquad t_{11} = \frac{L}{c_{11} - v},$$

$$c_{12} = \sqrt{c_1^2 + \left(\frac{v-u}{2}\right)^2} - \frac{v-u}{2}, \qquad t_{12} = \frac{L}{c_{12} + v}.$$

$t_1 = t_{11} + t_{12}$.

The speed of the reflected ray $c_{22}$ in tube 2 and $c_{21}$ in tube 1 and the corresponding times $t_{22}$ and $t_{21}$ the light travels in tubes 2 and 1, respectively, are

$$c_{22} = \sqrt{c_1^2 + \left(\frac{v-u}{2}\right)^2} + \frac{v-u}{2}, \qquad t_{22} = \frac{L}{c_{22} - v},$$

$$c_{21} = \sqrt{c_1^2 + \left(\frac{v+u}{2}\right)^2} - \frac{v+u}{2}, \qquad t_{21} = \frac{L}{c_{21} + v}.$$

$t_2 = t_{21} + t_{22}$.

The difference of times $t_2$ and $t_1$ is $\Delta t = t_2 - t_1$. The difference $\Delta t$ yields the fringe shift $N = c\Delta t/\lambda = 0.2353$ fringes.

## 4 Conclusions

The Fresnel drag coefficient that is the expression of the partial ether dragging theory applied to the derivation of the Fizeau experiment predicts a fringe shift of 0.2022 fringes. The complete ether dragging theory applied to the derivation of the Fizeau experiment predicts a fringe shift of 0.4597 fringes. The result of this study applied to the derivation of the Fizeau experiment predicts a fringe shift of 0.2353 fringes. The Fizeau experiment yields a fringe shift of 0.23016 fringes. Thus, the experimental result confirms the predictions of 0.2022 and 0.2353 fringes as possible truths and rejects the prediction of 0.4597 fringes of the complete ether dragging.

The dispersion of the fringe shift readings, taken by Fizeau in his experiment, covers an extensive range from 0.167 to 0.307 fringes, which overlaps both accepted predictions. The mean value of the fringe shift reading is 0.23016 fringes. Therefore, a Fizeau experiment performed with higher precision can offer a better distinction between the theoretical predictions.

The formula for the velocity of light through a moving medium, derived in this study, may not be complete. The Fresnel assumption that the modulus M of a medium has the same value $M_1 = \rho_1/(\varepsilon_1\mu_1)$ for any density means that $\rho$ and $\varepsilon\mu$ have to increase or decrease concurrently at the same rate, a conclusion that needs further investigation.

This study displays the complexity of obtaining a formula for the velocity of light through a moving medium. The propagation of light is also affected by factors not considered in this study. Nevertheless, the approach of this study is analytical and open to further research studies.

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