the general science journal

Derivation of the Michelson-Morley experiment for another geometry

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Abstract. This study starts with the geometry and derivation of the Michelson-Morley experiment for the initial position of the interferometer as presented by the authors, and then completes the derivation of the light paths and the fringe shift for the 90°, the 180°, and the 270° positions, when the interferometer is rotated in steps of 90° from its initial position. For this geometry, the rays that interfere cannot coincide. Starting with the rotation from the initial position to the 90° position, and continuing to the 180°, the 270°, and back to the initial positions, the predicted fringe shift is 0.8 fringes, -08 fringes, 0 fringes, and 0 fringes, respectively. The fringe shift from a position to another position of the interferometer is different from geometry to geometry. The result of the fringe shift for one geometry cannot be extended to any other geometry.

Keywords: Michelson-Morley experiment.

1 Introduction

In the Michelson-Morley experiment [1], there is a disagreement between the illustration and calculation presented with their intention to have the interfering ray coincide. The paper "A new derivation of the Michelson-Morley experiment" [2, 3], presents a new derivation of the light path to comply with the Michelson and Morley intention to have the interfering ray coincide. This paper keeps the illustration and calculation of the light path of Michelson-Morley derivation for the initial position, but the interference occurs at a cross point of the rays that interfere on the beam splitter, or in the space between beam splitter and screen, without the rays to coincide. The next sections present the derivation of the light paths and fringe shift for four positions.

2 Derivation of the light paths with the initial position of the interferometer

Figure 1 illustrates the initial position of the interferometer. The interferometer is moving with the Earth's inertial frame in the fixed frame.

Throughout this paper, points marked by a letter without an index are points as seen by an observer in the inertial frame. Points marked by a letter with an index are instances of inertial frame points in the fixed frame. For example, point A is a physical point on the beam splitter seen in the inertial frame and points A_0, A_1, A_2, A_3, and A_4 are different instances of point A in the fixed frame.

The velocity of the Earth's orbit around the Sun $v = 3 \times 10^4 m/s$, the velocity of light $c =$

$3 \times 10^8 m/s$, the wavelength of light $\lambda = 550 \times 10^{-9} m$, aberration angle $a = \arcsin(v/c)$, and length of the interferometer arms $L = 11m$.

In figure 1, each ray of light from the source is split into two light rays by the beam splitter. The transmitted ray, drawn in red, travels through the beam splitter to the mirror M_1, back to beam splitter, and then to the screen. The reflected ray, drawn in blue, travels at aberration angle to mirror M_2, back to beam splitter, and then to the screen. The geometry is depicted so that the mirror M1 to be perpendicular to the propagation direction of the transmitted rays, the reflected rays to travel with the aberration angle, and consequently, the transmitted and reflected rays that interfere do not coincide.

The derivation of the light paths starts when the wavefront of light from the source is at point A_1. The interference point under observation is point A.

The transmitted ray travels from point A_1 to mirror M_1 in time $t_{11} = L/(c - v)$ and back from mirror M_1 to point A_4 in time $t_{12} = L/(c + v)$. In time $t_1 = t_{11} + t_{12} = 2Lc/(c^2 - v^2)$, the light travels the path length $L_1 = ct_1 = 2Lc^2/(c^2 - v^2) = 2.200000022E + 01m$, and the beam splitter travels the distance $A_1A_4 = vt_1 = 2Lcv/(c^2 - v^2) = 2.200000022E - 03m$.

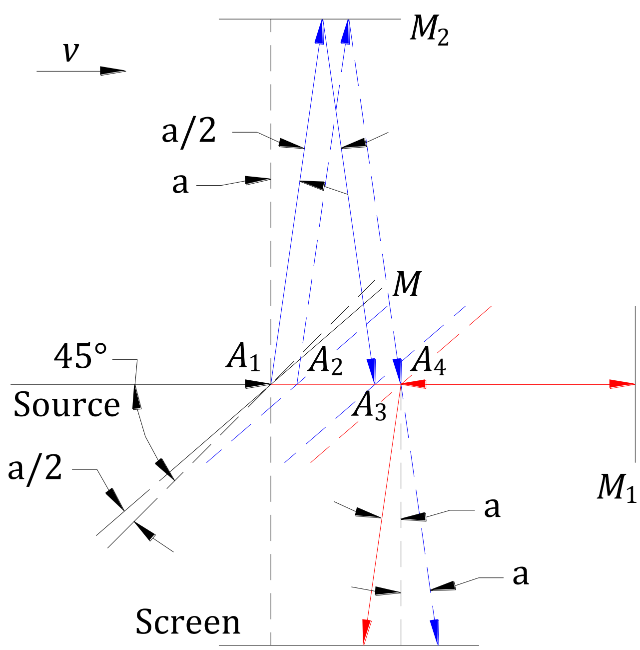

Figure 1. Light paths geometry with the initial position of the interferometer.

The reflected ray travels from point A_1 to mirror M_2 and back to point A_3 in time $t_2 = 2L/\sqrt{c^2 - v^2}$. In time t_2, the light travels the path length $L_2 = ct_2 = 2Lc/\sqrt{c^2 - v^2} = 2.200000011E + 01m$, and the beam splitter travels the distance $A_1A_3 = vt_2 = 2Lv/\sqrt{c^2 - v^2} = 2.200000011E - 03m$.

The difference ΔL_1 of the path length L_2 and the path length L_1, and the distance A_3A_4 for this position are:

$\Delta L_1 = L_1 - L_2 = 1.099999984E - 07m = 1.999999971E - 01\lambda$, and

$A_3A_4 = A_1A_4 - A_1A_3 = 1.099999995E - 11m = 1.999999992E - 05\lambda$.
The derivation for this position is similar to that of Michelson and Morley.

3 Derivation of the light paths with the interferometer rotated by 90° from its initial position

Figure 2 depicts the light paths of the interferometer rotated 90° clockwise from its initial position shown in figure 1.

The derivation of the light paths starts when point A on the beam splitter is at instance A_1, and the wavefront of light coming from the source is at line B_1C_1. Points A_1, B_1, C_1, and D_1, belong to the same instance, and points B_1, C_1, and D_1 belong to the same wavefront. Points B, and D are fixed points in the interferometer space and do not belong to the beam splitter as points A and C. The point under observation is point A.

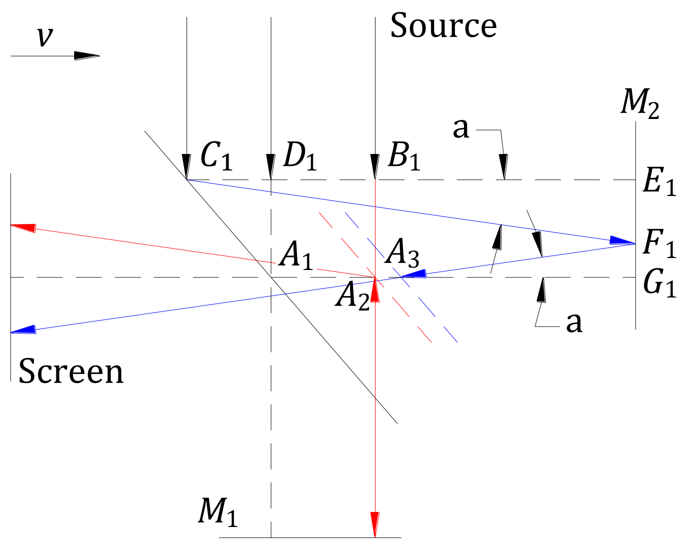

Figure 2. Light paths geometry when the interferometer is rotated by 90° from its initial position.

There is only one reflected ray that starts at point C that continuously intercepts point A. The light from C_1 intercepts point A at A_2 after traveling the path length L_2.

In triangle $C_1E_1F_1$, the speed along C_1E_1 is $c \times cos(a)$ and the speed along E_1F_1 is $c \times sin(a) = v$. The time t_{21}, in which the light travels from point C_1 to point F_1, is given by:

$$t_{21} = \frac{C_1D_1 + L}{c\,cos(a) - v} = \frac{E_1F_1}{v} \Rightarrow E_1F_1 = \frac{(C_1D_1 + L)v}{c\,cos(a) - v}.$$

The instance when the light from C_1 arrives at F_1 includes points E_1, F_1, and G_1. The light from F_1 travels to A_3.

In triangle $A_3F_1G_1$, the speed along G_1A_3 is $c \times cos(a)$, and the speed along F_1G_1 is $c \times sin(a) = v$. The time t_{22}, in which the light travels from point F_1 to point A_3, is given by:

$$t_{22} = \frac{L}{c\,cos(a) + v} = \frac{F_1G_1}{v} \quad \Rightarrow \quad F_1G_1 = \frac{Lv}{c\,cos(a) + v}.$$

In triangle $A_1C_1D_1$, angle $C_1A_1D_1$ is equal to $\pi/4 - a/2$. Thus,

$$tan(\pi/4 - a/2) \ = \frac{C_1D_1}{A_1D_1} \ \Rightarrow$$

$$C_1D_1 = A_1D_1\, tan(\pi/4 - a/2) = (E_1F_1 + F_1G_1)\, tan(\pi/4 - a/2).$$

By substituting the formula of E_1F_1 and F_1G_1 in the formula for C_1D_1:

$$C_1D_1 = \frac{(C_1D_1 + L)v\, tan(\pi/4 - a/2)}{c\, cos(a) - v} + \frac{Lv\, tan(\pi/4 - a/2)}{c\, cos(a) + v} \ \Rightarrow$$

$$C_1D_1\left(1 - \frac{v\, tan(\pi/4 - a/2)}{c\, cos(a) - v}\right) = Lv\, tan(\pi/4 - a/2)\left(\frac{1}{c\, cos(a) - v} + \frac{1}{c\, cos(a) + v}\right).$$

The distance C_1D_1 can be calculated, and then time t_2, distance A_1D_1, path length L_2, and distance $A_1A_3 = vt_2$ can be calculated:

$$t_2 = t_{21} + t_{22} = \frac{C_1D_1 + L}{c\, cos(a) - v} + \frac{L}{c\, cos(a) + v}, \ \ A_1D_1 = \frac{C_1D_1}{tan(\pi/4 - a/2)},$$

$L_2 = ct_2 = 2.2002200550E + 01\text{m}$, and $A_1A_3 = vt_2 = 2.200220055E - 03\text{m}$.

There is only one transmitted ray that starts at point B that continuously intercepts point A. The light from B_1 intercepts point A at A_2 after traveling the path length L_1.

The light from point B_1 travels to mirror M_1 and back to A_2 the light path $L_1 = A_1D_1 + 2L = 2.200220022E + 01$ in time $t_1 = L_1/c$, and the beam splitter travels the distance $A_1A_2 = vt_1 = 2.200220022E - 03$.

The difference ΔL_2 of the path length L_2 and the path length L_1, and the distanceA_2A_3 for this position are:

$\Delta L_2 = L_1 - L_2 = -3.300220043E - 07m = -6.000400079E - 01\lambda$, and

$A_2A_3 = A_1A_2 - A_1A_3 = -3.300220036E - 11m = -6.000400066E - 05\lambda$.

The derivation for this position is different from that of Michelson and Morley, by its geometry and the result of the paths length ΔL_2.

Rotating the interferometer from the initial position to the 90° position, the difference $\Delta L_{12} = \Delta L_1 - \Delta L_2 = 4.400220028E - 07\text{m}$. The fringe image is then expected to shift by $N = \Delta L_{12}/\lambda = 0.800$ fringes.

4 Derivation of the light paths with the interferometer rotated by 180° from its initial position

Figure 3 depicts the light paths of the interferometer rotated 180° clockwise from its initial position shown in figure 1.

The derivation of the light paths starts when point A of the beam splitter is at instance A_1 and the wavefront of light coming from the source is at line B_1C_1. Points A_1, B_1, and C_1 belong to the same instance, and points B_1, and C_1 belong to the same wavefront. Point B is a fixed point in the interferometer space and do not belong to the beam splitter as points A and C. The point under observation is point A.

There is only one reflected ray that starts at point C that continuously intercepts point A. The light from C_1 travels to M_2 then back to beam splitter and intercepts point A at A_2 after traveling the path length L_2.

In triangle $C_1E_1F_1$, the speed along C_1E_1 is $c \times cos(a)$, and the speed along E_1F_1 is $c \times sin(\text{a}) = v$. In triangle $A_2F_1G_1$, the speed along G_1A_2 is $c \times cos(a)$, and the speed along F_1G_1 is $c \times sin(a) = v$.

The time t_2, the light travels on vertical from C_1 to M_2 and back to A_2 the distance $C_1B_1 + 2L$, on horizontal from B_1 to the A_2 the distance B_1A_2, and the beam splitter travels the distance A_1A_2. Thus, $A_1B_1 = 2A_1A_2 = 2vt_2$, and

$$t_2 = \frac{C_1B_1}{c\ cos(a)} + \frac{2L}{c\ cos(a)} = \frac{A_1A_2}{v} = \frac{A_1B_1}{2v}.$$

In triangle $C_1A_1B_1$, angle $C_1A_1B_1$ is equal to $\pi/4 - a/2$. Thus,

$$\tan(\pi/4 - a/2) = \frac{C_1B_1}{A_1B_1} \quad \Rightarrow \quad C_1B_1 = A_1B_1 \tan(\pi/4 - a/2).$$

By substituting the formula of C_1B_1 in the formula for t_2:

$$\frac{C_1B_1}{c\ cos(a)} + \frac{2L}{c\ cos(a)} = \frac{A_1B_1}{2v} \quad \Rightarrow \quad \frac{A_1B_1 \tan(\pi/4 - a/2)}{c\ cos(a)} + \frac{2L}{c\ cos(a)} = \frac{A_1B_1}{2v} \quad \Rightarrow$$

$$A_1B_1\left(\frac{1}{2v} - \frac{\tan(\pi/4 - a/2)}{c\ cos(a)}\right) = \frac{2L}{c\ cos(a)}.$$

The distance A_1B_1 can be calculated and then the time t_2, the path length L_2 and the distance A_1A_2 can be calculated.

$t_2 = A_1B_1/2v$, $\quad L_2 = ct_2 = 2.200440055\text{E} + 01m$, $\quad$ and $\quad A_1A_2 = vt_2 = 2.200440055\text{E} - 03m$.

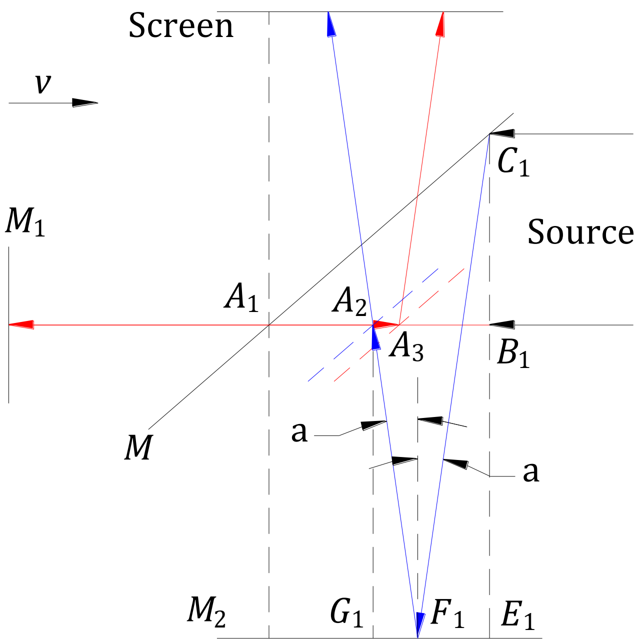

Figure 3. Light paths geometry when the interferometer is rotated by 180° from its initial position.

The light from B_1 travels to mirror M_1 in time $t_{11} = (A_1B_1 + L)/(c + v)$, and from mirror M_1 back to point A_3 in time $t_{12} = L/(c - v)$. Time in which the light travels the path length L_1 is $t_1 = t_{11} + t_{12}$. The path length L_1 and the distance A_1A_3 are:

$L_1 = ct_1 = 2.200440066\text{E} + 01m$, and $A_1A_3 = vt_1 = 2.200440066\text{E} - 03m$.

The difference ΔL_3 of the path length L_2 and the path length L_1, and the distance A_2A_3 for this position are:

$\Delta L_3 = L_1 - L_2 = 1.100000055\text{E} - 07m = 2.000000100\text{E} - 01\lambda$, and

$A_2A_3 = A_1A_3 - A_1A_2 = 1.100000038\text{E} - 11Em = 2.000000070E - 05\lambda$.

Rotating the interferometer from the 90° position to the 180° position, the difference $\Delta L_{23} = \Delta L_2 - \Delta L_3 = -4.400220099E - 07$. The fringes are then expected to shift by $N_{23} = \Delta L_{23}/\lambda = -0.800$ fringes.

5 Derivation of the light paths with the interferometer rotated by 270° from its initial position

Figure 4 depicts the light paths of the interferometer rotated 270° clockwise from its initial position of figure 1.

The derivation of the light paths starts when point A of the beam splitter is at instance A_1 and the wavefront of light coming from the source is at line B_1C_1. Points A_1, B_1, C_1, and D_1 belong to the same instance, and points B_1, C_1, and D_1 belong to the same wavefront. Points B and D are fixed points in the interferometer space and do not belong to the beam splitter as points A and C. The point under observation is point A.

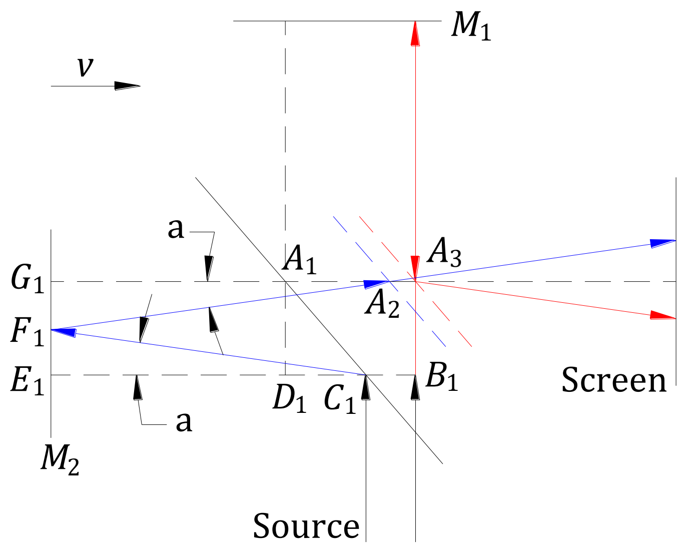

Figure 4. Light paths geometry when the interferometer is rotated by 270° from its initial position.

There is only one reflected ray that starts at point C that continuously intercepts point A. The light from C_1 intercepts point A at A_2 after traveling the path length L_2.

In triangle $C_1E_1F_1$, the speed along C_1E_1 is $c \times cos(a)$ and the speed along E_1F_1 is $c \times sin(\mathrm{a}) = v$. The time t_{21}, in which the light travels from point C_1 to point F_1, is given by:

$$t_{21} = \frac{C_1D_1 + L}{c\cos(a) + v} = \frac{E_1F_1}{v} \qquad \Rightarrow \qquad E_1F_1 = \frac{(C_1D_1 + L)v}{c\cos(a) + v}.$$

The instance when the light from C_1 arrives at F_1 includes points E_1, F_1, and G_1.

In triangle $A_2F_1G_1$, the speed along G_1A_2 is $c \times cos(a)$, and the speed along F_1G_1 is $c \times sin(\mathrm{a}) = v$. The time t_{22}, in which the light travels from point F_1 to point A_3, is given by:

$$t_{22} = \frac{L}{c\,cos(a) - v} = \frac{F_1G_1}{v} \qquad \Rightarrow \qquad F_1G_1 = \frac{Lv}{c\,cos(a) - v}.$$

In triangle $A_1C_1D_1$, angle $C_1A_1D_1$ is equal to $\pi/4 - a/2$. Thus,

$$tan(\pi/4 - a/2) \ = \frac{C_1D_1}{A_1D_1} \ \Rightarrow$$

$$C_1D_1 = A_1D_1 \, tan(\pi/4 - a/2) = (E_1F_1 + F_1G_1) \, tan(\pi/4 - a/2).$$

Substituting the formula for E_1F_1 and F_1G_1 in the formula for C_1D_1:

$$C_1D_1 = \frac{(C_1D_1 + L)v \, tan(\pi/4 - a/2)}{c \, co \, s(a) + v} + \frac{Lv \, tan(\pi/4 - a/2)}{c \, cos(a) - v} \qquad \Rightarrow$$

$$C_1D_1\left(1 - \frac{v \, tan(\pi/4 - a/2)}{c \, cos(a) + v}\right) = Lv \, tan(\pi/4 - a/2)\left(\frac{1}{c \, cos(a) + v} + \frac{1}{c \, cos(a) - v}\right).$$

The distance C_1D_1, and then the distance A_1D_1 and time t_2 can be calculated.

$$A_1D_1 = \frac{C_1D_1}{tan(\pi/4 - a/2)} \text{ and}$$

$$t_2 = t_{21} + t_{22} = \frac{C_1D_1 + \mathrm{L}}{\mathrm{c}\cos(a) + \mathrm{v}} + \frac{L}{c \, cos(a) - v}.$$

The light path length L_2 and the distance A_1A_2 are:

$L_2 = ct_2 = 2.200440055\mathrm{E} + 01Em$, and $A_1A_2 = vt_2 = 2.200{,}220{,}011E - 03m$.

The light from B_1 travels to mirror M_1 and back to point A_3 in time $t_1 = (A_1D_1 + 2L\ )/c$. The path length L_1 and the distance A_1A_3 are:

$L_1 = ct_1 = 2.200220022\mathrm{E} + 01m$, and $A_1A_3 = \ vt_1 = 2.200220022\mathrm{E} - 03m$.

The difference ΔL_4 of the path length L_2 and the path length L_1, and the distance A_2A_3 for this position are:

$\Delta L_4 = L_1 - L_2 = 1.099779964\mathrm{E} - 07m = 1.999599936\mathrm{E} - 01\lambda$, and

$A_2A_3 = A_1A_3 - A_1A_2 = 1.099779989\mathrm{E} - 11m = 1.999599980\mathrm{E} - 05\lambda$.

Rotating the interferometer from the 180° position to the 270° position, the difference $\Delta L_{34} = \Delta L_3 - \Delta L_4 = 2.200906124E - 11m$. The fringes are then expected to shift by $N_{34} = \Delta L_{34}/\lambda = 4.001E - 05$.

Rotating the interferometer from the 270° position to the 360° position, the difference $\Delta L_{41} = \Delta L_4 - \Delta L_1 - 2.200195581E - 11m$. The fringes are then expected to shift by $N_{41} = \Delta L_{41}/\lambda = -4.000E - 05$.

6 Conclusions

The fringe shift from a position to another position of the interferometer is different from geometry to geometry. We cannot study the fringe shift for one geometry and extend the result to any other geometry. We choose a geometry for derivation, but the experiment may have that geometry only by chance.

References:

[1] A. A. Michelson, and E. W. Morley, “On the Relative Motion of the Earth and the Luminiferous Ether,” American Journal of Science, v. 34, p. 333–345 (1887)

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