the general science journal

Derivation of the Michelson-Morley experiment for a particular geometry

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1 Introduction

The Michelson-Morley experiment [1] is a well-known experiment for over one hundred years but is still not thoroughly understood. The Michelson interferometer offers an infinite number of light path geometries. The derivation of the light paths presents a high degree of complexity that makes problematic to understand the light interference within the interferometer. This paper presents the particular geometry that is the least complex for studying and understanding the Michelson interferometer in which the beam splitter, the mirror of the transmitted beam of light, and the mirror of the reflected beam of light makes an angle of 45°, 90°, and 0°, respectively, with the direction of the velocity of light from the source. Other particular geometries are, for example [1, 2, 3], when the reflected beam of light travels at aberration angle and the light rays that interfere coincide.

2 Derivation of the light paths with the initial position of the interferometer

Figure 1 illustrates the particular geometry of the Michelson interferometer with the initial position. The interferometer is moving with Earth's inertial frame in the fixed frame. The velocity of the Earth's orbit around the Sun v and the velocity of light c from the source have the same direction. The beam splitter M makes an angle of 45°, mirror M_1 an angle of 90°, and mirror M_2 an angle of 0° with the direction of the velocities v and c.

Throughout this paper, the transmitted rays are drawn in red, the reflected rays are drawn in blue, and interfering rays are drawn in green. The points marked by a letter without an index are points as seen by an observer in the inertial frame. Points marked by a letter with an index are instances of inertial frame points in the fixed frame. For example, point A is a physical point on the beam splitter seen in the inertial frame and points A_1, A_2, and A_3 are different instances of point A in the fixed frame. Point A is dedicated to the ray from the source that coincides with the axis of the beam of light from the source.

The velocity of the Earth's orbit around the Sun $v = 3 \times 10^4 m/s$, the velocity of light $c = 3 \times 10^8 m/s$, and the length of the interferometer arms $L = 11m$.

The light from A_1 travels to M_1 in time $t_{11} = L/(c - v)$ and back to point A at its instance A_2 in time $t_{12} = L/(c + v)$, then the time $t_1 = t_{11} + t_{12} = 2Lc/(c^2 - v^2)$.

In time t_1, the light travels the path length L_1, and the beam splitter travels the distance A_1A_4.

$$L_1 = ct_1 = \frac{2Lc^2}{c^2 - v^2} = 2.200000022E + 01m, \quad \text{and}$$
$$A_1A_2 = vt_1 = \frac{2Lcv}{c^2 - v^2} = 2.200000022E - 03m.$$

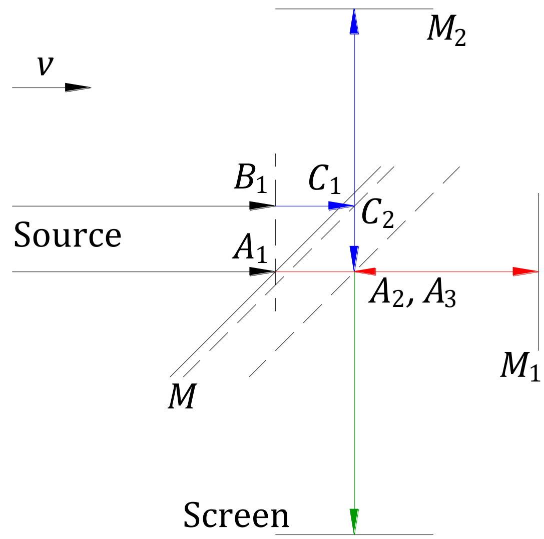

Figure 1. Derivation of the light paths with the initial position of the interferometer.

There is only one reflected ray that starts at point B that continuously intercepts point A. Points A_2 and A_3 are drawn as coinciding. The distance $B_1C_1 = A_1B_1 = A_3C_2 = l$. The light from B_1 travels to C_2 in time $t = l/(c - v)$, then $B_1C_2 = ct = lc/(c - v)$. The light from B_1 travels to C_2, then to M_2 and back to A_3 in time $t_2 = t + (2L - l)/c$.

The distance $A_1A_3 = B_1C_2$. In time t_2, the beam splitter travels from A_1 to A_3. Thus:

$$t_2 = t + \frac{2L - l}{c} = \frac{A_1A_3}{v}, \quad \Rightarrow \quad \frac{l}{c - v} + \frac{2L}{c} - \frac{l}{c} = \frac{lc}{v(c - v)} \quad \Rightarrow$$
$$l\left(\frac{c}{v(c - v)} - \frac{1}{c - v} + \frac{1}{c}\right) = \frac{2L}{c} \quad \Rightarrow \quad l = \frac{2Lv}{c + v}.$$

The times t and t_2 are: $t = \frac{l}{c}$, and $t_2 = \frac{2Lc}{c^2 - v^2}$.

In time t_2, the light travels the path length L_2, and the beam splitter travels the distance A_1A_3.

$$L_2 = ct_2 = \frac{2Lc^2}{c^2 - v^2} = 2.200000022E + 01m, \quad \text{and}$$
$$A_1A_3 = vt_2 = \frac{2Lcv}{c^2 - v^2} = 2.200000022E - 03m.$$

The distance $A_1A_2 = A_1A_3$. Thus, points A_2 and A_3 coincide, and the two rays interfere in phase. The difference of the path length L_2 and the path length L_1 is $\Delta L_1 = L_2 - L_1 = 0m$.

The interference image is predicted to be just one spot at maximum brightness. The rays that interfere along the beam splitter are in phase when the arms of the interferometer are equal or when their difference is a multiple of λ; otherwise, the rays that interfere along the beam splitter have the same constant difference of phase.

3 Derivation of the light paths when the interferometer is rotated by 90° from its initial position

Figure 2 illustrates the interferometer rotated by 90° from its initial position. At the initial instance, the wavefront of light is at line A_1B_1.

There is only one transmitted ray that starts at point B that continuously intercept point A. The light from B_1 travels to M_1, then back to beam splitter and intercept point A at A_2. Thus points A_2 and B_1 coincide. The light from C_1 travels through B_2 to M_1, then back to beam splitter and intercept point A at A_3. Thus points A_3 and B_2 coincide.

The light from A_1 travels to M_1 and back to point A_1 in time $t_1 = 2L/c$. In time t_1, the light from B_1 travels to M_1 and back to point A_2 the path length L_1, and the beam splitter travels the distance A_1A_2.

$L_1 = ct_1 = 2\text{L} = 2.200000000E + 01m,$ and

$A_1A_2 = vt_1 = \dfrac{2Lv}{\text{c}} = 2.200000000E - 03m.$

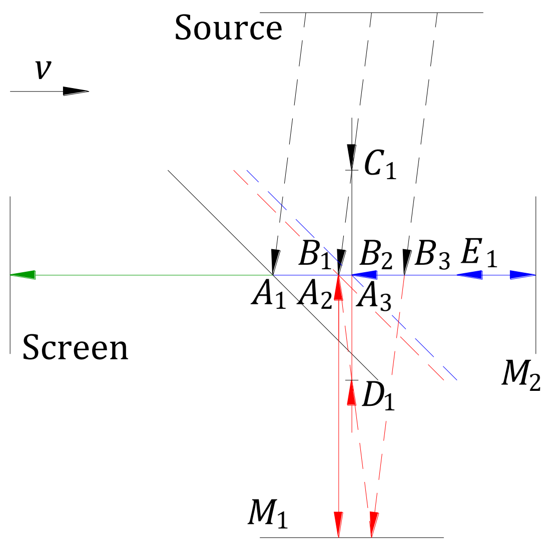

Figure 2. Derivation of the light paths when the interferometer is rotated by 90° from its initial position.

The light from A_1 travels to M_2 in time $t_{21} = L/(c - v)$, and then back to point A at its instance A_3 in time $t_{22} = L/(c + v)$. The time $t_2 = t_{21} + t_{22} = 2Lc/(c^2 - v^2)$. In time t_2, the light travels the path length L_2, and the beam splitter travels the distance A_1A_3.

$L_2 = ct_2 = \dfrac{2Lc^2}{c^2 - v^2} = 2.200000022E + 01m,$ and

$A_1A_3 = vt_2 = \dfrac{2Lcv}{c^2 - v^2} = 2.200000022E - 03m.$

The difference of the path length L_2 and the path length L_1 is:

$\Delta L_2 = L_2 - L_1 = 2\text{L} - \dfrac{2Lc^2}{c^2 - v^2} = -\dfrac{2Lv^2}{c^2 - v^2} = 2.200000004\text{E} - 07\text{m}.$

The solid line arrows represent the wavefront of light rays, and the dashed line arrows represent full rays of light observed in the fixed frame.

When the wavefront of light from B_1 return to A_2, the point B is at B_3, the wavefront of light from point C_1 is at D_1, and the reflected wavefront of light from point A_1 is at E_1. The distance $B_2C_1 = A_3D_1 = A_3E_1$. While the beam splitter travels from A_2 to A_3, the light from D_1 and E_1 travel to A_3 the distance ΔL_2. The reflected ray from A_1 and the transmitted ray from C_1 interfere at A_3 with a difference of phase equivalent to ΔL_2.

By rotating the interferometer from the initial position to the 90° position, the difference $\Delta L_{12} = \Delta L_2 - \Delta L_1$ gives the fringe shift $N_{12} = \Delta L_{12}/\lambda = 0.400$ fringes. Thus, the interference image is one spot, one fringe, dimmed from the maximum brightness of the initial position.

4 Derivation of the light paths when the interferometer is rotated by 180° from its initial position

Figure 3 illustrates the interferometer rotated by 180° from its initial position. At the initial instance, the wavefront of light is at line A_3B_1.

There is only one reflected ray that starts at point B that continuously intercepts point A. The light from B_1 travels to M_2, then back to beam splitter and intercept point A at A_3. The distance $A_1A_3 = A_3B_1 = l$. In time t_2, the light from B_1 travels to M_2 and back to A_3 the path length L_1, and the beam splitter travels the distance $A_1A_3 = l$. Thus:

$t_2 = \frac{2L + l}{c} = \frac{A_1A_3}{v}, \quad \Rightarrow \quad \frac{2L}{c} + \frac{l}{c} = \frac{l}{v} \quad \Rightarrow \quad l\frac{c - v}{cv} = \frac{2L}{c} \quad \Rightarrow \quad l = \frac{2Lv}{c - v},$ and then

$t_2 = \frac{2L + l}{c} = \frac{2L}{c - v}.$

In time t_2, the light travels the path length L_2, and the beam splitter the distance A_1A_3.

$L_2 = ct_2 = \frac{2Lc}{c - v} = 2.200220022E + 01m,$ and

$A_1A_3 = vt_2 = l = \frac{2Lv}{c - v} = 2.200220022E - 03m.$

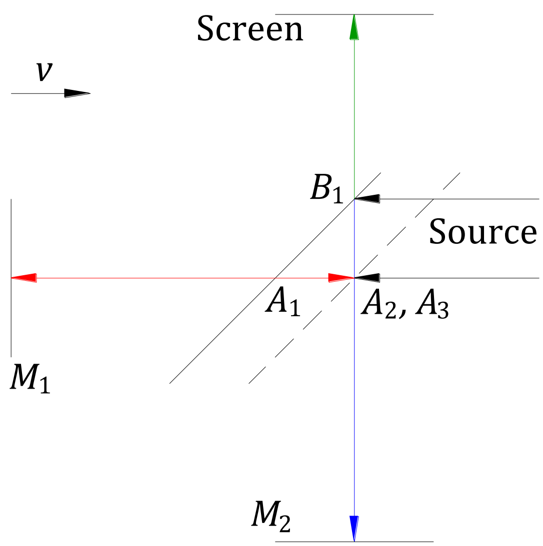

Figure 3. Derivation of the light paths when the interferometer is rotated by 180° from its initial position.

The light from A_1 travels to M_1 in time $t_{11} = (L + l)/(c + v)$ and back from M_1 to point A at its instance A_2 in time $t_{12} = L/(c - v)$. Thus,

$$t_1 = t_{11} + t_{12} = \frac{L + l}{c + v} + \frac{L}{c - v} = \frac{2L}{c - v}.$$

In time t_1, the light travels the path length L_1, and the beam splitter travels the distance A_1A_2.

$$L_1 = ct_1 = \frac{2Lc}{c - v} = 2.200220022E + 01m, \text{ and}$$

$$A_1A_2 = vt_1 = \frac{2Lv}{c - v} = l = 2.200220022E - 03m.$$

The distance $A_1A_2 = A_1A_3$. Thus, points A_2 and A_3 coincide, and the two rays interfere in phase. The difference of the path length L_2 and the path length L_1 is $\Delta L_3 = L_2 - L_1 = 0m$.

The interference image is predicted to be just one spot at the maximum brightness.

By rotating the interferometer from the 90=0° position to the 180° position, the difference $\Delta L_{23} = \Delta L_3 - \Delta L_2$ gives the fringe shift $N_{23} = \Delta L_{23}/\lambda = -0.400$ fringes, so the brightness is recovering to the maximum brightness.

5 Derivation of the light paths when the interferometer is rotated by 270° from its initial position

Figure 4 illustrates the interferometer rotated by 270° from its initial position. At the initial instance, the wavefront of light is at line A_1B_1.

There is only one transmitted ray that starts at point B that continuously intercept point A. The light from B_1 travels to M_1, then back to beam splitter and intercept point A at A_2. Thus points A_2 and B_1 coincide. The light from C_1 travels through B_2 to M_1, then back to beam splitter and intercept point A at A_3. Thus points A_3 and B_2 coincide.

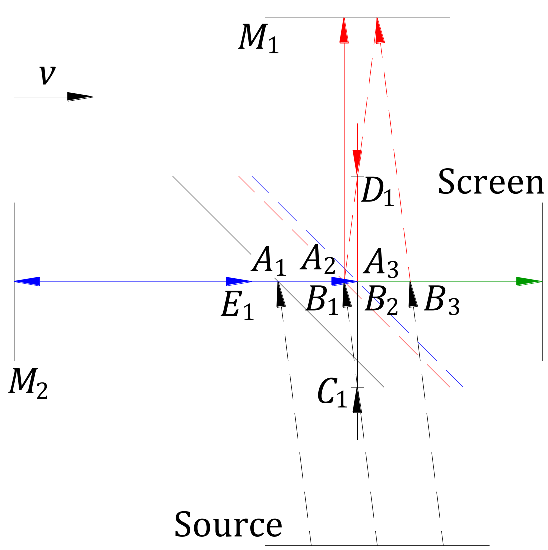

Figure 4. Derivation of the light paths when the interferometer is rotated by 270° from its initial position.

The light from A_1 travels to M_1 and back to point A_1 in time $t_1 = 2L/c$. In time t_1, the light from B_1 travels to M_1 and back to point A_2 the path length L_1, and the beam splitter travels the distance A_1A_2.

$L_1 = ct_1 = 2\text{L} = 2.200000000E + 01m$, and

$A_1A_2 = vt_1 = \frac{2Lv}{\text{c}} = 2.200000000E - 03m$.

The light from A_1 travels to M_2 in time $t_{21} = L/(c + v)$, and then back to point A at its instance A_3 in time $t_{22} = L/(c - v)$. The time $t_2 = t_{21} + t_{22} = 2Lc/(c^2 - v^2)$. In time t_2, the light travels the path length L_2, and the beam splitter travels the distance A_1A_3.

$L_2 = ct_2 = \frac{2Lc^2}{c^2 - v^2} = 2.200000022E + 01m$, and

$A_1A_3 = vt_2 = \frac{2Lcv}{c^2 - v^2} = 2.200000022E - 03m$.

The difference of the path length L_2 and the path length L_1 is:

$\Delta L_4 = L_2 - L_1 = 2\text{L} - \frac{2Lc^2}{c^2 - v^2} = -\frac{2Lv^2}{c^2 - v^2} = 2.200000004\text{E} - 07m$.

When the wavefront of light from B_1 return to A_2, the point B is at B_3, the wavefront of light from point C_1 is at D_1, and the reflected wavefront of light from point A_1 is at E_1. The distance $B_2C_1 = A_3D_1 = A_3E_1$. While the beam splitter travels from A_2 to A_3, the light from D_1 and E_1 travel to A_3 the distance ΔL_4. The reflected ray from A_1 and the transmitted ray from C_1 interfere at A_3 with a difference of phase equivalent to ΔL_4.

By rotating the interferometer from the 180° position to the 270° position, the difference $\Delta L_{34} = \Delta L_4 - \Delta L_3$ gives the fringe shift $N_{34} = \Delta L_{34}/\lambda = 0.400$ fringes. Thus, the interference image is one spot, one fringe, dimed from the maximum brightness of the 180° position.

By rotating the interferometer from the 270° position to the initial position, the difference $\Delta L_{41} = \Delta L_1 - \Delta L_4$ gives the fringe shift $N_{41} = \Delta L_{41}/\lambda = -0.400$ fringes, so the brightness at the initial position is recovering to maximum brightness.

References

[1] A. A. Michelson, and E. W. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," American Journal of Science, v. 34, p. 333–345 (1887)

[2] F. Dambi, "A new derivation of the Michelson-Morley experiment" (2018) http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7384

[3] F. Dambi, "A new derivation of the Michelson-Morley experiment-Four positions of the interferometer" (2018) http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7578