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A new derivation of the Michelson-Morley experiment
Four positions of the interferometer

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1. Introduction

The Michelson-Morley experiment [1] is a well-known experiment, but not well understood in details. The paper "A new derivation of the Michelson-Morley experiment [2] presents in details the first two positions of Michelson's interferometer. This paper completes the derivation of the light paths and the fringe shift in Michelson-Morley experiment for four positions of the interferometer.

2. Derivation of the light paths with the initial position of the interferometer

Figure 1 illustrates the initial position of the interferometer. The interferometer is moving with the Earth's inertial frame in a fixed frame.

Throughout this paper, points marked by a letter without an index are points as seen by an observer in the inertial frame. Points marked by a letter with an index are instances of inertial frame points in the fixed frame. For example, point A is a physical point on the beam splitter seen in the inertial frame and points A0, A1, A2, A3, A4, and A5 are different instances of point A in the fixed frame.

The velocity of the Earth's orbit around the Sun $v = 3 \times 10^4 m/s$, the velocity of light $c = 3 \times 10^8 m/s$, the wavelength of light $\lambda = 550 \times 10^{-9} m$, aberration angle $a = \arcsin(v/c)$, and length of the interferometer arms $L = 11m$.

In figure 1, each ray of light from the source is split into two light rays by the beam splitter. The light rays that travel through the beam splitter M toward mirror M_1 are called transmitted rays and are drawn in red. The light rays that are reflected by the beam splitter M toward mirror M_2 at the aberration angle are called reflected rays and are drawn in blue. The interfering rays are drawn in green. The light path geometry is depicted so that the reflected rays travel with the aberration angle, and the transmitted and reflected rays that interfere coincide.

The derivation of the light paths starts when the wavefront of light from the source is at line A_1B_1. Points A, B, D, and G are aligned vertically. Point B is a fixed point in the interferometer space and does not belong to the beam splitter as point A. The point under observation is point A.

There is only one transmitted ray that starts at point B that continuously intercepts point A. The light from B_1 intercepts point A at A_4 after traveling the path length L_1. The light at A_4 is reflected toward the screen, making the angle $C_1A_4G_2$, which is equal to $90° - a$, and therefore coincides with the reflected ray at A_4.

The instance when the light from B_1 arrives at E_1 includes points E_1, F_1, C_1, and A_3. In time t_{11}, the light travels from B_1 to E_1 and point A travels from A_1 to A_3. In time t_{12}, the light travels from E_1 to A_4 and point A travels from A_3 to A_4.

The time $t_{11} = (L - F_1C_1)/(c - v) = (L - E_1F_1 \tan(a))/(c - v)$, and the time $t_{12} = (L - E_1F_1 \tan(a))/(c \cos(2a) + v)$. The distance $E_1F_1 = ct_{12}\, sin(2a)$ and then $t_{12} = (L - ct_{12}\, sin(2a) \tan(a))/(c \cos(2a) + v) \Rightarrow t_{12} = L/\,(c \cos(2a) + \mathrm{c}\, sin(2a) \tan(a) + v)$. The distance E_1F_1 can be calculated and then the time $t_{11} = (L - E_1F_1 \tan(a))/(c - v)$.

The time $t_1 = t_{11} + t_{12} = 7.333{,}333{,}333{,}333{,}330E - 08s$. In time t_1, the light travels from point B_1 to point A_4 and the beam splitter travels the distance A_1A_4. The path length L_1 and the distance A_1A_4 are:

$L_1 = ct_1 = 2.200{,}000{,}000\mathrm{E} + 01m = 4.000{,}000{,}000\mathrm{E} + 07\lambda$ and

$A_1A_4 = vt_1 = 2.200{,}000{,}000\mathrm{E} - 03m = 4.000{,}000{,}000\mathrm{E} + 03\lambda$.

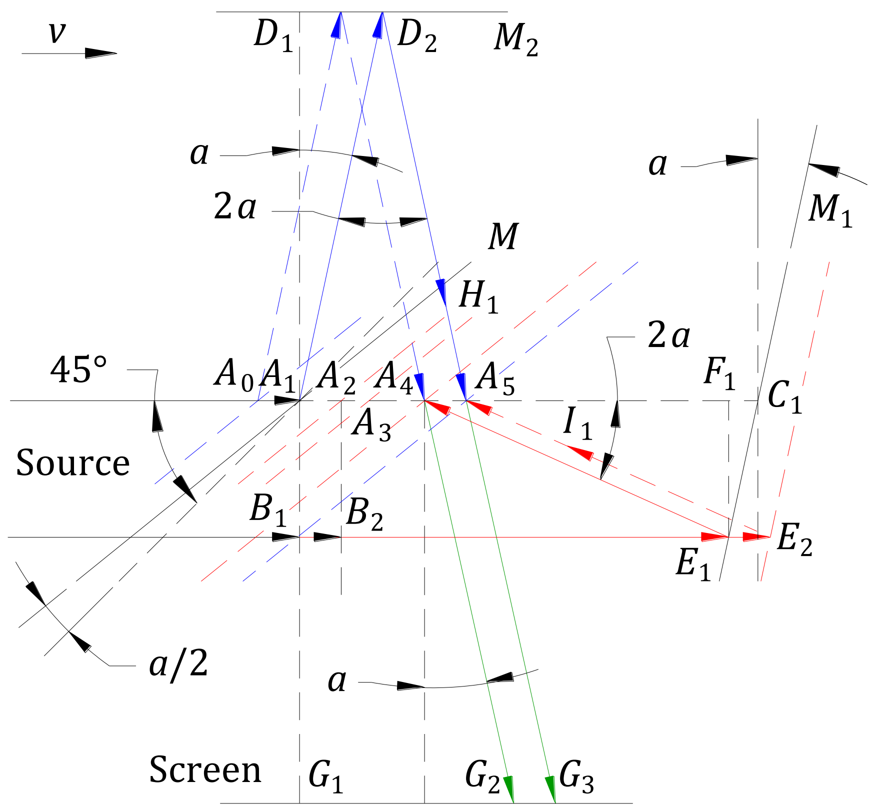

Figure 1. Light path geometry with the initial position of the interferometer.

The reflected light rays travel at an angle a equal to the aberration angle; thus, the reflected ray that starts at point A intercepts the same point A continuously. The light from A_1 intercepts point A_5 after traveling the path length L_2.

In triangle $A_1D_1D_2$, the speed along A_1D_1 is $c \times cos(a)$. In time t_2, the light travels from point A_1 to point D_1 then back to point A_5, and the beam splitter travels the distance A_1A_5.

The time t_2 is defined as: $t_2 = 2L/(c\, cos(a)) = 7.333{,}333{,}370{,}000{,}000\mathrm{E} - 08\mathrm{s}$.

The path length L_2 and distance A_1A_5 are:

$L_2 = ct_2 = 2.200{,}000{,}011\mathrm{E} + 01m = 4.000{,}000{,}020\mathrm{E} + 07\lambda$ and

$A_1A_5 = vt_2 = 2.200{,}000{,}011\text{E} - 03m = 4.000{,}000{,}020\text{E} + 03\lambda$.

The difference ΔL_1 of the path lengths L_2 and L_1 and the distance A_4A_5 are:

$\Delta L_1 = L_2 - L_1 = 1.100\text{E} - 07m = 2.000\text{E} - 01\lambda$ and

$A_4A_5 = A_1A_5 - A_1A_4 = 1.100\text{E} - 11m = 2.000\text{E} - 05\lambda$.

3. Derivation of the light paths with the interferometer rotated 90° from its initial position

Figure 2 depicts the light paths of the interferometer rotated 90° clockwise from its initial position shown in figure 1.

The distances *AP* and *AH* are the arm length equals to L. The derivation of the light paths starts when point *A* of the beam splitter is at instance A_1 and the wavefront of light coming from the source is at line B_1C_1. Points A_1, E_1, B_1, C_1, D_1, J_1, U_1, and V_1 belong to the same instance, and points B_1, and C_1 belong to the same wavefront. Points *B*, and *E* are fixed points in the interferometer space and do not belong to the beam splitter as points *A* and *C*. Points *A*, *D*, and *P* are vertically aligned. The point under observation is point *A*.

There is only one reflected ray that starts at point *C* that continuously intercepts point *A*. The light from C_1 intercepts point A at A_4 after traveling the path length L_2.

In triangle $C_1F_1G_1$, the speed along C_1G_1 is $c \times cos(a)$ and the speed along G_1F_1 is $c \times sin(\text{a}) = v$. The time t_{21}, in which the light travels from point C_1 to point F_1, is given by:

$$t_{21} = \frac{C_1D_1 + L}{c\,cos(a) - v} = \frac{F_1G_1}{v} \Rightarrow F_1G_1 = \frac{(C_1D_1 + L)v}{c\,cos(a) - v}.$$

The instance when the light from C_1 arrives at F_1 includes points F_1, G_1, and H_1.

In triangle $A_4F_1H_1$, the speed along H_1A_4 is $c \times cos(a)$, and the speed along F_1H_1 is $c \times sin(a) = v$. The time t_{22}, in which the light travels from point F_1 to point A_4, is given by:

$$t_{22} = \frac{L}{c\,cos(a) + v} = \frac{F_1H_1}{v} \quad \Rightarrow \quad \text{F}_1\text{H}_1 = \frac{Lv}{c\,cos(a) + v}.$$

Triangle $C_1A_1D_1$ gives: $tan(\pi/4 - a/2) = \dfrac{C_1D_1}{A_1D_1} \Rightarrow$

$C_1D_1 = A_1D_1\,tan(\pi/4 - a/2) = (F_1G_1 + F_1H_1)\,tan(\pi/4 - a/2)$.

Substituting the formula for F_1G_1 and F_1H_1 in the formula for C_1D_1:

$$C_1D_1 = \frac{(C_1D_1 + L)v\,tan(\pi/4 - a/2)}{c\,cos(a) - v} + \frac{Lv\,tan(\pi/4 - a/2)}{c\,cos(a) + v} \quad \Rightarrow$$

$$C_1D_1\left(1 - \frac{v\,tan(\pi/4 - a/2)}{c\,cos(a) - v}\right) = Lv\,(tan(\pi/4 - a/2)\left(\frac{1}{c\,cos(a) - v} + \frac{1}{c\,cos(a) + v}\right).$$

The distance C_1D_1 can be calculated, and then distance A_1D_1 and time t_2 can be calculated as follows:

$$A_1D_1 = \frac{C_1D_1}{tan(\pi/4 - a/2)} \text{ and}$$

$$t_2 = t_{21} + t_{22} = \frac{C_1D_1 + L}{c\,cos(a) - v} + \frac{L}{c\,cos(a) + v} = 7.334{,}066{,}850{,}025{,}670E - 08s.$$

The light path length L_2 and distance A_1A_4 are:

$L_2 = ct_2 = 2.200{,}220{,}055\text{E} + 01m = 4.000{,}400{,}100\text{E} + 07\lambda$ and

$A_1A_4 = vt_2 = 2.200{,}220{,}055\text{E} - 03m = 4.000{,}400{,}100\text{E} + 03\lambda$.

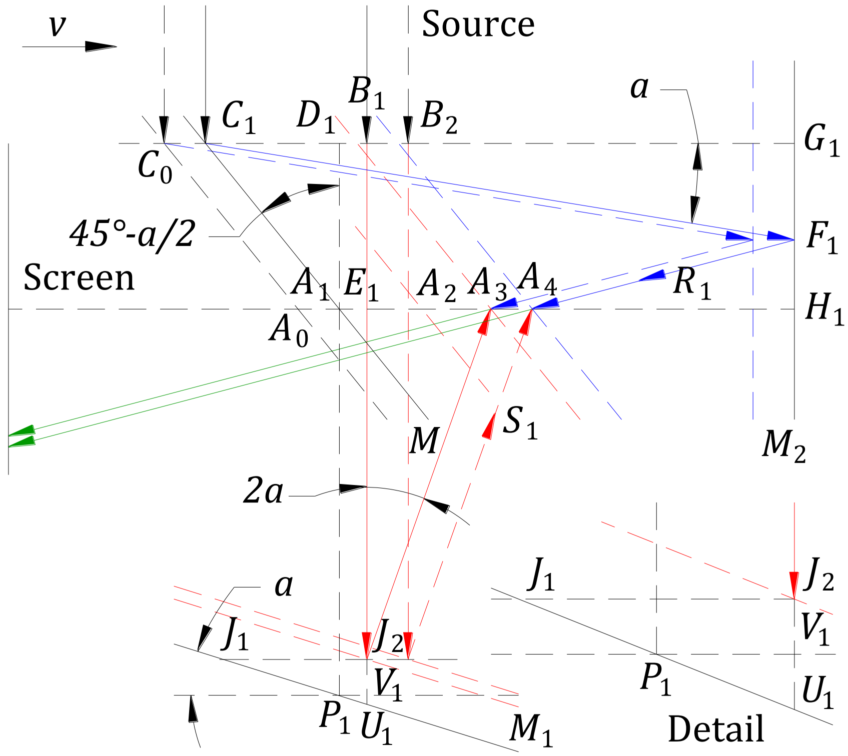

Figure 2. Light path geometry when the interferometer is rotated by 90° from its initial position.

There is only one transmitted ray that starts at point B that continuously intercepts point A. The light from B_1 intercepts point A at A_3 after traveling a path length of L_1.

In time t_{11}, the light from point B_1 travels to point J_2, point A travels from A_1 to A_2, and point J travels from J_1 to J_2. In triangle $J_1J_2U_1$, point J travels along J_1J_2 with the speed v, and the speed along U_1J_2 is $v' = v\,tan(a)$. A ray from the source sees the mirror M_1 coming toward it at the speed v'. In triangle $P_1U_1V_1$, $U_1V_1 = P_1V_1\tan(a)$. The time $t_{11} = (A_1D_1 + L + U_1V_1)/(c + v\,tan(a))$, then $J_2U_1 = vt_{11}\tan(a)$, and $A_1A_2 = vt_{11}$.

The time t_{12}, in which the light travels from point J_2 to point A_3, is given as:

$t_{12} = \dfrac{(L - J_2U_1 + U_1V_1)}{c\cos(2a)}$, then $A_2A_3 = vt_{12}$, and $A_3E_1 = ct_{12}\sin(2a)$.

The distance $P_1V_1 = A_1E_1 = A_1A_2 + A_2A_3 - A_3E_1 \Rightarrow U_1V_1 = P_1V_1\tan(a) = A_1A_2\tan(a) + A_2A_3\tan(a) - A_3E_1\tan(a) = vt_{11}\tan(a) + vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a)$. Substituting the formula for J_2U_1 and U_1V_1 in the formula of time t_{12} $\Rightarrow$

$$t_{12} = \frac{(L - vt_{11}\tan(a) + vt_{11}\tan(a) + vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{c\cos(2a)} \Rightarrow$$

$$t_{12} = \frac{L}{(c\cos(2a) + c\sin(2a)\tan(a) - v\tan(a))}.$$

The time t_{12} can be calculated, then

$$t_{11} = \frac{(A_1D_1 + L + vt_{11}\tan(a) + vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{(c + v\,tan(a))} \Rightarrow$$

$$t_{11} = \frac{(A_1D_1 + L + vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{c}.$$

The time $t_1 = t_{11} + t_{12} = 7.334{,}066{,}740{,}018{,}340E - 08s$, and the path length L_1 and distance A_1A_3 are:

$L_1 = ct_1 = 2.200{,}220{,}022\mathrm{E} + 01m = 4.000{,}400{,}040\mathrm{E} + 07\lambda$ and

$A_1A_3 = vt_1 = 2.200{,}220{,}022\mathrm{E} - 03m = 4.000{,}400{,}040\mathrm{E} + 03\lambda$.

The difference ΔL_2 of the path lengths and distance A_3A_4, for this position, are:

$\Delta L_2 = L_2 - L_1 = 3.300 - 07m = 6.000\mathrm{E} - 01\lambda$ and

$A_3A_4 = A_1A_4 - A_1A_3 = 3.300\mathrm{E} - 11m = 6.000\mathrm{E} - 05\lambda$.

Rotating the interferometer from the initial position to the 90° position, the difference $\Delta L_{12} = \Delta L_2 - \Delta L_1 = 2.200E - 07m = 4.000E - 01\lambda$. The fringe image is then expected to shift by $N_{12} = \Delta L_{12}/\lambda = 0.400$ fringes in the positive direction from the initial position. The positive direction is chosen arbitrarily.

4. Derivation of the light paths with the interferometer rotated by 180° from its initial position

Figure 3 depicts the light paths of the interferometer rotated 180° clockwise from its initial position shown in figure 1. The distances AG and AD are the arm length, equals to L.

The derivation of the light paths starts when point A of the beam splitter is at instance A_1 and the front of light coming from the source is at the line C_1B_1. The point under observation is point A.

There is only one reflected ray that starts at point C that continuously intercepts point A. The light from C_1 travels to M_2 then back to beam splitter, and intercepts point A at A_4 after traveling the path length L_2.

In time t_2, the light travels from point C_1 to point A_4, and the beam splitter travels the distance A_1A_4. The speed of light along the vertical is $c \times cos(a)$, and the speed along the horizontal is $c \times sin(a) = v$. Thus, the distance H_1A_4 is equal to the distance A_1A_4 and $A_1H_1 = 2A_1A_4 = 2vt_2$.

In triangle $C_1A_1H_1$, angle $C_1A_1H_1$ is equal to $\pi/4 - a/2$. Thus,

$$\tan(\pi/4 - a/2) = \frac{C_1H_1}{A_1H_1} \quad \Rightarrow \quad C_1H_1 = A_1H_1\tan(\pi/4 - a/2).$$

The time t_2 is given as:

$$t_2 = \frac{C_1H_1}{c\,cos(a)} + \frac{2L}{c\,cos(a)} = \frac{A_1A_4}{v} \quad \Rightarrow \quad \frac{A_1H_1\tan(\pi/4 - a/2)}{c\,cos(a)} + \frac{2L}{c\,cos(a)} = \frac{A_1H_1}{2v} \quad \Rightarrow$$

$$A_1H_1\left(\frac{1}{2v} - \frac{\tan(\pi/4 - a/2)}{c\,cos(a)}\right) = \frac{2L}{c\,cos(a)}.$$

The distance A_1H_1 can be calculated and then the time $t_2 = A_1H_1/2v = 7.334{,}800{,}183{,}355{,}340E - 08$. The path length L_2 and the distance A_1A_4 can be calculated.

$L_2 = ct_2 = 2.200{,}440{,}055E + 01m = 4.000{,}800{,}100E + 07\lambda$, and

$A_1A_4 = vt_2 = 2.200{,}440{,}055E - 03m = 4.000{,}800{,}100E + 03\lambda$.

There is only one transmitted ray that starts at point B that continuously intercepts point A. The light from B_1 intercepts point A at A_3 after traveling the path length L_1. The light at A_3 is reflected towards the screen, making angle $G_1A_3K_1$, which is equal to $90° - a$, and therefore coincides with the reflected ray at A_3.

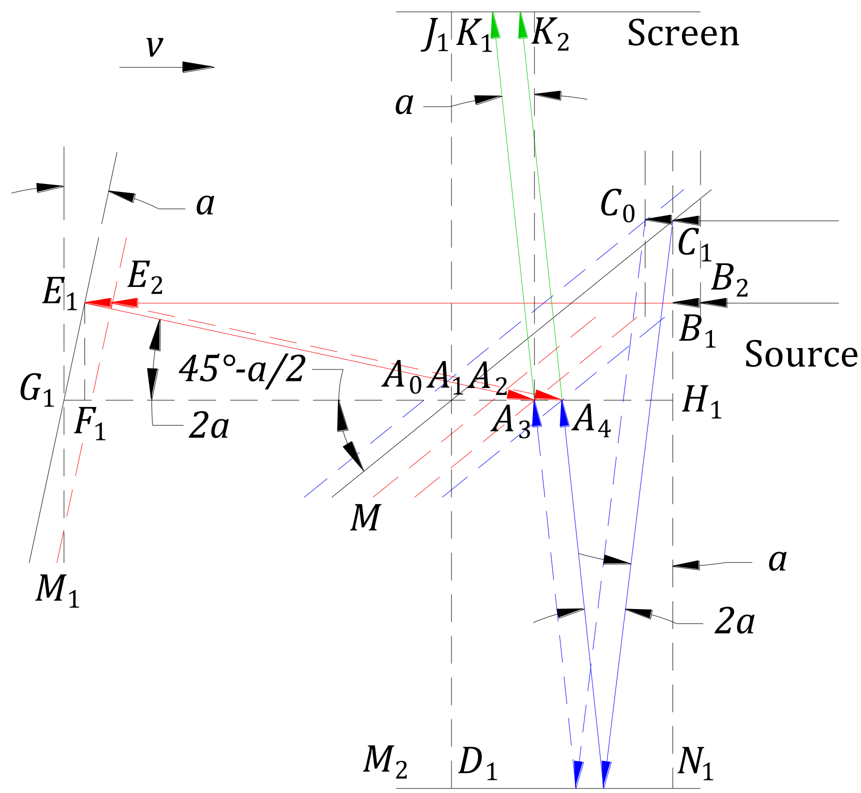

Figure 3. Light path geometry when the interferometer is rotated by 180° from its initial position.

The instance when the light from B_1 arrives at E_1 includes points E_1, F_1, G_1, and A_2. In time t_{11}, the light travels from B_1 to E_1, and point A travels from A_1 to A_2. In time t_{12}, the light travels from E_1 to A_3 and point A travels from A_2 to A_3.

From the geometry of this path, triangles $A_3E_1F_1$ and $E_1F_1G_1$ give:

$$tan(2a) = \frac{E_1F_1}{A_3F_1} \;\Rightarrow\; E_1F_1 = A_3F_1\, tan(2a), \qquad tan(a) = \frac{F_1G_1}{E_1F_1} \;\Rightarrow\; E_1F_1 = \frac{F_1G_1}{tan(a)},$$

thus $$E_1F_1 = A_3F_1 \tan(2a) = \frac{F_1G_1}{\tan(a)} \qquad \Rightarrow \qquad F_1G_1 = A_3F_1 \tan(a)\tan(2a).$$

In triangle $A_3E_1F_1$, the speed along F_1A_4 is $c \times cos(2a)$. Times t_{11} and t_{12} are defined as:

$$t_{11} = \frac{L - F_1G_1 + A_1H_1}{c+v} = \frac{B_1E_1}{c} \qquad \text{and} \qquad t_{12} = \frac{L - F_1G_1}{c\,cos(2a) - v} = \frac{A_3F_1}{c\,cos(2a)}.$$

Substituting the formula for F_1G_1 in the formulas of time t_{12}:

$$\frac{L - A_3F_1 \tan(a)\tan(2a)}{c\,cos(2a) - v} = \frac{A_3F_1}{c\,cos(2a)} \qquad \Rightarrow$$

$$A_3F_1\left(\frac{1}{c\,cos(2a)} + \frac{\tan(a)\tan(2a)}{c\,cos(2a)+v}\right) = \frac{L}{c\,cos(2a)-v}.$$

The distances A_1H_1 is known from the calculation of t_1. The distances A_3F_1 and F_1G_1 can be calculated and then t_{11} and t_{12} . The time $t_1 = t_{11} + t_{12} = 7.334,800,146,688,670E - 08s$. In time t_1, the light travels from point B_1 to point A_3 and the beam splitter travels the

distance A_1A_3. The path length L_1 and the distance A_1A_3 are:
$L_1 = ct_1 = 2.200{,}440{,}044E + 01m = 4.000{,}800{,}080E + 07\lambda$, and
$A_1A_3 = vt_1 = 2.200{,}440{,}044E - 03m = 4.000{,}800{,}080E + 03\lambda$.

The difference ΔL_3 of the paths length L_2 and L_1, and the distance A_3A_4 are:
$\Delta L_3 = L_2 - L_1 = 1.100E - 07m = 2.000E - 01\lambda$, and
$A_3A_4 = A_1A_4 - A_1A_3 = 1.100E - 11m = 2.000E - 05\lambda$.

Rotating the interferometer from the 90° position to the 180° position, the difference $\Delta L_{23} = \Delta L_3 - \Delta L_2 = -2.200{,}219E - 07m = -0.400.399\lambda$. The fringes are then expected to shift by $N_{23} = \Delta L_{23}/\lambda = -0.400$ fringes.

5. Derivation of the light paths with the interferometer rotated by 270° from its initial position

Figure 4 depicts the light paths of the interferometer rotated 270° clockwise from its initial position of figure 1.

The distances AP and AH are the arm length equals to L. The derivation of the light paths starts when point A of the beam splitter is at instance A_1 and the front of light coming from the source is at the line B_1C_1. Points A_1, E_1, B_1, C_1, D_1, J_1, and P_1 belong to the same instance and points B_1 and C_1 belong to the same wavefront. The point under observation is point A.

There is only one reflected ray that starts at point C that continuously intercepts point A. The light from C_1 intercepts point A at A_4 after traveling the path length L_2.

In triangle $C_1F_1G_1$, the speed along C_1G_1 is $c \times cos(a)$ and the speed along G_1F_1 is $c \times sin(\text{a}) = v$. The time t_{21}, in which the light travels from point C_1 to point F_1 is:

$$t_{21} = \frac{C_1D_1 + L}{c\cos(a) + v} = \frac{G_1F_1}{v} \qquad \Rightarrow \qquad G_1F_1 = \frac{(C_1D_1 + L)v}{c\cos(a) + v}.$$

The instance when the light from C_1 arrives at F_1 includes points F_1, G_1, and H_1.

In triangle $A_4F_1H_1$, the speed along H_1A_4 is $c \times cos(a)$, and the speed along F_1H_1 is $c \times sin(\text{a}) = v$. The time t_{22}, in which the light travels from point F_1 to point A_4, is given by:

$$t_{22} = \frac{L}{c\,cos(a) - v} = \frac{F_1H_1}{v} \qquad \Rightarrow \qquad F_1H_1 = \frac{Lv}{c\,cos(a) - v}.$$

Triangle $C_1A_1D_1$ gives: $tan(\pi/4 - a/2) = C_1D_1/A_1D_1 \Rightarrow$
$C_1D_1 = A_1D_1\,tan(\pi/4 - a/2) = (G_1F_1 + F_1H_1)\,tan(\pi/4 - a/2)$.

Substituting the formula for G_1F_1 and F_1H_1 in the formula for C_1D_1:

$$C_1D_1 = \frac{(C_1D_1 + L)v\,tan(\pi/4 - a/2)}{c\,co\,s(a) + v} + \frac{Lv\,tan(\pi/4 - a/2)}{c\,cos(a) - v} \qquad \Rightarrow$$

$$C_1D_1\left(1 - \frac{v\,tan(\pi/4 - a/2)}{c\,cos(a) + v}\right) = Lv\,tan(\pi/4 - a/2)\left(\frac{1}{c\,cos(a) + v} + \frac{1}{c\,cos(a) - v}\right).$$

The distance C_1D_1, and then the distance A_1D_1 and time t_2 can be calculated.

$$A_1D_1 = \frac{C_1D_1}{tan(\pi/4 - a/2)} \text{ and}$$

$$t_2 = t_{21} + t_{22} = \frac{C_1D_1 + \text{L}}{\text{c}\cos(a) + \text{v}} + \frac{L}{c\,cos(a) - v} = 7.334{,}066{,}703{,}344{,}340E - 08s.$$

The light path length L_2 and the distance A_1A_4 are:
$L_2 = ct_2 = 2.200{,}220{,}011E + 01m = 4.000{,}400{,}020E + 07\lambda$, and
$A_1A_4 = vt_2 = 2.200{,}220{,}011E - 03m = 4.000{,}400{,}020E + 03\lambda$

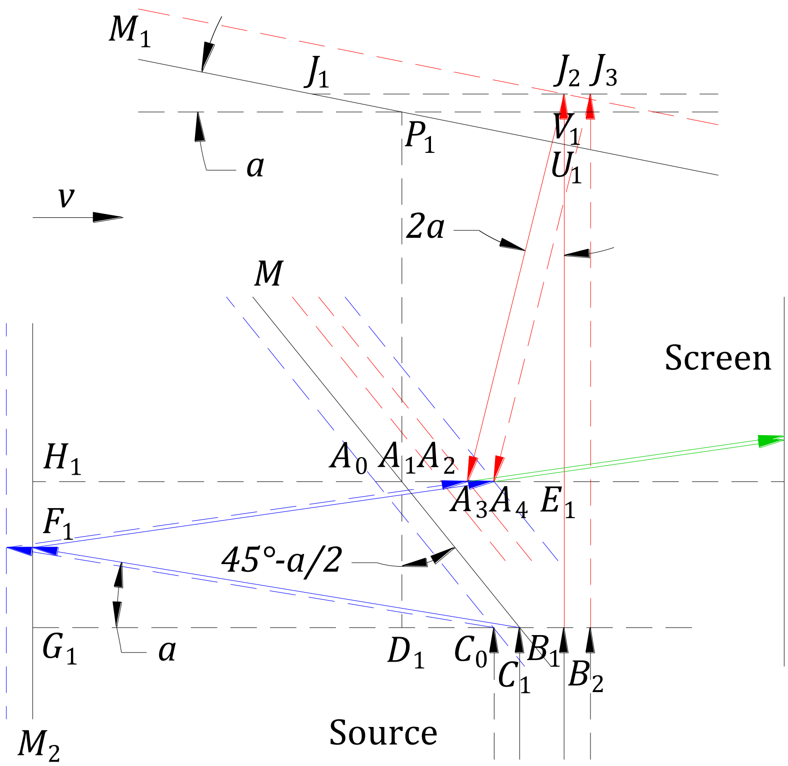

Figure 4. Light path geometry when the interferometer is rotated by 270° from its initial position.

There is only one transmitted ray that starts at point B that continuously intercepts point A. The light from B_1 intercepts point A at A_3 after traveling the path length L_1.

In time t_{11}, the light from point B_1 travels to point J_2, point A travels from A_1 to A_2, and point J travels from J_1 to J_2. In triangle $J_1J_2U_1$, point J travels along J_1J_2 with the speed v, and along U_1J_2 with the speed $v' = v\,tan(a)$. A ray from the source sees the mirror M_1 departing from it at the speed v'. In triangle $P_1U_1V_1$, $U_1V_1 = P_1V_1\tan(a)$. The time $t_{11} = (A_1D_1 + L - U_1V_1)/(c - v\,tan(a))$, then $J_2U_1 = vt_{11}\tan(a)$, and $A_1A_2 = vt_{11}$.

The time t_{12}, in which the light travels from point J_2 to point A_3, is given as:

$t_{12} = \dfrac{(L + J_2U_1 - U_1V_1)}{c\cos(2a)}$, then $A_2A_3 = vt_{12}$, and $A_3E_1 = ct_{12}\sin(2a)$.

The distance $P_1V_1 = A_1E_1 = A_1A_2 + A_2A_3 + A_3E_1 \Rightarrow U_1V_1 = P_1V_1\tan(a) = A_1A_2\tan(a) + A_2A_3\tan(a) + A_3E_1\tan(a) = vt_{11}\tan(a) + vt_{12}\tan(a) + ct_{12}\sin(2a)\tan(a)$. Substituting the formula for J_2U_1 and U_1V_1 in the formula of time t_{12} $\Rightarrow$

$$t_{12} = \frac{(L + vt_{11}\tan(a) - vt_{11}\tan(a) - vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{c\cos(2a)} \Rightarrow$$

$$t_{12} = \frac{L}{(c\cos(2a) + c\sin(2a)\tan(a) + v\tan(a))}.$$

The time t_{12} can be calculated, then

$$t_{11} = \frac{(A_1D_1 + L - vt_{11}\tan(a) - vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{(c - v\,tan(a))} \Rightarrow$$

$$t_{11} = \frac{(A_1 D_1 + L - vt_{12}\tan(a) - ct_{12}\sin(2a)\tan(a))}{c}.$$

The time $t_1 = t_{11} + t_{12} = 7.334{,}066{,}593E - 08s$. The path length L_1 and distance A_1A_3 are:

$L_1 = ct_1 = 2.200{,}219{,}978\text{E} + 01m = 4.000{,}399{,}960\text{E} + 07\lambda$ and

$A_1A_3 = vt_1 = 2.200{,}219{,}978\text{E} - 03m = 4.000{,}399{,}960\text{E} + 03\lambda$.

The difference ΔL_4 of the paths length and the distance A_3A_4, for this position, are:

$\Delta L_4 = L_2 - L_1 = 3.300E - 07m = 6.000E - 01\lambda$, and

$A_3A_4 = A_1A_4 - A_1A_3 = 3.300E - 11m = 6.000E - 05\lambda$.

The distances B_1B_2, J_2J_3, C_0C_1, and A_0A_1 are equal to A_3A_4. Employing similar reasoning as in the previous section, the difference ΔL_4 is the difference of the path length of the rays that interfere in any instance of point A.

Rotating the interferometer from the 180° position to the 270° position, the difference $\Delta L_{34} = \Delta L_4 - \Delta L_3 = 2.200E - 07m = 4.000E - 01\lambda$. The fringes are then expected to shift by $N_{34} = \Delta L_{34}/\lambda = 0.400$ fringes.

Rotating the interferometer from the 270° position to the 360° position, the difference $\Delta L_{41} = \Delta L_1 - \Delta L_4 = -2.200E - 07m = -4.000E - 01\lambda$. The fringes are then expected to shift by $N_{41} = \Delta L_{41}/\lambda = -0.400$ fringes.

References

[1] A. A. Michelson, and E. W. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," American Journal of Science, v. 34, p. 333–345 (1887)

[2] F. Dambi, "A new derivation of the Michelson-Morley experiment" (2018) http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7384