MOTION OF AN ELECTRON IN CLASSICAL AND RELATIVISTIC ELECTRODYNAMICS AND AN ALTERNATIVE ELECTRODYNAMICS

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Abstract
For an electron of mass \( m \) and charge \(-e\) moving with velocity \( \mathbf{v} \) and acceleration \( \frac{d\mathbf{v}}{dt} \) in an electric field of magnitude \( E \), the accelerating force is put as vector \( \mathbf{F} = e\mathbf{E}(\mathbf{e} - \mathbf{v})/c = m\frac{d\mathbf{v}}{dt} \), where \( \mathbf{e} - \mathbf{v} \) is the relative velocity between the electrical force propagated with velocity of light \( c \) and the electron. The electron is accelerated to the speed of light \( c \) or it revolves in a circle at a constant speed. The relativistic mass-velocity formula is correct for circular revolution and “mass” in that formula is the ratio of electrostatic force \(-e\mathbf{E}\) to acceleration \(-\mathbf{v}/r\) in a circle of radius \( r \), which is infinitely large for rectilinear motion. An electrodynamics is developed for an electron accelerated to the speed of light at constant mass and with emission of radiation, contrary to classical and relativistic electrodynamics. Radiation occurs if there is a change in kinetic energy.

Keywords: Aberration, acceleration, charge, field, force, mass, radiation, relativity, velocity

1. Introduction
There are now three systems of electrodynamics in physics. Classical electrodynamics is applicable to electrically charged particles moving at a speed that is much slower than that of light. Relativistic electrodynamics is for particles moving at a speed comparable to that of light. Quantum electrodynamics is for atomic particles moving at very high speeds. There should be one system of electrodynamics applicable to all particles at speeds up to that of light. Classical electrodynamics is based on the second law of motion, originated by Galileo Galilei in 1638 [1], but enunciated by Isaac Newton [2]. The theory of special relativity was formulated in 1905 mainly by Albert Einstein [3, 4]. The quantum theory was devised by Max Planck [5], Louis de Broglie [6] and others. Relativistic electrodynamics reduces to classical electrodynamics at low speeds as normally encountered. The relativity and quantum theories are incompatible at high speeds. Both the relativity and quantum theories, therefore, cannot be correct. One of the theories or both theories may be wrong. Indeed, special relativity is under attack by physicists: Beckmann [7] and Renshaw [8]. Relativity is the bone of contention in this paper. The paper introduces an alternative electrodynamics, applicable to an electrically charged particle, like an electron, moving in an electric field at speeds up to that of light, with mass of a moving particle remaining constant and with emission of radiation.

1.1 Newton’s Second Law of Motion
For a body moving with velocity \( \mathbf{v} \) at time \( t \), Newton’s second law of motion, which includes the first and third laws, relates the rate of change of velocity or acceleration \( \frac{d\mathbf{v}}{dt} \) produced on a body of mass \( m \), to the impressed force \( \mathbf{F} \), in the vector equation:

\[
\mathbf{F} = m \frac{d\mathbf{v}}{dt}
\]

(1)

According to equation (1), where mass \( m \) is a constant independent of velocity \( \mathbf{v} \), the acceleration becomes zero, and the body moves in a straight line with constant speed, if the accelerating force \( \mathbf{F} \) reduces to zero or if the mass \( m \) becomes infinitely large. With the advent of the theory of special relativity, where \( m \) increases with \( \mathbf{v} \), Newton’s second law of motion was modified. The law now relates force \( \mathbf{F} \) to the rate of change of momentum \( m\mathbf{v} \), thus:
This paper assumes the validity of Newton’s second law of motion, but where mass \( m \) remains constant and accelerating force \( F \) reduces to zero at the velocity of light \( c \), of magnitude \( c \).

### 1.2 Coulomb’s Law of Electrostatic Force
Coulomb’s law, first published in 1785, is the most important principle in physics. It gives the force \( F \) of attraction between an electron of charge \( -e \) and a positive electric charge \( Q \), separated by a distance \( r \) in space, as vector:

\[
F = \frac{-eQ}{4\pi\varepsilon_0 r^2} = -eE
\]

where \( E \) is the electrostatic field intensity due to charge \( Q \) and \( \varepsilon_0 \) is the permittivity of space. The current problem of physics lies in making Coulomb’s law independent of velocity of the electron, of charge \( -e \), moving with velocity \( v \) in the electrostatic field of intensity \( E \).

### 1.3 Relativistic Mass-velocity Formula
According to Newton’s second law of motion, a force can accelerate particle to a speed greater than that of light, with its mass remaining constant. But experiments with accelerators have shown that no particle, not even the electron, the lightest particle known in nature, can be accelerated beyond the speed of light. The theory of special relativity explains this limitation by positing that the mass of a particle increases with its speed, becoming infinitely large at the speed of light. That since an infinite mass cannot be accelerated any faster by any finite force, the speed of light becomes the limit to which a body can be accelerated. This is a plausible proposition. The relativistic mass-velocity formula is:

\[
m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0
\]

where \( m \) is the mass of a particle moving with speed \( v \) relative to an observer, \( m_0 \) is the rest mass, \( c \) is the speed of light in a vacuum and \( \gamma \) is the Lorentz factor. Equation (4), where \( m \) is a physical quantity, becoming infinitely large at the speed of light, is the bone of contention in this paper. The difficulty with infinite mass, at the speed of light \( c \), in equation (4), is the Achilles’ heel of the theory of special relativity. Resolving this difficulty, by allowing a moving particle to reach the speed of light with its mass remaining constant, is the purpose of this paper. Such a resolution, giving the ultimate speed without infinite mass, would bring great relief to physicists all over the world.

The proponents of special relativity just ignore the problem with equation (4). They say that it is the momentum, not the mass, which increases with speed. They avoid the difficulty altogether by arguing that the speed never really reaches that of light \( c \), or that particles moving at the speed of light (photons) have zero rest mass. But electrons are easily accelerated and have been accelerated to practically the speed of light as demonstrated by William Bertozzi in 1964 [9], using a linear accelerator of 15 MeV energy. Electron accelerators, betatrons and electron synchrotrons of over \( 10^6 \) MeV, have been built and operated with electrons moving at the speed of light for all practical purposes.

A most remarkable demonstration of the existence of a universal limiting speed, equal to the speed of light \( c \), was in an experiment by William Bertozzi of the Massachusetts Institute of Technology [9]. The experiment (see Table1) showed that electrons accelerated through energies of 15 MeV or over, attain, practically, the speed of light \( c \). Bertozzi measured the heat energy \( J \) developed when a stream of accelerated electrons hit an aluminium target at the end of their flight path, in a linear accelerator. He found the heat energy released \( J \) to be nearly
equal to the potential energy $P$ lost, to give $P = J = K$, where $K$ was the kinetic energy lost. Bertozzi identified $J$ as solely due to the kinetic energy $K$ lost by the electrons, on the assumption that the accelerating force on an electron of charge $-e$ moving in an electric field of magnitude $E$, is $-eE$, independent of the speed of the electron.

Bertozzi might have made a mistake in equating the potential energy $P$ with the kinetic energy $K$ of the electrons. The energy equation should have been $P = J + R = K + R$, where $R$ was the energy radiated. Radiation is propagated at the speed of light with maximum in a direction perpendicular to the acceleration of the electrons. The transverse radiation had no heating effect, as there was no component impinging at the same point or on the same target as the accelerated electrons. This radiation is a result of aberration of electric field.

1.4 Larmor Formula for Radiation Power

Larmor formula of classical electrodynamics, described by Griffith [10], gives the radiation power $R_\circ$ of an accelerated electron as proportional to the square of its acceleration. For an electron revolving with constant speed $v$ in a circle of radius $r$ with centripetal acceleration of magnitude $v^2/r$, Larmor classical formula gives $R_\circ = (e^2/6\pi\varepsilon_o r^3)v^4/c^3$, where $\varepsilon_o$ is the permittivity of space. Special relativity adopted this formula [10] and gives radiation power $R = \gamma^4 R_\circ$, where the Lorentz factor $\gamma$ is defined in equation (4). The relativistic factor $\gamma^4$ means that the radiation power increases explosively as the speed $v$ approaches that of light $c$.

According to Larmor formula, the hydrogen atom, consisting of an electron revolving round a heavy positively charged nucleus, would radiate energy as it accelerates and spirals inward to collide with the nucleus, leading to the collapse of the atom. But atoms are the most stable entities known in nature. Use of Larmor formula was unfortunate as it led physics astray early in the 20th century. It required the brilliant hypotheses of Niels Bohr’s [11] quantum mechanics to stabilize and retain the Rutherford’s [12] nuclear model of the hydrogen atom.

In the alternative electrodynamics, there is no need for Bohr’s quantum theory to stabilize the nuclear model of the hydrogen atom. In this paper it is shown that circular revolution of an electron round a nucleus, with constant speed, is without irradiation. Radiation comes only if there is a change in the potential energy or kinetic energy of a charged particle moving in an electric field.

1.5 Aberration of Electric Field

Figure 1 depicts an electron of charge $-e$ and mass $m$, moving at a point $P$ with velocity $\mathbf{v}$, in an electrostatic field of intensity $\mathbf{E}$ due to a stationary source charge $+Q$ at an origin $O$. For motion at an angle $\theta$ to the accelerating force $\mathbf{F}$, the electron is subjected to aberration of electric field. This is a phenomenon similar to aberration of light discovered by the English astronomer James Bradley in 1725 [13]. In aberration of electric field, as in aberration of light, the direction of the electric field, indicated along $\mathbf{PN}$ by the velocity vector $\mathbf{e}$, as shown in Figure 1, appears shifted by an aberration angle $\alpha$, from the instantaneous line $\mathbf{PO}$, such that:

$$\sin \alpha = \frac{v}{c} \sin \theta$$

(5)

where the speeds $v$ and $c$ are the magnitudes of the velocities $\mathbf{v}$ and $\mathbf{e}$ respectively. The reference direction is the direction of the accelerating force $\mathbf{F}$. Equation (5) was first derived by astronomer James Bradley with respect to light radiation from a star. Aberration of electric field, which is missing in classical and relativistic electrodynamics, is used in the formulation of the alternative electrodynamics.

The result of aberration of electric field is that the accelerating force on a moving electron depends on the velocity of the electron in an electric field. If the accelerating force is reduced to zero at the speed of light, that speed becomes the ultimate limit, in accordance with...
Newton’s first law of motion. Also, the difference between the accelerating force $F$ (on a moving electron) and the electrostatic force $-eE$ (on a stationary electron) gives the radiation reaction force, from which the radiation power is derived, in contrast to Abraham-Lorentz formula and Larmor formula of classical electrodynamics.

2. Equations of Rectilinear Motion in Classical electrodynamics

The accelerating force $F$ exerted on an electron of charge $-e$ and mass $m$ moving at time $t$ with velocity $v$ and acceleration $(dv/dt)$, in an electrostatic field of intensity $E$, in accordance with Coulomb’s law (equation 3) and Newton’s 2\textsuperscript{nd} law of motion (equation 1), is:

$$ F = -eE = m \frac{dv}{dt} \quad (6) $$

For an electron accelerated in the opposite direction of a uniform electrostatic field of constant intensity $E = E\hat{u}$ in the direction of unit vector $\hat{u}$, equation (6) becomes:

$$ F = -eE\hat{u} = -m \frac{dv}{dt}\hat{u} \quad (7) $$

Equation (7) is a first order differential equation with solution:

$$ v = at $$

$$ \frac{v}{c} = \frac{at}{c} \quad (8) $$

where speed $v = 0$ at time $t = 0$ and $a = eE/m$ is a constant. The speed $v$ may reach any value.

An electron moving with velocity $v$ in the positive direction of the field, suffers a deceleration and the equation of motion becomes:

$$ F = -eE\hat{u} = m \frac{dv}{dt}\hat{u} \quad (9) $$

The solution of equation (9) for an electron decelerated from speed of light $c$ by a uniform field of magnitude $E$, is:
The electron is decelerated to a stop in time \( t = c/a \).

3. Equations of Rectilinear Motion in Relativistic Electrodynamics

In relativistic electrodynamics, the accelerating force \( \mathbf{F} \) exerted on an electron of charge \( -e \) and mass \( m \), moving with velocity \( \mathbf{v} \) at time \( t \) in an electrostatic field of intensity \( \mathbf{E} \), is:

\[
\mathbf{F} = -e\mathbf{E}\hat{\mathbf{u}} = -\frac{d}{dt}(mv)\hat{\mathbf{u}}
\]

where mass \( m \) increases with speed \( v \) in accordance with equation (4), so that

\[
\mathbf{F} = -e\mathbf{E}\hat{\mathbf{u}} = -\frac{d}{dt} m\sqrt{1 - \frac{v^2}{c^2}} \hat{\mathbf{u}}
\]

For a constant field of magnitude \( E \), equation (11) is also a first order differential equation with solution as:

\[
v = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}}
\]

where speed \( v = 0 \) at time \( t = 0 \) and \( a = eE/m_0 \) is a constant. Equation (12) makes the speed of light \( c \) the ultimate limit as time \( t \to \infty \), in contrast to equation (8).

In relativistic electrodynamics, an electron moving at the speed of light \( c \) cannot be decelerated and stopped by any finite force. Such a moving electron continues to move at the speed of light, gaining potential energy without losing kinetic energy.

4. Equations of Motion in the Alternative Electrodynamics

The force exerted on an electron, moving with velocity \( \mathbf{v} \), by an electrostatic field, is propagated at the velocity of light \( c \) relative to the source charge and transmitted with velocity \( (c - \mathbf{v}) \) relative to the moving electron. The electron can be accelerated to the velocity of light \( c \) and no faster. In Figure 1 the electron may be accelerated in the direction of the force with \( \theta = 0 \) or it may be decelerated against the force with \( \theta = \pi \) radians or it can revolve in a circle, at constant speed, perpendicular to the accelerating field, with \( \theta = \pi/2 \) radians.

The accelerating force \( \mathbf{F} \) (Figure 1), on an electron of charge \( -e \) and mass \( m \) moving at time \( t \) with velocity \( \mathbf{v} \) and acceleration \( (d\mathbf{v}/dt) \), in an electrostatic field of magnitude \( E \), is proposed as given by the vector equation and Newton’s second law of motion, thus:

\[
\mathbf{F} = \frac{eE}{c} (\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt}
\]

where \( \mathbf{c} \) is the velocity of light, of magnitude \( c \), at aberration angle \( a \) to the accelerating force \( \mathbf{F} \) and \( (\mathbf{c} - \mathbf{v}) \) is the relative velocity of transmission of the force with respect to the moving electron. The force, propagated at velocity of light \( \mathbf{c} \), cannot “catch up” and “impact” on an electron also moving with velocity \( \mathbf{v} = \mathbf{c} \). With no force on the electron, it continues to move with constant speed \( c \), in accordance with Newton’s first law of motion. Equation (13) may be regarded as an extension, amendment or modification of Coulomb’s law of electrostatic force between two electric charges (equation 3), taking into consideration the relative velocity between the charges. In equation (13), the electric field experienced by a moving charged particle may also be regarded as dependent on velocity of the particle in the field.

Equation (5) linking the angle \( \theta \) with the aberration angle \( a \) (Figure 1) and equation (13) are the twin equations of the alternative electrodynamics. Equation (13) is the basic expression.
of the alternative electrodynamics. Expanding equation (13) by taking the *modulus* of the vector \((c - v)\), with respect to the angles \(\theta\) and \(\alpha\) (Figure 1), gives:

\[
F = \frac{eE}{c} (c - v) = \frac{-eE}{c} \sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}} \hat{u} = m \frac{dv}{dt} \tag{14}
\]

where \((\theta - \alpha)\) is the angle between the vectors \(c\) and \(v\).

### 4.1 Equations of Rectilinear Motion

For an electron accelerated in a straight line, where \(\theta = 0\), equations (5) and (14) give:

\[
F = -eE \left(1 - \frac{v}{c}\right) \hat{u} = -m \frac{dv}{dt} \hat{u} \tag{15}
\]

This is a first order differential equation. The solution of equation (15) for an electron accelerated by a uniform electric field of constant magnitude \(E\), from initial speed \(u\), is:

\[
v = c - \left\{c - u\right\}\left\{\exp\left(-\frac{at}{c}\right)\right\} \tag{16}
\]

For acceleration from zero initial speed \((u = 0)\), equation (16) becomes:

\[
\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \tag{17}
\]

where \(a = eE/m\) is a constant. Fig. 2.C1 is a graph of \(v/c\) against \(at/c\) for equation (17). The electron will be accelerated, by the electric field, to an ultimate speed equal to that of light \(c\).

The distance \(x = \int v(dt)\), covered in time \(t\) by an electron accelerated from zero initial speed, is obtained by integrating equation (17), to give:

\[
x = ct + \frac{c^2}{a} \left\{\exp\left(-\frac{at}{c}\right) - 1\right\} \tag{18}
\]

For an electron decelerated in a straight line, where \(\theta = \pi\) radians, equations (5) and (14) give the decelerating force \(F\) as:

\[
F = -eE \left(1 + \frac{v}{c}\right) \hat{u} = m \frac{dv}{dt} \hat{u} \tag{19}
\]

Solving the differential equation (19) for an electron decelerated from speed \(u\), by a uniform electric field of magnitude \(E\), gives:

\[
v = \left\{c + u\right\}\left\{\exp\left(-\frac{at}{c}\right)\right\} - c \tag{20}
\]

For an electron decelerated from speed of light \(c\), equation (20) becomes:

\[
\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right) - 1 \tag{21}
\]

Figure 2.C2 is a plot of \(v/c\) against \(at/c\) according to equation (21). The electron will be decelerated to a stop \((v = 0)\) in time \(t = (c/a)\ln 2 = 0.693c/a\), having lost kinetic energy \(0.5mc^2\), equal to the potential energy gained plus the energy radiated. Energy is radiated whenever there is a change in the kinetic energy or potential energy of a moving electron.

Figure 2 shows a graph of \(v/c\) against \(at/c\) for an electron accelerated from zero initial speed or an electron decelerated from speed of light \(c\), by a uniform electric field: the solid lines, \(A1\) and \(A2\) according to classical electrodynamics (equations 8 and 10), the dashed curve \(B1\) (for equation 12) and line \(B2\) according to relativistic electrodynamics and the dotted curves \(C1\) and \(C2\) according to equations (17) and (21) of the alternative electrodynamics. At low speeds the three systems of electrodynamics coincide for accelerated electrons but there is a marked departure for electrons decelerated from the speed of light.
Figure 2. $v/c$ (speed in units of $c$) against $at/c$ (time in units of $c/a$) for an electron of charge $-e$ and mass $m = m_o$, accelerated from zero initial speed or decelerated from the speed of light $c$, by a uniform electrostatic field of magnitude $E$, where $a = eE/m$; the lines $A1$ and $A2$ according to classical electrodynamics (equations 8 and 10), the dashed curve $B1$ (equation 12) and line $B2$ according to relativistic electrodynamics and the dotted curves $C1$ and $C2$ according to equations (17) and (21) of the alternative electrodynamics.

The distance $x = \int v(\text{d}t)$, covered in time $t$ by an electron decelerated from the speed of light $c$, is obtained by integrating equation (21) to give:

$$x = \frac{2c^2}{a} \left\{ 1 - \exp \left( \frac{-at}{c} \right) \right\} - ct \quad (22)$$

In equations (21) and (22) it is seen that an electron entering a uniform decelerating field at a point with speed $c$, comes to a stop in time $t = 0.693c/a$, at a distance $X = 0.307c^2/a$ from the point of entry, having lost kinetic energy equal to $0.5mc^2$, gained potential equal to $eEX = 0.307mc^2$ and radiated energy equal to $0.193mc^2$. The electron will come back to the starting point ($x = 0$) in time $t = 1.594c/a$, with speed $0.594c$ and kinetic energy $0.176mc^2$, having lost potential energy equal to $eEX = 0.307mc^2$ and radiated energy equal to $0.131mc^2$. The electron will then be accelerated to the speed of light $c$, as the ultimate limit, with emission of radiation. These results are not obtainable from classical or relativistic electrodynamics.

### 4.2 Equations of Circular Motion

For $\theta = \pi/2$ radians we get revolution is in a circle of radius $r$ with constant speed $v$ and centripetal acceleration $(-v^2/r)\hat{u}$. Equations (5) and (14), with mass $m = m_o$ (rest mass) and noting that $\cos(\pi/2 - \alpha) = \sin \alpha = v/c$, give the accelerating force $F$ as:

$$F = -eE \sqrt{1 - \frac{v^2}{c^2}} \hat{u} = -m \frac{v^2}{r} \hat{u} = -m_o \frac{v^2}{r} \hat{u}$$
\[ eE = \frac{m_0 v^2}{r} = \zeta \frac{v^2}{r} \]  
\[ \zeta = \frac{e E r}{v^2} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

Equation (1) for relativistic mass ‘\( m \)’ and equation (24) for \( \zeta \) (zeta ratio) are identical but obtained from two different points of view. In equation (1), of relativistic electrodynamics, the quantity ‘\( m \)’ increases with speed \( v \), becoming infinitely large at speed \( c \). In equation (24), of the alternative electrodynamics, mass \( m \) remains constant at the rest mass \( m_o \), and the quantity \( \zeta = [(eE)/(v^2/r)] \) is the ratio of magnitude of the radial electrostatic force \( -(eE) \) on a stationary electron, to the centripetal acceleration \( -(v^2/r) \) in circular motion. This quantity \( \zeta \), the zeta ratio, may become infinitely large at the speed of light \( c \), without any difficulty. At the speed of light, the electron moves with zero acceleration in an arc of a circle of infinite radius, which is a straight line, to make the ratio \( \zeta \) also infinite without any problem. Equating \( \zeta \) with physical mass ‘\( m \)’ , which has a weight, is an expensive case of mistaken identity.

In classical electrodynamics, radius \( r \) of circular revolution for an electron of charge \( -e \) and mass \( m \), in a radial electric field of magnitude \( E \) due to a positively charged nucleus, is:

\[ r = \frac{mv^2}{eE} = \frac{m_0 v^2}{eE} = r_o \]  

where \( m = m_o \) is a constant and \( r_o \) is the classical radius. In relativistic electrodynamics, where mass \( m \) varies with speed \( v \) in accordance with equation (4), the radius of revolution becomes:

\[ r = \frac{mv^2}{eE} = \frac{m_0 v^2}{eE \sqrt{1 - \frac{v^2}{c^2}}} = \gamma r_o \]  

In the alternative electrodynamics, where \( m = m_o \) is a constant, the radius \( r \) of revolution, obtained from equation (24), becomes:

\[ r = \frac{m_0 v^2}{eE \sqrt{1 - \frac{v^2}{c^2}}} = \frac{m v^2}{eE \sqrt{1 - \frac{v^2}{c^2}}} = \gamma r_o \]  

Relativistic electrodynamics and the alternative electrodynamics give the same expression for radius of revolution in circular motion as \( r = \gamma r_o \), but for different reasons. This increase in radius with speed was misconstrued in special relativity as increase in mass with speed.

5. **Radiation Reaction Force and Radiation Power**

The accelerating force on a moving electron is less than the electrostatic force \( -(eE) \) on a stationary electron. The difference between the accelerating force \( \mathbf{F} \) and the electrostatic force \( -(eE) \) is the radiation reaction force \( \mathbf{R}_f = \mathbf{F} - -(eE) \), that is always present when a charged particle is accelerated by an electric field. This is analogous to a frictional force, which always opposes motion. A simple and useful expression for radiation reaction force \( \mathbf{R}_f \) is missing in classical and relativistic electrodynamics and it makes all the difference. The direction of maximum emission of electromagnetic radiation, from an accelerated charged particle, is perpendicular to the direction of acceleration. For rectilinear motion, with \( \theta = 0 \) (Figure 1), equation (13) gives the radiation reaction force \( \mathbf{R}_f \) in the direction of unit vector \( \hat{u} \), as:

\[ \mathbf{R}_f = -\frac{eE}{c} (c-v) \hat{u} + eE \hat{u} = \frac{eE}{c} \mathbf{v} = -\frac{eE}{c} \mathbf{v} \]  

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In decelerated rectilinear motion, with $\theta = \pi$ radians, $\mathbf{R}_f = -(eE/c)\hat{u} = -(eE/c)$, same as (28).

Radiation power is $R_p = -v \cdot \mathbf{R}_f$, the scalar product of $\mathbf{R}_f$ and velocity $v$. The scalar product is obtained, with reference to Figure 1, as:

$$R_p = -v \cdot \mathbf{R}_f = -v \left( \frac{eE}{c} (c - v) + eE \right)$$

$$R_p = eEv \left( \cos \theta - \cos \left( \theta - \alpha \right) + \frac{v}{c} \right)$$

(29)

For rectilinear motion with $\theta = 0$ or $\theta = \pi$ radians, equations (5) for $\theta$ and $\alpha$ and equation (29) give radiation power as:

$$R_p = -v \cdot \mathbf{R}_f = eEv \frac{v^2}{c}$$

(30)

Positive radiation power, as given by equation (30), means that energy is radiated in accelerated and decelerated motions.

In circular revolution, where $v$ is orthogonal to $E$ and $\mathbf{R}_f$, the radiation power $R_p$ (scalar product of $v$ and $\mathbf{R}_f$) is zero, as can be ascertained from equations (5) and (29) with $\theta = \pi/2$ radians and $\cos(\theta - \alpha) = \sin \alpha = v/c$. Equation (29) is significant in the alternative electrodynamics. It makes circular revolution of an electron, round a central force of attraction, as in Rutherford’s nuclear model of the hydrogen atom, stable, outside Bohr’s quantum theory.

Equations (28), (29) and (30) are the radiation formulas of the alternative electrodynamics. These equations are in contrast to those of classical electrodynamics where radiation force is proportional to the rate of change of acceleration (Abraham-Lorentz formula) and the radiation power is proportional to the square of acceleration (Larmor formula). There is no formula for radiation reaction force in relativistic electrodynamics. Special relativity adopted a modified Larmor formula: $R = \gamma^2 R_p$. We now consider change in potential energy and radiation for an accelerated or decelerated electron in three systems of electrodynamics

6. **Potential Energy in Classical Electrodynamics**

In classical electrodynamics, the magnitude of accelerating force on an electron of charge $-e$ and constant mass $m$, moving at time $t$ with speed $v$ and acceleration of magnitude $dv/dt$, in the opposite direction of an electrostatic field of magnitude $E$, is given, in accordance with Coulomb’s law of electrostatic force and Newton’s second law of motion, by equation (6):

$$eE = m \frac{dv}{dt}$$

For rectilinear motion in the direction of a displacement $x$, we obtain the differential equation:

$$eE = m \frac{dv}{dt} = mv \frac{dv}{dx}$$

(31)

The potential energy $P$ lost by the electron or work done on the electron, in being accelerated with constant mass $m = m_0$, through distance $x$ from an origin ($x = 0$), to a speed $v$ from rest, is:

$$P = \int_0^x eE \, dx = m_0 \int_0^x mv \, (dv)$$

(32)

Integrating, equation (32) gives:

$$P = \int_0^x eE \, dx = \frac{1}{2} mv^2$$

This is equal to the kinetic energy $K$ gained by the electron.

$$\frac{P}{mc^2} = \frac{1}{2} \left( \frac{v}{c} \right)^2$$

(33)
Here, with no consideration of radiation, the potential energy lost is equal to the kinetic energy gained by an accelerated electron. A graph of $\frac{v}{c}$ against $P/mc^2$ is shown as $A1$ in Figure 3.

In classical electrodynamics, an electron, moving at the speed of light $c$, can be decelerated to a stop and may be accelerated in the opposite direction to reach a speed greater than $-c$. The potential energy $P$ gained, equal to the kinetic energy $K$ lost, in decelerating an electron from the speed of light $c$ to a speed $v$, within a distance $x$ in a field of magnitude $E$, without radiation, is:

$$P = -\int_0^x E\,dx = m\int_v^c \frac{v}{c}\,dv = \frac{1}{2} m\left(v^2 - c^2\right)$$

where $m_0$ is the rest mass (at $v = 0$) and $c$ the speed of light in a vacuum. The amount of kinetic energy is supposed to be accounted for by the increase in mass. Bertozzi’s experiment was conducted to verify equation (35) and it did so in a remarkable way. A graph of $v/c$ against $P/mc^2$ is shown as $A2$ in Figure 3.

# Potential Energy in Relativistic Electrodynamics

In relativistic electrodynamics, the kinetic energy $K$ gained by an electron or the work done, in being accelerated by an electric field $E$, through a distance $x$, to a speed $v$ from rest, is the potential energy $P$ lost. There is no consideration of energy radiation in this situation. The kinetic energy $K$ of a particle of mass $m$ and rest mass $m_o$, moving with speed $v$, is given by the relativistic equation:

$$K = P = mc^2 - m_o c^2$$

where $m_o$ is the rest mass (at $v = 0$) and $c$ the speed of light in a vacuum. The amount of kinetic energy is supposed to be accounted for by the increase in mass. Bertozzi’s experiment was conducted to verify equation (35) and it did so in a remarkable way. A graph of $v/c$ against $P/mc^2$, for equation (35), is shown as $B1$ in Figure 3.

In relativistic electrodynamics, an electron moving at the speed of light $c$ (with infinite mass), cannot be stopped by any decelerating force. The electron continues to move at the same speed $c$, (line $B2$ in Figure 3) gaining potential energy without losing kinetic energy. This is the point of departure between relativistic and the alternative electrodynamics.

# Potential Energy and Radiation in the Alternative Electrodynamics

In the alternative electrodynamics, the force $\mathbf{F}$ (Figure 1), is given by equations (5) and (14). For an electron accelerated in a straight line, the equations with $\theta = 0$, give:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{u} = -m\frac{dv}{dt}\hat{u}$$

The scalar equation is:

$$eE\left(1 - \frac{v}{c}\right) = m\frac{dv}{dt} = mv\frac{dv}{dx}$$
Potential energy $P$ lost in accelerating an electron through distance $x$, to a speed $v$ from rest, is:

$$P = \int_0^x eE \, dx = \int_0^x mv \frac{dv}{1 - \frac{v}{c}}$$

(38)

Resolving the right-hand integral into partial fractions, we obtain:

$$P = mc \int_0^x \left( \frac{1}{1 - \frac{v}{c}} - 1 \right) \, dv$$

(39)

$$P = -mc^2 \ln \left( 1 - \frac{v}{c} \right) - mcv$$

(40)

$$\frac{P}{mc^2} = -\ln \left( 1 - \frac{v}{c} \right) - \frac{v}{c}$$

(41)

The energy radiated $R$ in acceleration is obtained by subtracting the kinetic energy gained, $K = \frac{1}{2} mv^2$, from potential energy $P$ lost, thus:

$$R = P - K = -mc^2 \ln \left( 1 - \frac{v}{c} \right) - mcv - \frac{1}{2} mv^2$$

$$R = -mc^2 \left\{ \ln \left( 1 - \frac{v}{c} \right) + \frac{v}{c} + \frac{v^2}{2c^2} \right\}$$

(42)

In acceleration, equation (41), for the alternative electrodynamics, should be compared with equation (35) for relativistic electrodynamics and equation (33) for classical electrodynamics.

For a decelerated electron, equations (5) and (14), with $\theta = \pi$ radians, give:

$$F = -eE \left[ 1 + \frac{v}{c} \right] \frac{d\hat{u}}{dt} = m \frac{dv}{dt} \hat{u}$$

(43)

$$eE \left[ 1 + \frac{v}{c} \right] = -m \frac{dv}{dt} = -mv \frac{dv}{dx}$$

(44)

Potential energy $P$ gained in decelerating the electron through distance $x$, from speed $c$ to $v$, is:

$$P = -\int_c^x eE \, dx = \int_c^x -mv \frac{dv}{1 + \frac{v}{c}}$$

(45)

Resolving the integrand into partial fractions and integrating, the potential energy gained is:

$$P = mc \int_c^x \left( \frac{1}{1 + \frac{v}{c}} - 1 \right) \, dv$$

(46)

$$P = mc^2 \ln \frac{1}{2} \left( 1 + \frac{v}{c} \right) + mc^2 \left( 1 - \frac{v}{c} \right)$$

(47)

Graphs of $P/mc^2$ against $v/c$ are shown as $C1$ and $C2$, in Figure 3 for equations (41) and (47).

Energy radiated $R$ is kinetic energy lost minus potential energy gained, thus:

$$R = \frac{1}{2} m(c^2 - v^2) - mc^2 \ln \frac{1}{2} \left( 1 + \frac{v}{c} \right) - mc^2 \left( 1 - \frac{v}{c} \right)$$

$$R = -mc^2 \left\{ \frac{1}{2} + \frac{v^2}{2c^2} + \ln \frac{1}{2} \left( 1 + \frac{v}{c} \right) - \frac{v}{c} \right\}$$

(48)
9. Speed Versus Potential Energy in Bertozzi’s Experiment

In the experiment performed by William Bertozzi [9], the speed \( v \) of high-energy electrons was determined by measuring the time \( T \) required for them to traverse a distance of 8.4 metres after having been accelerated through a potential energy \( P \) inside a linear accelerator. Bertozzi’s experimental data, reproduced in Table 1, clearly demonstrate that electrons accelerated through potential energy of 15 MeV attain, practically, the speed of light.

TABLE 1. RESULTS OF BERTOZZI’S EXPERIMENTS WITH ELECTRONS ACCELERATED IN TIME \( T \) THROUGH 8.4 M AND ENERGY \( P \) IN AN ACCELERATOR

\[(m_e c^2 = 0.5 \text{ MeV}, v = 8.4/T \text{ m/sec})\]

<table>
<thead>
<tr>
<th>( P ) MeV</th>
<th>( P/m_e c^2 )</th>
<th>( T \times 10^{-8} ) sec.</th>
<th>( v \times 10^8 ) m/sec</th>
<th>( v/c ) Experiment</th>
<th>( v/c ) Classical Equation 33</th>
<th>( v/c ) Relativistic Equation 35</th>
<th>( v/c ) Alternative Equation 41</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1*</td>
<td>3.23</td>
<td>2.60</td>
<td>0.87*</td>
<td>1.41</td>
<td>0.866</td>
<td>0.842</td>
</tr>
<tr>
<td>1.0</td>
<td>2*</td>
<td>308</td>
<td>2.73</td>
<td>0.91*</td>
<td>2.00</td>
<td>0.943</td>
<td>0.947</td>
</tr>
<tr>
<td>1.5</td>
<td>3*</td>
<td>2.92</td>
<td>2.88</td>
<td>0.96*</td>
<td>2.45</td>
<td>0.968</td>
<td>0.981</td>
</tr>
<tr>
<td>4.5</td>
<td>9</td>
<td>2.84</td>
<td>2.96</td>
<td>0.99</td>
<td>4.24</td>
<td>0.990</td>
<td>1.000</td>
</tr>
<tr>
<td>15.0</td>
<td>30</td>
<td>2.80</td>
<td>3.00</td>
<td>1.00</td>
<td>7.75</td>
<td>0.999</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Figure 3. \( v/c \) (speed in units of \( c \)) against \( P/mc^2 \) (potential energy in units of \( mc^2 \)) for an electron of mass \( m \) accelerated from zero initial speed or decelerated from the speed of light \( c \), the solid lines (\( A1 \) and \( A2 \)) according to classical electrodynamics (equation 33 and 34), the dashed curve (\( B1 \)) according to relativistic electrodynamics (equation 35) and the dotted curves (\( C1 \) and \( C2 \)) according to the alternative electrodynamics (equations 41 and 47). The solid squares are the result of Bertozzi’s experiment (Table 1).
10. **Speed and Kinetic Energy in 3 Systems of Electrodynamics**

In classical electrodynamics, kinetic energy gained by an electron of mass \( m \) in being accelerated, to a speed \( v \), is \( K = \frac{1}{2} m v^2 \), same as equation (33) for potential energy \( P \) lost.

\[
\frac{K}{mc^2} = \frac{v^2}{2c^2}
\]  

(49)

In classical electrodynamics, kinetic energy lost in deceleration from the speed of light \( c \) to a speed \( v \), is \( K = \frac{1}{2} m(c^2 - v^2) \), same as equation (34) for potential energy \( P \) gained.

\[
\frac{K}{mc^2} = \frac{1}{2} \left( 1 - \frac{v^2}{c^2} \right)
\]

(50)

Equations (49) and (50) are the same for classical and relativistic electrodynamics. These are respectively illustrated as curve \((A1 & C1)\) and curve \((A2 & C2)\) in Figure 4.

![Figure 4](image)

**Figure 4.** \( v/c \) (speed in units of \( c \)) against \( K/mc^2 \) (kinetic energy lost or gained in units of \( mc^2 \)) for an electron accelerated from zero initial speed or decelerated from the speed of light \( c \), under 3 systems of electrodynamics

In relativistic electrodynamics, kinetic energy gained by an electron of rest mass \( m_o \) in being accelerated to a speed \( v \), from rest, is:

\[
K = mc^2 - m_o c^2 = m_o c^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)
\]

\[
\frac{K}{m_o c^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1
\]

(51)

In relativistic electrodynamics, an electron moving at the speed of light \( v = c \), with infinite momentum, cannot be stopped by any force. It is supposed to continue moving at speed \( c \).
Figure 4 shows graphs of \( \frac{v}{c} \) (speed in units of \( c \)) against \( \frac{K}{mc^2} \) (kinetic energy, gained or lost, in units of \( mc^2 \)) for an electron of charge \(-e\) and mass \( m\) accelerated from zero initial speed or decelerated from the speed of light \( c\), by a uniform field of magnitude \( E\), where \( a = \frac{eE}{m_0} \) is a constant. The dotted curves \((A1 & C1)\) and \((A2 & C2)\) are in accordance with the classical electrodynamics and the alternative electrodynamics (equations 49 and 50). The dashed curve \( B1\) is in accordance with relativistic electrodynamics (equation 51). No curve is obtainable in relativistic electrodynamics for an electron decelerated from the speed of light \( c\).

11. Speed Versus Energy Radiation

Figure 5 shows graphs of \( \frac{v}{c} \) (speed in units of \( c \)) against \( \frac{R}{mc^2} \) energy radiated (in units of \( mc^2 \)) for an electron of mass \( m\) and charge \(-e\) accelerated by an electric field from zero initial speed or decelerated from the speed of light \( c\) according to the alternative electrodynamics. The curve \( C1\) is for an accelerated electron (equation 42) and curve \( C2\) for a decelerated electron (equation 48). Energy is always radiated, under acceleration or deceleration. There are no such energy radiation graphs from the points of views of classical electrodynamics and relativistic electrodynamics. Radiation, the most common phenomenon in nature, makes the difference between the three systems of electrodynamics.
12. Concluding Remarks

Quantum electrodynamics is not required in describing the motion of an electron in an electric field. Relativistic electrodynamics and the alternative electrodynamics appear to be in agreement for an accelerated electron in rectilinear motion (equations 12 and 17), as depicted in Fig. 2. Bertozzi’s experimental results appear to be in agreement with relativistic electrodynamics (equation 35) and the alternative electrodynamics (equation 41) for an accelerated electron in rectilinear motion, as depicted in Figure 3. Relativistic and the alternative systems of electrodynamics demonstrate clearly the speed of light \( c \) as a limit; relativistic electrodynamics on the basis of mass of a moving particle increasing to become infinitely large at the speed of light and the alternative electrodynamics on the basis of accelerating force or accelerating field reducing to zero at the speed of light. Actually, it is the accelerating field \( \vec{E} \) experienced, as well as the accelerating force \( -e \vec{E} \), which depends on the speed of a charged particle in the field, as charge \( e \) is a constant.

Relativistic electrodynamics and the alternative electrodynamics give the same expression for radius of circular revolution as \( \gamma r_o \) for an electron round a centre of force of attraction, where Lorentz factor \( \gamma \) is defined in equation (1) and \( r_o \) is the classical radius as expressed in equation (27). In circular revolution of a charged particle, decrease in accelerating force with speed, in accordance with the alternative electrodynamics, has the same effect (increase in radius) as apparent increase of mass with speed in accordance with relativistic electrodynamics. This may explain the apparent agreement between relativistic electrodynamics and revolution of charged particles (electrons and protons) in cyclic accelerators. At the speed of light the accelerating force on a revolving charged particle reduces to zero, in accordance with the alternative electrodynamics, and it moves in a circle of infinite radius, which is a straight line. This is in contrast to relativistic electrodynamics where increase in radius is misinterpreted as being the result of mass increasing with speed.

In the alternative electrodynamics, the force exerted on an electric charge moving in an electric field, depends on the velocity of the charge. This is tantamount to modifying Coulomb’s law of electrostatic force, taking into consideration the relative velocity between two electric charges moving in space. This is not the case in classical and relativistic electrodynamics where the Coulomb force is independent of the relative velocities.

The question now is: “Which one of the electrodynamics is correct?” The answer may be found in the motion of electrons decelerated from the speed of light \( c \). According to classical electrodynamics, an electron of mass \( m \) entering a retarding field at a point \( x = 0 \), with speed of light \( c \), is brought to rest after losing kinetic energy \( 0.500mc^2 \), equal to the potential energy gained, without energy radiation. The electron may then be accelerated backwards to reach the point of entry with speed \(-c\) and may reach a speed greater than \(-c\) without radiation after a long time.

According to relativistic electrodynamics, an electron moving at the speed of light (with infinitely large mass, momentum and energy), cannot be stopped by any finite force. The electron is supposed to move at the speed of light gaining potential energy without losing kinetic energy, contrary to the principle of conservation of energy.

In the alternative electrodynamics an electron moving at the speed of light (with kinetic energy \( 0.500mc^2 \)) on entering a retarding field at a point \( x = 0 \), is easily brought to rest after gaining potential energy equal to \( 0.307mc^2 \) and radiating energy equal to \( 0.193mc^2 \) (Figures 2 and 5). The kinetic energy \( K \) lost minus the potential energy \( P \) gained is equal to the energy \( R \) radiated \( (K - P = R) \). The electron is then accelerated backwards to return to the point of entry \( (x = 0) \) at speed \( v = -0.594c \), losing potential energy equal to \( 0.307mc^2 \), gaining kinetic energy \( \frac{1}{2} mv^2 = 0.176mc^2 \) and radiating energy equal to \( 0.131mc^2 \) (Figure 3). The potential energy \( P \) lost minus the kinetic energy \( K \) gained is equal to the energy \( R \) radiated \( (P - K = R) \). Energy radiated in the round trip, entering with speed \( c \) and returning back with speed \( 0.594c \), is
The electron may then be accelerated backwards to reach an ultimate speed \(-c\) with radiation of energy.

For electrons accelerated by an electric field, classical electrodynamics is a failure but relativistic electrodynamics, the alternative electrodynamics and Bertozzi’s experiment appear to be in agreement (Figures 2 and 3). The picture is completely different for decelerated electrons. It is energy radiation that makes all the difference. In the alternative electrodynamics, an electron moving at the speed of light \(c\) is easily brought to rest on entering a decelerating field at a point. In an experiment with a narrow burst of electrons accelerated as near to the speed of light as possible and made to enter a decelerating field at a point, the electrons being stopped invalidates relativistic electrodynamics. Such electrons being stopped and turned back on their tracks, to return to the point of entry with speed equal to \(-0.594c\), invalidates relativistic electrodynamics and inaugurates the alternative electrodynamics.

12. References