

**Mass and Energy:**  
*A Brief Analysis of the Mass-Energy Relation*

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**Abstract:**

In this investigation, experiments, by J. J Thomson, on cathode rays, are briefly reviewed. And the treatment of the charge-to-mass ratio and the relation between mass and energy generally are analyzed in detail, within the theoretical framework of classical physics and relativistic physics, respectively. And accordingly, it's concluded that precise procedures and rigorous mathematical formulas for transforming velocity values from classical physics to relativistic physics, and vice versa, undermine and ultimately render the available experimental evidence for increasing mass with increasing velocity, in the final analysis, entirely insufficient and inconclusive.

**Keywords:**

Charge-to-mass ratio; kinetic energy; momentum; rest mass; relativistic mass; charged particles; magnetic deflection; Lorentz force law; velocity transformation; magnetic field; electric field; mass-

energy equivalence; cathode rays.

## **Introduction:**

Throughout the last decade of the 19<sup>th</sup> century, J. J. Thomson carried out a series of successful experiments to demonstrate the corpuscular nature of cathode rays, and to determine the mass-to-charge ratio of the electron [*Ref. #1.a & 1.b*].

Undoubtedly, the discovery of the electron, or as J. J. Thomson used to call it the '*electrified particle*', has been a major setback for the idea of aetherial electricity and to the old hypothesis of indivisible atoms for all the elements in the periodic table, at the same time.

Furthermore, and although it ranks third among the major milestones in the timeline of the nuclear age [*Ref. #7*], J. J. Thomson's discovery of the electron marks, in fact, the true starting point of all subsequent theoretical and experimental investigations of the structure of the electron, the electromagnetic mass, and the mass-energy relation in general.

Shortly after the aforementioned discovery by J. J. Thomson, several quantitative predictions have been made, on the basis of a number of physical theories, which were still under construction, with regard to the exact nature of the newly discovered electron and the dynamical aspects of its shape and mass.

The first quantitative prediction was computed in *1899*, by H. Lorentz on the basis of his modified aether theory, in accordance with which, the electromagnetic mass of the newly discovered electron varies with its velocity in the longitudinal direction and in the transversal direction as well.

The second quantitative prediction was computed in *1902*, within the framework of Max Abraham's theory, according to which, the electron is a perfectly rigid sphere whose purely electromagnetic mass has a longitudinal component as well as a transversal component, at all times.

And the third quantitative prediction was put forward, in *1905*, on the basis of Einstein's special relativity, in which variations in mass with speed are generalized and made independent of direction; and at the same time, mass and energy are assumed to be equivalent and convertible into each other, although no physical mechanism has ever been given or specified.

And, of course, within the theoretical framework of Newton-Maxwell physics, the mass of J. J. Thomson's '*electrified particle*' must remain absolutely constant and cannot, under any conceivable circumstances, vary with its velocity at all.

In any case, on *November 30, 1905*, W. Kaufmann declared, in a published report, that the final results of his beta-ray experiments support the prediction of M. Abraham's theory over the predictions of the modified aether theory of H. Lorentz and the theory of special relativity of A. Einstein.

However, because, in Abraham's theory and the Lorentz theory as well as in Einstein's special theory of relativity, the velocity of electrons is not allowed to exceed the canonical speed of light in vacuum  $c$ ,

M. Planck was able to point out that Kaufmann's experimental report implies velocity values greater than  $c$  for at least one of its data points; and therefore, the reported experimental results are inconsistent with the three theories, in question, [Ref. #4.b].

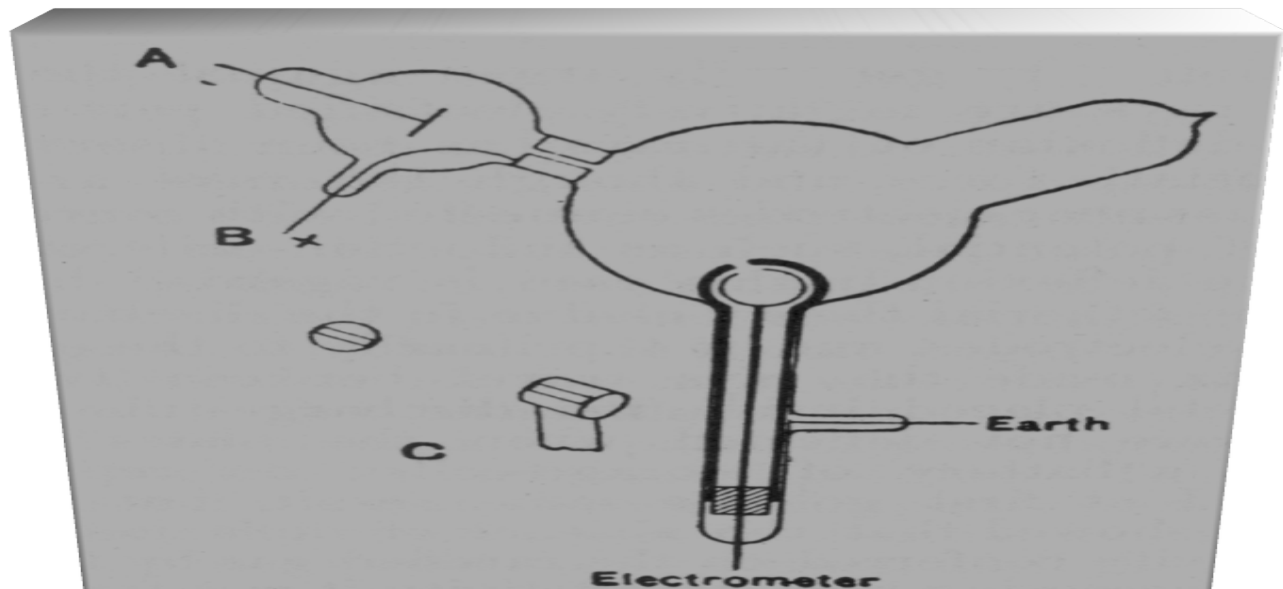
W. Ritz, too, was able, in his review of Kaufman's experimental report, to reach the same conclusion as that of M. Planck, and to demonstrate that most of the reported data points form the classical parabolic curve of the Newtonian equation for computing kinetic energy, along with its unrestricted range of velocity values [Ref. #5].

## **1. Thomson's Cathode-Ray Experiments:**

To measure the mass-to-charge ratio of the electron, J. J. Thomson carried out two important experiments on the '*electrified particle*' of cathode rays [Ref. #1.a]:

### **I. The First Experiment:**

In this experiment, cathode rays, from Cathode *C*, as shown in the Illustration below, pass through a horizontal slit, cut in a metal plug *B*, and strike two coaxial cylinders at the end of the tube. An electrometer is connected to the inner cylinder; and the outer cylinder is connected with the ground.



**Illustration: Thomson's Experimental Apparatus**

Thin strips of iron and copper are fastened to iron and copper wires connected to a galvanometer. The strips of iron and copper are weighed; and the thermal capacity in one set of junctions is  $5 \times 10^{-3}$ , and in another set is  $3 \times 10^{-3}$ . From the deflection of the galvanometer, therefore, the kinetic energy of the cathode rays,  $W$ , can be determined:

$$W = \frac{1}{2} Nmv^2$$

where  $m$  is the electron's mass;  $v$  is the electron's velocity; and  $N$  is the number of electrons.

## II. The Second Experiment:

In this experiment, two aluminum plates are connected to a battery to produce a uniform electric field. And a coil  $C$  is placed between two large coils to generate a uniform magnetic field. Based upon the deflection of cathode rays in the uniform electric field and the deflection of the same rays in the uniform magnetic field, the electric force and the magnetic force can be balanced; and hence, electron's velocity and mass-to-charge ratio can be measured:

$$\frac{m}{e} = \frac{B^2 r}{E}$$

where  $m$  is the electron's mass;  $e$  is the electron's charge;  $E$  is the electric field;  $B$  is the magnetic field; and  $r$  is the radius of curvature due to magnetic deflection.

### **2. The Classical Derivation of Specific-Charge Equation:**

One of the main objectives of the aforementioned experiments is to measure the mass-to-charge ratio of the cathode-ray particles (*e.g. electrons*).

However, since the numerical value of electron's electric charge is much larger than the numerical value of its mass, it's much more convenient to calculate its charge-to-mass ratio instead.

If only the magnetic field  $B$  is applied to the cathode rays, in the experiments under discussion, then the magnetic force  $F_B$ , on these cathode rays, can be computed, in accordance with Newtonian mechanics, through the use of the following equation:

$$F_B = evB$$

where  $e$  is the electric charge; and  $v$  is the velocity of the cathode rays.

Since, according to Newton's second law:

$$F = ma$$

where  $a$  stands for acceleration;

we set the magnetic force equal to the centripetal force:

$$evB = \frac{mv^2}{r}$$

where  $v^2/r$  is the centripetal acceleration due to the magnetic field.

And accordingly:

$$\frac{e}{m} = \frac{v}{Br}$$

where  $r$  is the radius of the curved cathode-ray path.

By adjusting the electric field  $E$ , until the path of cathode rays becomes a straight line and free of deflection, the force  $F_B$  due to the magnetic field is effectively balanced by the force  $F_E$  due to the electric field:

$$F_B = F_E = evB = eE$$

where  $E$  is the electric field.

And subsequently:

$$v = \frac{E}{B}$$

where  $v$  is the velocity of the cathode rays.

By inserting this value of  $v$  into the charge-to-mass equation above, therefore, we obtain:

$$\frac{e}{m} = \frac{E}{B^2 r}$$

where  $e/m$  is the specific charge of the cathode-ray particle (*e.g. the electron*).

### **3. The Relativistic Derivation of Specific-Charge Equation:**

Although no mention of any relativistic derivation, in this regard, is made in the surveyed literature, it seems, at first glance, that there are two possible derivations of the relativistic formula for calculating the charge-to-mass ratio of J. J. Thomson's cathode-ray particle: The momentum-based derivation and the kinetic-energy-based derivation, in accordance with the basic rules of relativistic physics.

#### ***A. The Momentum-Based Derivation:***

According to the special theory of relativity, if any particle (*electrically charged or not*) with rest mass  $m_0$  is moving at a velocity  $v$ , then its relativistic momentum  $p$  can be computed by using the following equation:

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where  $m_0$  is the rest mass of the moving particle, as defined within the framework of relativistic physics; and  $c$  is the speed of light in vacuum.

Let's, now, rearrange this classical force equation:

$$evB = \frac{mv^2}{r}$$

into its linear-momentum form:

$$eB = \frac{mv}{r}$$

where  $mv^2/r$  is the centripetal force due to deflection by the magnetic field  $B$ .

By replacing the classical momentum, in the above equation, with the relativistic momentum, we can readily obtain this corresponding relativistic equation:

$$eB = \frac{m_0 v}{r \left( \sqrt{1 - \frac{v^2}{c^2}} \right)}$$

where  $r$  is the radius of the curved cathode-ray path; and  $m_0$  is the relativistic rest mass.

And consequently, we obtain:

$$\frac{e}{m_0} = \frac{v}{Br \left( \sqrt{1 - \frac{v^2}{c^2}} \right)}$$

where the relativistic rest mass  $m_0$  is the same as the Newtonian mass  $m$ ; i.e.,

$$m_0 = m$$

and where  $r$  is the radius of the curved cathode-ray path.

By adjusting the electric field  $E$ , until the path of cathode rays becomes a straight line and free of deflection, the force  $F_B$  due to the magnetic field  $B$  is balanced by the force  $F_E$  due to the electric field  $E$ ; and the following relation can be obtained:

$$F_B = F_E = evB = eE$$

where  $E$  is the electric field.

And subsequently:

$$v = \frac{E}{B}$$

where  $v$  is the velocity of the cathode rays.

And finally, by inserting this value of  $v$  into the charge-to-mass equation above, therefore, we obtain the following relativistic momentum-based equation for calculating the charge-to-mass ratio of J. J. Thomson's cathode-ray particle:

$$\frac{e}{m_0} = \frac{E}{B^2 r \left( \sqrt{1 - \frac{E^2}{B^2 c^2}} \right)}$$



where  $e/m_0$  is the specific charge of the cathode-ray corpuscle (*the electron*).

### ***B. The Kinetic-Energy-Based Derivation:***

According to Einstein's special theory of relativity, if a particle with rest mass  $m_0$  is moving at a velocity  $v$ , then its relativistic kinetic energy  $E_k$  can be calculated through the use of this equation:

$$E_k = m_0 c^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

where  $m_0$  is the rest mass of the moving particle; and  $c$  is the speed of light in vacuum.

And so, now, we have to rearrange this Newtonian magnetic-force-centripetal-force equation:

$$evB = \frac{mv^2}{r}$$

into the following classical kinetic-energy form:

$$eB = \frac{2 \left( \frac{1}{2} mv^2 \right)}{vr}$$

where the term  $\frac{1}{2}mv^2$  stands, in the above equation, for the Newtonian kinetic energy of J. J. Thomson's moving cathode-ray particle.

And hence, by replacing the classical kinetic energy, in the above equation, with the relativistic kinetic energy, we can obtain this corresponding relativistic equation:

$$eB = \frac{2m_0c^2}{vr} \left( \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)$$

where  $r$  is the radius of curvature.

And accordingly, we obtain:

$$\frac{e}{m_0} = \frac{2c^2}{Bvr} \left( \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)$$

where  $m_0$  is the relativistic rest mass; and  $r$  is the radius of the curved cathode-ray path.

By adjusting the electric field  $E$ , until the path of cathode rays becomes a straight line and free of deflection, the force  $F_B$  due to the magnetic field  $B$  is balanced by the force  $F_E$  due to the electric field  $E$ ; and hence the following relation can be obtained:

$$F_B = F_E = evB = eE$$

where  $e$  is the electric charge.

And subsequently:

$$v = \frac{E}{B}$$

where  $v$  is the velocity of the cathode-ray corpuscle.

And therefore, by inserting the latter value of  $v$  into the charge-to-mass equation above, we obtain the following relativistic kinetic-energy-based equation for calculating the charge-to-mass ratio of J. J.

Thomson's cathode-ray corpuscle:

$$\frac{e}{m_0} = \frac{2c^2}{Er} \left( \frac{1}{\sqrt{1 - \frac{E^2}{B^2c^2}}} - 1 \right)$$

where  $e/m_0$  is the specific charge of the cathode-ray particle.

### ***C. Analytical Evaluation of the Two Relativistic Specific-Charge Formulas:***

The primary method of evaluation, in this regard, is to divide the charge-to-mass ratio of J. J. Thomson's cathode-ray corpuscle  $R_k$ , as given by the relativistic kinetic-energy-based equation, by the charge-to-mass ratio of the same cathode-ray corpuscle  $R_m$ , as given by the relativistic momentum-based equation.

And therefore, if:

$$\frac{R_k}{R_m} = 1$$

then the two relativist formulas are correct.

But if:

$$\frac{R_k}{R_m} \neq 1$$

then, at least, one of the two derivations, above, should be deemed incorrect.

Let  $R_m$  denote the charge-to-mass ratio of J. J. Thomson's cathode-ray particle  $e/m_0$ , as computed by using the following relativistic momentum-based equation:

$$\frac{e}{m_0} = \frac{E}{B^2 r \left( \sqrt{1 - \frac{E^2}{B^2 c^2}} \right)}$$

And let  $R_k$  denote the charge-to-mass ratio of the same cathode-ray particle,  $e/m_0$ , as calculated by using this relativistic kinetic-energy-based equation:

$$\frac{e}{m_0} = \frac{2c^2}{Er} \left( \frac{1}{\sqrt{1 - \frac{E^2}{B^2 c^2}}} - 1 \right)$$

where  $c$  is the speed of light in vacuum.

By dividing  $R_k$  by  $R_m$ , therefore, we obtain the following result:

$$\frac{R_k}{R_m} = \frac{2B^2 c^2}{E^2} \left( 1 - \sqrt{1 - \frac{E^2}{B^2 c^2}} \right)$$

where  $E$  is the electric field;  $B$  is the magnetic field; and  $c$  is the speed of light in vacuum.

And hence:

$$\frac{R_k}{R_m} \neq 1$$

It follows, therefore, that the relativistic momentum-based equation and the relativistic kinetic-energy equation for calculating the charge-to-mass ratio of J. J. Thomson's cathode-ray corpuscle, as derived above, are incompatible; and accordingly, at least, one of those two derivations is incorrect.

The big question, now, is this:

What went wrong?

Upon closer examination, it should become clear that the above attempt at deriving the relativistic equations, for computing the charge-to-mass ratio of J. J. Thomson's electrified particle, is bound to go wrong for two major reasons:

- In Einstein's theory of special relativity, there is no mathematically equivalent relation to this Newton-second-law equation:

$$F = ma$$

And consequently, no direct connection between the classical derivation of specific-charge equations and any possible derivation of relativistic equivalent equations, can be made.

- Even though, the velocity of J. J. Thomson's cathode rays is denoted by the same algebraic symbol  $v$ , within the framework of Newtonian physics as well as within the framework of Einsteinian physics, its numerical values are not the same. In particular, the velocity values, in the former, can approach infinity; while, in the latter, the velocity of cathode rays, in Thomson's experiments, cannot exceed, under any circumstances, the canonical value of the speed of light in vacuum  $c$ . That is a considerable difference with far-reaching implications, for the treatment of J. J. Thomson's cathode rays, on the basis of these two different kinds of physics.

Nonetheless, the relativistic momentum-based equation above is very close, in most respects, to the conventional relativistic treatment of  $e/m$ , for charged particles in general, according to which the charge-to-mass ratio is, first of all, computed classically; and then and only then only the classical mass  $m$  is replaced with the relativistic mass  $m_{rl}$ , as given by this relativistic formula:

$$m_{rl} = \frac{m_0}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $v_{rl}$  is the restricted velocity of the cathode-ray particle, in accordance with relativistic physics.

#### 4. The Far-reaching Implications of $c$ Versus Infinity:

As already mentioned above, although the concept of velocity in Maxwell-Newton physics and Einstein-Lorentz physics is theoretically the same, its numerical values, in practice, are vastly different.

Based on classical physics of Newton and Maxwell, the velocity of J. J. Thomson's cathode rays can have any numerical value within the following range:

$$-\infty < v_{cl} < +\infty$$

where  $v_{cl}$  is the velocity of cathode rays according to classical physics.

On the basis of modified physics of Lorentz and Einstein, by contrast, the velocity of J. J. Thomson's cathode rays can have any numerical value only within this highly restricted range:

$$-c < v_{rl} < +c$$

where  $v_{rl}$  is the velocity of cathode rays according to relativistic physics.

Why is the range of numerical values, for cathode rays' velocity, in J. J. Thomson's experiments, is always limited and extremely restricted, within the framework of Einstein-Lorentz physics?

Certainly, the theory of Lorentz does not have any explicit assumption or axiom named the '*postulate of constancy*' as that of Einstein's theory. And so, how can its velocity range become very limited and restricted just like that of Einstein's special theory of relativity?

Well; in physical theories, such as the theory of Lorentz, the theory of Larmor, and the theory of Abraham, the velocity range of the electron is limited, not because of their implementation of any postulate of constancy, but because of their extensive utilization of this so-called '*gamma factor*':

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $v_{rl}$  is the velocity of cathode rays in accordance with relativistic physics.

And so, whether those theories choose explicitly to place a limit on the velocity of J. J. Thomson's cathode rays or not, the numerical results of their gamma factor necessarily become imaginary for all velocity values greater than  $+c$  or less than  $-c$ .

Now, since the velocity range of cathode rays, on the basis of classical physics, is completely different from their velocity range, on the basis of relativistic physics; is there any mathematically precise method for transforming velocity values from classical physics to relativistic physics; and vice versa?

It's definitely possible to transform any velocity values from Newton's physics to Einstein's physics; and the reverse is, also, true, in these two main cases:

### **1. The Case of Experimentally Measured Linear Momentum:**

Let's assume that the momentum of J. J. Thomson's cathode-ray particle  $p_{xp}$  is measured experimentally by some practical method independent of any physical theory.

Since the measured quantity of  $p_{xp}$  is going to be, at the same time in this special case, the left-hand term of this classical momentum equation:

$$p_{xp} = mv_{cl}$$

and the left-hand term of this relativistic momentum equation:

$$p_{xp} = \frac{m_0 v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

the right-hand terms of the two equations, in the case under discussion, must be equal; i.e.,

$$mv_{cl} = \frac{m_0 v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $v_{cl}$  is the velocity of cathode rays, in accordance with classical physics; and  $v_{rl}$  is the velocity of the same cathode rays, in accordance with relativistic physics.

And since the relativistic rest mass  $m_0$  is always equal to the classical mass  $m$ :

$$m_0 = m$$

the two quantities cancel each other out.

It follows, therefore, that the relativistic velocity  $v_{rl}$  can be transformed to the classical velocity  $v_{cl}$  by using the following equation:

$$v_{cl} = \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $c$  is the speed of light in vacuum.

And likewise, the classical velocity  $v_{cl}$  can be readily transformed to the relativistic velocity  $v_{rl}$  by using this equation:

$$v_{rl} = \frac{v_{cl}}{\sqrt{1 + \frac{v_{cl}^2}{c^2}}}$$



where  $v_{rl}$  and  $v_{cl}$  are the velocity of J. J. Thomson's cathode rays, according to relativistic physics and classical physics, respectively.

## **2. The Case of Experimentally Measured Kinetic Energy:**

Now, let's assume, once again, that the kinetic energy of J. J. Thomson's cathode rays  $E_{xp}$  is measured experimentally by some practical means independent of any physical theory.

Because, in this case, the measured quantity of  $E_{xp}$  is, simultaneously, the left-hand term of the classical kinetic-energy equation:

$$E_{xp} = \frac{1}{2}mv_{cl}^2$$

and the left-hand term of the relativistic kinetic-energy equation:

$$E_{xp} = m_0c^2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)$$

the right-hand terms of the two equations above, in this particular case, must be equal; i.e.,

$$\frac{1}{2}mv_{cl}^2 = m_0c^2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)$$

And because the relativistic rest mass  $m_0$  is always equal to the classical mass  $m$ :

$$m_0 = m$$

these two quantities cancel each other out.

And it follows, therefore, that the relativistic velocity  $v_{rl}$  can be transformed to the classical velocity  $v_{cl}$  by using the following equation:

$$v_{cl} = c \sqrt{2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)}$$

where  $c$  is the speed of light in vacuum.

And correspondingly, the classical velocity  $v_{cl}$  can be transformed to the relativistic velocity  $v_{rl}$  by using this equation:

$$v_{rl} = c \sqrt{1 - \left( \frac{1}{1 + \frac{v_{cl}^2}{2c^2}} \right)^2} = v_{cl} \left( \frac{\sqrt{1 + \frac{v_{cl}^2}{4c^2}}}{1 + \frac{v_{cl}^2}{2c^2}} \right)$$

where  $v_{rl}$  and  $v_{cl}$  are the velocity of J. J. Thomson's cathode rays, according to the basic rules of relativistic physics and classical physics, respectively.

The following points should be made clear with regard to the velocity transformations above:

- Given the linear momentum, the computed velocity, on the basis of relativistic physics, is always less, in its numerical magnitude, than the computed velocity, in accordance with classical physics.

- Given the kinetic energy, the calculated velocity, in accordance with relativistic physics, is always less, numerically, than the calculated velocity, within the theoretical framework of classical physics.
- If only the linear momentum is measured, then the computed kinetic energy, on the basis of classical physics, is always greater than the computed kinetic energy, on the basis of relativistic physics.
- If only the kinetic energy is measured, then the calculated linear momentum, on the basis of Einsteinian physics, is always greater than the calculated linear momentum, on the basis of Newtonian physics.
- For any given velocity, the computed linear momentum, according to relativistic physics, is always greater, in its magnitude, than the calculated linear momentum, on the basis of classical physics.
- For any given velocity, the calculated kinetic energy, within the theoretical framework of relativistic physics, is always greater, numerically, than the computed kinetic energy, according to the rules of classical physics.
- In the absence of any experimental measurements, it's absolutely impossible to transform velocity values from relativistic physics to classical physics; and vice versa.
- In the presence of experimental measurements, it's mathematically impossible to make relativistic physics and classical physics share the same measured linear momentum and the same measured kinetic energy, at the same time.
- In J. J. Thomson's experiments, under investigation, the velocity of cathode rays  $v_{cl}$  as given by the following equation of classical physics:

$$v_{cl} = \frac{E}{B}$$

even though, in principles, it's obtained by the experimental method of balancing the electric field  $E$  and the magnetic field  $B$ , that velocity cannot be used correctly without transformation in any computations, within the theoretical framework of relativistic physics; because it's necessarily based upon this classical equation:

$$ev_{cl}B = eE$$

where  $e$  is the electric charge;  $E$  the applied electric field; and  $B$  is the applied magnetic field.

- In J. J. Thomson's experiments, in question, when the velocity of cathode rays  $v_{cl}$ , as given by the following equation of classical physics:

$$v_{cl} = \frac{E}{B}$$

is correctly transformed to relativistic physics, the path radius of cathode rays  $r_{rl}$  due to the deflection by the magnetic field  $B$ , as calculated by using the following classical equation:

$$r_{cl} = \frac{mv_{cl}}{eB}$$

and the path radius of cathode rays  $r_{rl}$  due to the deflection by the same magnetic field  $B$ , as calculated by using the following relativistic equation:

$$r_{rl} = \frac{m_0 c_{rl}}{eB \sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

become equal; and hence the theoretical difference in length between the two radii, as computed in accordance with classical physics and relativistic physics, respectively, evaporates and disappears. And as a result, the experimental evidence for any mass increase with increasing velocity, as reported by W. Kaufmann and others, instantly evaporates and vanishes as well.

## 5. Magnetic Deflection and Mass-Energy Relation:

As pointed out pointed earlier, in this discussion, within the theoretical frame of Newtonian physics, the velocity of cathode rays  $v_{cl}$ , in J. J. Thomson's experiments, can, in principle, have any numerical value within the following range:

$$-\infty < v_{cl} < +\infty$$

where  $v_{cl}$  is the velocity of cathode rays according to classical physics.

While, by contrast, in accordance with the rules of relativistic physics, the velocity values of those same cathode rays can never exceed the speed of light in vacuum; i.e.,

$$-c < v_{rl} < +c$$

where  $v_{rl}$  is the velocity of cathode rays according to relativistic physics.

It should be immediately clear, therefore, that because of its immense numerical range, the independent variable  $v_{cl}$ , alone and by itself, is sufficient to account for any amount of increase in the path radius  $r$ , due to the effect of the magnetic field  $B$ , in direct proportion to cathode rays' velocity, as given by this classical equation:

$$r = \frac{mv_{cl}}{eB}$$

where  $e$  is the electric charge; and  $m$  stands for mass, as defined in classical physics.

And it should be, also, obvious, at once, that, in relativistic physics, this so-called '*gamma factor*':

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

must be always included, because the independent variable  $v_{rl}$ , due to its very narrow and restricted range, cannot, alone and by itself, sufficiently account for the total amount of increase in the same path radius  $r$ , due to the deflection by the same magnetic field  $B$ , as calculated through the use of the following relativistic formula:

$$r_{rl} = \frac{m_0 c_{rl}}{eB \sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $m_0$  is the rest mass, as defined in relativistic physics.

Although the factor gamma, traditionally, is associated and always paired, in the above equation, with the relativistic rest mass  $m_0$ :

$$m_0 \gamma = \frac{m_0}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

it's quite easy and legitimate as well, from purely algebraic standpoint, to pair it with the relativistic velocity variable  $v_{rl}$ ; in the following way:

$$v_{rl} \gamma = \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

However, the latter algebraic pair, unlike the former one, breaks, for certain, the relativistic speed limit,

and formally imports this velocity range of Newtonian physics:

$$-\infty < v_{cl} < +\infty$$

where  $v_{cl}$  is the velocity of cathode rays, as dictated by the rules of classical physics.

As a matter of fact, the latter relation, as demonstrated earlier, is just the momentum-based equation for transforming a given Einsteinian velocity to its Newtonian counterpart; i.e.,

$$v_{cl} = \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where  $v_{cl}$  is the classical velocity; and  $c$  the velocity of light in vacuum.

We conclude, therefore, that the notion of mass increase, based upon reported radius-of-path measurements in the case of cathode rays, beta rays, ... etc., is nothing more than a theoretical peculiarity, within the framework of relativistic physics; and with no physical basis whatsoever in the experiments under discussion or within the theoretical framework of Newtonian physics.

And so, now, the most relevant question, in that regard, is this:

If the radius-of-path measurements, in all reported cases, constitute no experimental evidence for the theoretical mass increase, as predicted by relativistic physics, then how can the other reported observations related to the relativistic mass-energy equivalence and, in particular, the mathematical terms of this relativistic energy equation itself:

$$E_{rl} = \sqrt{p_{rl}^2 c^2 + m_0^2 c^4} = m_0 c \sqrt{\left( \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} \right)^2 + c^2} = \frac{m_0 c^2}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

where the relativistic momentum  $p_{rl}$  is defined in accordance with this equation:

$$p_{rl} = \frac{m_0 v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

be interpreted, incorporated, or justifiably dismissed, in accordance with the general principles of Newtonian physics and its formal rules?

Clearly, the above relativistic energy equation is composed of two parts: The kinetic-energy part  $E_k$  and the rest-energy part  $E_0$ :

$$E_{rl} = E_k + E_0$$

where  $E_k$  is computed by using this equation:

$$E_k = m_0 c^2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)$$

And in which  $E_0$  is defined by this equation:

$$E_0 = m_0 c^2$$

where  $m_0$  is the rest mass; and  $c$  is the speed of light in vacuum.

The kinetic-energy part is easily accounted for, on the basis of Newtonian physics, by inserting into this



classical kinetic-energy equation:

$$E_k = \frac{1}{2}mv_{cl}^2$$

the classical velocity  $v_{cl}$  as obtained by transforming the relativistic velocity  $v_{rl}$  through the use of the following kinetic-energy-based equation:

$$v_{cl} = c \sqrt{2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)}$$

where  $c$  is the speed of light in vacuum.

As for the rest-energy part, its traditional interpretation, on the basis of relativistic physics, in terms of mass-energy equivalence and actual conversion of mass into energy and vice versa, is, at first glance, totally unacceptable, within the theoretical framework of classical physics, since, by definition, the conservation of mass is one of its basic physical principles.

However, if light turns out to have mass, as assumed, for example, in Newton's corpuscular theory of light, then this relativistic rest-energy equation:

$$E_0 = m_0c^2$$

can be re-interpreted, quite easily, in terms of kinetic energy of light.

For instance, if the total mass of two fused protons is less than the total mass of two free protons by an amount  $\Delta m$ ; and if this  $\Delta m$  is emitted in the form light, then the emitted light must have total mass of  $\Delta m$  and kinetic energy of  $E_k$ :

$$E_k = \frac{1}{2} \Delta mc^2$$

where  $c$  is the muzzle speed of emitted light.

In addition, the emitted light must make the two fused protons recoil with an equal amount of linear momentum, in the opposite direction, and gain, as a result of that recoil, an amount of kinetic energy equals exactly to  $E_k$ ; i.e.,

$$E_k = \frac{1}{2} \Delta mc^2$$

And accordingly, the total kinetic energy in both direction  $E_{tot}$  is:

$$E_{tot} = 2E_k = \Delta mc^2$$

where  $c$  is the speed of light.

Obviously, the above classical interpretation makes better sense, from physical standpoint, than its relativistic counterpart, with regard to fusion reactions, in which the total sum of kinetic energy, in all directions, is important.

## **6. Concluding Remarks:**

The main conclusions of the current investigation, can be summarized in the following points:

- Deflections of cathode rays by magnetic fields, as observed in the aforementioned J. J. Thomson's experiments, Kaufmann's experiments, and other similar experiments, can be interpreted equally well either through the unrestricted velocity range, in Newtonian physics; or through the assumption of increasing mass with increasing velocity, within the theoretical framework of Einsteinian physics.
- The equation for converting the velocity values of  $v_{rl}$ , in relativistic physics, to the velocity values of  $v_{cl}$ , in classical physics:

$$v_{cl} = \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

and the equation for converting the velocity values of  $v_{cl}$ , in classical physics, to the velocity values of  $v_{rl}$ , in relativistic physics:

$$v_{rl} = \frac{v_{cl}}{\sqrt{1 + \frac{v_{cl}^2}{c^2}}}$$

render any experimental evidence, based upon linear-momentum measurements, for increasing mass with increasing velocity, inconclusive.

- Similarly, the equation for converting the velocity values of  $v_{rl}$ , in relativistic physics, to the velocity values of  $v_{cl}$ , in classical physics:

$$v_{cl} = c \sqrt{2 \left( \frac{1}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}} - 1 \right)}$$

and the equation for converting the velocity values of  $v_{cl}$ , in classical physics, to the velocity values of  $v_{rl}$ , in relativistic physics:

$$v_{rl} = c \sqrt{1 - \left( \frac{1}{1 + \frac{v_{cl}^2}{2c^2}} \right)^2} = v_{cl} \left( \frac{\sqrt{1 + \frac{v_{cl}^2}{4c^2}}}{1 + \frac{v_{cl}^2}{2c^2}} \right)$$

render any experimental evidence, based upon kinetic-energy measurements, for increasing mass with increasing velocity, inconclusive.

- In J. J. Thomson's experiments, the equation for converting the velocity values of  $v_{rl}$ , in relativistic physics, to the velocity values of  $v_{cl}$ , in classical physics:

$$v_{cl} = \frac{v_{rl}}{\sqrt{1 - \frac{v_{rl}^2}{c^2}}}$$

and the equation for converting the velocity values of  $v_{cl}$ , in classical physics, to the velocity values of  $v_{rl}$ , in relativistic physics:

$$v_{rl} = \frac{v_{cl}}{\sqrt{1 + \frac{v_{cl}^2}{c^2}}}$$

render any experimental evidence, based upon radius-of-path measurements, for increasing mass with increasing velocity, inconclusive.

Now, is there any reliable method for finding out experimentally whether the charge-to-mass ratio of J. J. Thomson's cathode-ray particle varies with velocity because of relativistic mass variations, as predicted by Einsteinian physics; or it remains always constant, as predicted by Newtonian physics?

In principle, there are, at least, two main methods for testing experimentally the two diametrically opposed theoretical predictions above:

### **A. The Time-of-Flight Measurements:**

In accordance with this experimental method, the time of flight of J. J. Thomson's cathode rays over a specific distance is measured directly; and hence the velocity of those cathode rays can be determined practically and independently of any physical theory. And since, for any experimentally measured velocity value, the equations of relativistic physics, for calculating linear momentum and kinetic energy, always give larger values than the values given by the equations of classical physics, for

computing linear momentum and kinetic energy; it's possible, in principle, to test decisively, in the laboratory, these two predictions of relativistic physics and classical physics, respectively.

In practice, however, it's extremely difficult to carry out time-of-flight measurements over a specific path at the required level of precision for determining theory-free velocity values and testing the two predictions, in question.

For example, if one tries to measure the precise start of time of flight, then one cannot measure the exact end of the same time of flight; because these two crucial time measurements cannot be done on the same cathode-ray particles twice. That is on one hand.

On the other hand, if one makes an educated guess about the time of emission, for J. J. Thomson's cathode rays, and then uses it as the star of time of flight, then the level of precision, in time-of-flight measurements, can no longer be labeled as precise and sufficient for deciding between the aforementioned predictions experimentally.

Furthermore, it's virtually impossible to find, in any current physics laboratory, reliable time measuring instruments completely free of any built-in theoretical considerations.

But it's reasonable to assume that if experimental physicists work a little harder and direct their best efforts towards ridding their experimental methods and their experimental instruments of any implicit theoretical assumptions and built-in theoretical biases, they can measure theory-free velocity values and test decisively by experimental means these two diametrically opposed predictions of classical physics and relativistic physics and many other diametrically opposed predictions as well.

## **B. The Momentum and Kinetic-Energy Measurements:**

As demonstrated earlier, in the current discussion, the equations of classical physics and the equations of relativistic physics can give the same measured momentum or the same measured kinetic energy, for the same cathode rays in question; but not both at the same time.

And therefore, it's possible, in principle, to test decisively the predictions of classical physics and the predictions of relativistic physics, in this regard, by measuring the momentum and the kinetic energy of the same cathode rays under investigation, through the use of experimental means independent of any theoretical assumptions explicitly expressed or otherwise.

But once again, however, it's almost impossible to find, in modern physics laboratories, any theory-free instruments or any well-tested experimental procedures capable of measuring momentum and kinetic energy of cathode rays practically and independently of any physical theory.

Nevertheless, it is not unrealistic to expect that if experimental physicists just try a little harder to purge their working procedures and instruments from any theoretical influence, it may well be within their grasp to decide experimentally and once and for all between the aforementioned theoretical predictions of relativistic physics and classical physics.

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