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INERTIA AND INERTNESS

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There are quite a lot of works dedicated to revealing the physical essence of a body's inertia [1-4]. They present the historical development of the concept, with various manifestations and practical use of inertia in operating mechanisms. The notions of inertia and inertness are introduced and some authors use them interchangeably. By inertia, it is most often meant the attempt of a body to preserve its state unchanged in relation to an inertial (motionless) reference system. The most advanced formulation of inertial properties is given in [5]: “Every body offers resistance to attempts to put it in motion or to change the modulus or direction of its motion”. Otherwise, if no external forces act on a body from other bodies or the environment, or these forces balance each other, the body preserves the state of rest or steady rectilinear motion. This expression is, in essence, Newton’s first law.

As is well known, Newton’s first law reads: every body is in a state of rest or uniform rectilinear motion until applied forces produce changes in this state [6]. Newton supposed that in the absolute vacuum of space, bodies – planets, stars, comets and other parts and particles of bodies can move freely. To a certain extent, such a situation assumes some “freedom of will”: after a primary impulse a body starts moving steadily in a straight line (as if without assistance) in space.

Experimental observations show that Newton’s first law can be obeyed only in some special abstract conditions, deprived of gravitational and other fields. It is well known that any space in the cosmos is filled with the gravitational field. Under gravitation the paths of cosmic bodies – comets, asteroids, planets, stars etc. are distorted and most often acquire elliptic orbits. In such an orbit the Moon travels about the Earth, the Earth and the Moon move round the Sun. The Sun together with the planets of the solar system travels around the galaxy centre with the velocity of about 250 *km/s* [7]. The motion of celestial as well as terrestrial bodies cannot be straight. Therefore the notions of “uniform linear motion” can only be based on certain assumptions, such as disregarding the gravitational fields that penetrate the entire universe. On examining the motion of some terrestrial bodies in practice, one can concede that a body moves straight and uniformly on a certain short path where the curvature of its trajectory is moderately visible due to the gravitation effect.

On the other hand, it is intuitively obvious that if the gravitational force disappeared, all celestial bodies - comets, asteroids, planets, stars etc. would move uniformly along straight lines in outer space. In such a hypothetical space, Newton’s first law would be fully obeyed. Free motion of the body along a straight path would change only through collision with another body. The cluster of substance that represents a physical body has such an imperative property that if it is not subject to external forces it will preserve uniform linear motion. This ability is due to the body’s mass. A moving physical body possesses kinetic energy besides its mass [4].

Unlike the power that a moving body possesses, motionless bodies can possess potential energy. But these bodies must be in some gradient force field, for instance, in a gravitational, magnetic or some other field. It was mentioned above, that outer space is penetrated by the vastest and most long-ranging fields of gravitational forces. So all bodies that are limited in free motion possess potential energy. A body can be considered motionless if its potential energy is not realized through motion. If a body has no relationship with other bodies that restrict its movement, its potential energy induces the body to move, which turns into kinetic energy.

In classical mechanics developed by the works of Galileo, Newton, D'Alembert, Lagrange, Euler et al., the inertia of physical bodies is considered a motion of a material point in relation to the coordinate system, which is motionless (absolute) or mobile, for instance, related to planet Earth. Within the approach using a motionless coordinate system, D'Alembert (in his studies of the movement of a constrained point) introduced vector $\boldsymbol{D}$ in mechanics equal to the product of the mass m of a point body by its acceleration $\boldsymbol{a}$ taken with the opposite sign [2],

$$\boldsymbol{D} = - m\boldsymbol{a}, \tag{1}$$

This vector was later called d'Alembert's force. According to Newton's third law, this force must balance the force accelerating the body. On this approach, the interaction of the force and body mass described by Newton's second law can be reduced to a static task. A more general principle that allows solving complex equations of dynamics is called D'Alembert-Lagrange principle: "In the motion of a system with ideal relations (i.e. such relations in which reactions cannot produce work – without friction, losses of energy and such) at every instant of time, the total elementary work of all active forces applied (i.e. not reactions) and all inertia forces with any possible movement of the system, will be equal to zero" [4]. This principle allows setting up an equation of motion for any mechanical system and finding the forces acting inside the system.

For the mobile coordinate system relative to some "absolute", the great Euler offered additional inertia forces – transportable $\boldsymbol{P}$ and coriolis $\boldsymbol{Q}$ [8]. The complete dynamical equation of a point body with regard for these inertia forces takes the form:

$$m\boldsymbol{a} = \boldsymbol{F} + \boldsymbol{P} + \boldsymbol{Q}, \tag{2}$$

where $\boldsymbol{F}$ is an acting physical force.

An interpretation of the notion of "force" at present is not unequivocal. As is considered in mechanics [3], Eq. (2) has the structure of the initial equation of absolute motion in which the terms $\boldsymbol{P}$ and $\boldsymbol{Q}$ have a force dimension. But in the author's opinion [3], these terms do not produce any mechanical effect; therefore they are not physical forces. These forces are considered to be inertia forces. A force is the vector measure of the mechanical manifestation of all bodies' interactions. Periodically some controversy arises among specialists in classic mechanics as to whether the forces $\boldsymbol{P}$ and $\boldsymbol{Q}$ are real or not. The same doubt appears about d'Alembert's force $\boldsymbol{D}$. Most likely these forces are the body's responses to the effort to change its state in accordance with Newton's first law. These forces are considered to be inertia forces. It has been suggested that the forces that are exhibited during a mechanical contact between bodies, should be called physical forces.

In our opinion, some general interpretation of the "inertia" concept should be formulated. Such an interpretation or determination can be found when analyzing the situations in which inertia is immediately manifested when bodies interact.

The simplest way is to consider the inertia and energy of two impinging bodies. The body's inertia is known to be proportional to its mass; it greatly increases by increasing the body's velocity. Let us consider a perfectly inelastic impact using the collision of balls as an example [5]. Assume that the balls move along the straight line connecting their centres with velocities V_1 and V_2. In this case the impact is considered to be central. Let us denote the general velocity of the balls after the collision by V. The kinetic energies of the system before the impact K_1 and after the impact K_2 will be respectively:

$$K_1 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 , \quad K_2 = \frac{1}{2} (m_1 + m_2) V^2 , \tag{3}$$

where m_1 and m_2 are the masses of the balls.

The difference $K_1 - K_2 = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} (V_1 - V_2)^2$ is the part of energy that was absorbed at collision. At nonelastic collision of two identical bodies approaching each other with the same speed, their mutual speed after the collision is equal to zero and the entire energy is spent for breaking-down the bodies (for instance, two cars). With a perfectly elastic impact, the internal energy of bodies do not change [5], i.e. the total energy of every body before the impact is equal to the total energy of these bodies after the impact.

It is very simple to calculate the quantitative characteristic of a body's inertia estimating its change at the collision of a small body with a body of a much greater mass M. Let us estimate the energy that will be released with this collision. From (3) it can be derived that if mass m of one body is much less than the mass of the other body ($V = V_2 = 0$), the kinetic energy released at impact will be equal to (if the second-order effects are disregarded):

$$E_k = \frac{1}{2} m V^2 , \; kg{\cdot}m^2/s^2. \tag{4}$$

This formula was first obtained by Oliver Heaviside and independently, by William Thomson [9]. It follows that at collision, the entire energy released is at the cost of the small body. The body with mass m moving at a speed of V after a collision with the body with mass M will have the speed $V = 0$. After the collision the small body loses all inertia in relation to body M. According to Eq. (4), its motion energy will be equal to zero. This example shows that the body's inertia can be set equal to the amount of kinetic energy accumulated in its mass and speed.

The above allows the conclusion that the inertia or kinetic energy of a body can be estimated only in relation to another body relative to which it moves. If no abstract coordinate system is introduced then inertia as well as kinetic energy is inherent in the body that can be considered to be moving relative to another body. Within the assumption of a possibility for uniform linear motion one should realize the relativity of notions of “motionless” and “moving” bodies. For instance, a body does not possess inertia in relation to another body relative to which it does not move. When a body is at rest in relation to another body it does not possess inertia in relation to that other body. One can envision two bodies moving in one direction with the same speed. They will not possess inertia in relation to each other. But these two bodies will possess inertia in relation to a third body moving relative to them with some speed different from zero.

Therefore the inertia (potential) measure of every specific body moving independently has a great number of values depending on its relation to another body by which it is estimated. The inertia of one and the same body or its kinetic energy will be different in relation to bodies moving with various speeds. It is exhibited only in collision with another body. Some external object is necessary to estimate the inertial measure or for the inertia of a physical body to be mechanically manifested. So this type of inertia should be called external inertia.

Unlike a free inertial body's motion which can be (conventionally) estimated as uniform and straight, a body rotating around some axis possesses a special type of inertia. This type can be considered to be the internal inertia of a body. Internal inertia and its estimation do not depend on external bodies. It has one certain meaning. Internal inertia and the body rotational energy are necessarily accompanied by stresses and strains inside the body at the cost of the appearance of centrifugal forces. Internal inertia is accompanied by the gyroscopic effect. The centripetal force equal to the centrifugal one ensures stability in the body's rotation. Since the centripetal force is constantly applied to a moving body, then, according to Newton's second law, the body is considered to be in accelerated motion. At the same time, the body moves uniformly, i.e. the linear velocity of its every individual point remains constant. Since this body moves uniformly, then, according to Newton's first law, one can assume that it moves without acceleration. Most likely, the last mentioned statement has more validity since the body's quantitative characteristics do not change when it is in uniform circular motion. The motion energy and the body's inertia remain constant.

There is a strict correspondence between the body's mass, the angular velocity of its rotation, internal stresses and strains. The potential inertia of a rotating body can be numerically estimated by the moment of inertia. The moment of inertia of a material point relative to some axis is equal to the product of its mass by the squared distance from the point to this axis [6]:

$$J = mR^2 \tag{5}$$

The moment of the body's inertia is the sum of the moments of inertia of the material points constituting this body. The kinetic energy of a rotating body is equal to [6]:

$$E_r = \frac{1}{2} J\omega^2 \ , \tag{6}$$

where ω is the angular velocity, rad.

Planet Earth rotating round its axis has internal inertia. Due to this inertia the geoid shape, in the first approximation, represents a flattened ellipsoid with the larger radius located in the equatorial plane. The Earth-Moon and Sun-Earth systems have internal inertia as does every pair of star-planets. The body's internal inertia can be found experimentally. For instance, measurements of equatorial and polar radii of the globe reveal deformations of Earth's shape due to its rotation around its axis. Coriolis force (see expression (2)) deflecting the threads of streams westward in the north hemisphere and eastward in the southern one points to the manifestation of the Earth's internal inertia. Indoors, the Foucault pendulum clearly demonstrates the presence of this internal inertia. Since one can reveal the fact of a body's rotation in an isolated space experimentally, the Galilean relativity principle is inapplicable for rotating bodies. Thus, strictly speaking, the Galilean principle cannot be followed under the earthly conditions.

One can give a lot of vivid examples of a system of bodies that have great internal inertia. One of them is the solar-planetary system. In it, the planets move along the elliptic orbits close to circular ones. This motion is possible only with the constant centripetal force F_c applied to the moving body in the direction perpendicular to the direction of motion. During circular motion, this force is applied from one centre (a simple case). To make this body move along a circular trajectory this force should be equal to [6]:

$$F_c = \frac{1}{R} mV^2, \; kg{\cdot}m^2/s^2, \quad (7)$$

where R is radius, m is mass, V is linear velocity of the body movement along the orbit. Concerning a body that is in earth orbit, the role of force F_c is played by the earth's gravity.

Within mechanics determined by Newton’s first and second laws, besides inertia the second characteristic that is closely related to it should be singled out, i.e. inertness. Inertness and inertia are different categories. A number of well-known works [1-4] indicate that the inertness measurement of a body is its mass. “By convention, the mass of an elementary particle is determined by the fields related to it – electromagnetic, nuclear etc., but a quantitative theory of mass has not been created as yet” [4]. The differences in masses of various bodies are revealed in attempting to change their state, for instance, to accelerate or decelerate, to change the direction of their motion.

Newton’s second law establishes the measure of a body's mass (for simplicity, we do not take into account the vector nature of force and acceleration):

$$F = ma, \quad (8)$$

where F is the force inducing the body to change its state, a – is the acceleration that the body acquires. Newton’s second law shows that with one and the same force F, the greater the mass m, the less acceleration is acquired by the body.

Inertness is inherent in every body that possesses mass. Inertness is the property of physical bodies to show resistance in attempting to set them in motion, to change the modulus or direction of this motion. Thus, inertness is also inherent in motionless bodies. Inertness, as well as mass, determines the ability of a body to move freely in a free space. According to Newton’s second law the categories of inertness and mass are one and the same property of physical bodies. In this connection the category “inertness” can be convincingly considered to be a synonym to the category “mass”.

As mentioned above, unlike inertness the body inertia is related to its motion. There are two main kinds of inertia – external and internal. External inertia is the inertia of a body moving (within some assumptions) uniformly and linearly. The inertia of a moving body does not manifest itself in any way until it collides with another body.

Internal inertia is inherent in rotating bodies. Unlike bodies possessing external inertia, bodies possessing internal inertia experience internal strain and stress. If a rotating body is deprived of external effects it will rotate for a long time without limit maintaining one and the same rotational energy.

These notions of internal and external inertia have long been known in one form or another. Even Galileo differentiated these kinds of inertia. In other formulations they can be interpreted as “rotatory” and “linear” inertia. In our opinion, a clear separation of inertial types into internal and external, an emphasis on distinction between inertia and inertness will provide better insight into the essence of these very important physical categories. On the other hand, the equality of inertness and mass of a body has long been accepted in classical mechanics [3]. In our opinion, a detailed analysis of these notions presented above is necessary to gain a clearer understanding of their physical essence.

On the basis of the above, it can be noted that Newton's first law characterizes the rest and motion of a free body that has, within our definitions, only external inertia. It is unsuitable for characterizing the internal inertia. In this connection the demand arises for formulating the combining law that would characterize both types of inertia. In our mind, this formulation might be as follows: “a body set in motion that is free from external relations will move for an indefinitely long period, conserving motion energy”. This formulation combines both types of inertia. It is inclusive and can be called the law of inertia.

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