A Scalar Equation of Motion

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Abstract

In classical mechanics, this paper presents a scalar equation of motion, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The Scalar Equation of Motion

If we consider two particles A and B of mass m_a and m_b respectively, then the scalar equation of motion, is given by:

$$\frac{1}{2}\, m_a m_b \left[(\mathbf{v}_a - \mathbf{v}_b)^2 + (\mathbf{a}_a - \mathbf{a}_b) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right] = \frac{1}{2}\, m_a m_b \left[2 \int \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot d(\mathbf{r}_a - \mathbf{r}_b) + \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

where $\mathbf{v}_a$ and $\mathbf{v}_b$ are the velocities of particles A and B, $\mathbf{a}_a$ and $\mathbf{a}_b$ are the accelerations of particles A and B, $\mathbf{r}_a$ and $\mathbf{r}_b$ are the positions of particles A and B, and $\mathbf{F}_a$ and $\mathbf{F}_b$ are the net forces acting on particles A and B.

This scalar equation of motion can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces. In addition, this scalar equation of motion is invariant under transformations between reference frames.

Appendix

Conservation of Energy

A system of particles forms a system of biparticles. For example, the system of particles A, B, C and D forms the system of biparticles AB, AC, AD, BC, BD and CD.

The total energy of a system of biparticles, is given by:

$$\sum_i \sum_{j>i} \frac{1}{2}\, m_i m_j \left[(\mathbf{v}_i - \mathbf{v}_j)^2 + (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) - 2 \int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot d(\mathbf{r}_i - \mathbf{r}_j) - \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right] = 0$$

where m_i and m_j are the masses of the i-th and j-th particles, $\mathbf{v}_i$ and $\mathbf{v}_j$ are the velocities of the i-th and j-th particles, $\mathbf{a}_i$ and $\mathbf{a}_j$ are the accelerations of the i-th and j-th particles, $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of the i-th and j-th particles, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net forces acting on the i-th and j-th particles.

Therefore, from the above equation it follows that the total energy of a system of biparticles is always in equilibrium.

On the other hand, the above equation would be valid even if Newton's third law were false.

Bibliography

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