On Universal Mechanics and Superluminal Velocities in ST

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Abstract
It was argued in previous works [1,2] that a concept of time which is consistent with motion requires time and distance to be of the same dimension. The mechanical system of units must thus contain only two basic units, MS for mass and $TS = LS$ for time and distance. In this work we discuss the concepts of the dimensionless universal and inertial velocities, which are intimate to the structure of the scaling theory. The inertial velocity which characterizes the relative magnitudes of the simultaneous source's displacement and the corresponding length of the light trip to the observer remains bounded and does not exceed the velocity of light. The universal velocity which has the same meaning as the familiar velocity in mechanics is unbounded, and accommodates consistently possible particles' superluminal velocities. The naturally arising reduced system of units $RSU \equiv \{LS = TS, MS\}$ reveals that mass and energy are equivalent and related by Einstein's mass-energy equivalence formula [3]. The mechanics constructed on the bases of the new concepts, named universal mechanics, admits superluminal velocities, but yet, almost coincides with the relativistic mechanics in its basic dynamical components and their inter-relations.

1.The Euclidean Form of the Scaling Transformations

Assume that a body b is moving in the universal frame S at velocity $\boldsymbol{u} = u\boldsymbol{i}$ and $B \in S$ is a body at rest in S with a geometric time distance T *sec* from the observer $O \in S$. Suppose that at an instant $t = 0$ corresponding to b passing by $B \in S$, a spherical light wave emanates from observer $O \in S$. The wave arrives at B at an instant T and intercepts the source at a point $b' \in S$ an instant t. Because the geometry of the universal frame is Euclidean by hypothesis, the sides' lengths of the triangle OBb' satisfy all triangle relations in Euclidean trigonometry. In particular, the relation, $\boldsymbol{Ob'} = \boldsymbol{OB} + \boldsymbol{Bb'}$, has its familiar meaning in Euclidean geometry. Since O and B are at rest in S, we have $\boldsymbol{OB} = cT(-\boldsymbol{e})$, but since the body b is moving in S we set provisionally $\boldsymbol{Bb'} = a\boldsymbol{u}t$ and $\boldsymbol{Ob'} = act(-\boldsymbol{e}_L)$, where a is a factor that depends on the magnitude of the velocity of the body b and must go to 1 as u goes to zero. The sides vectors of the aforementioned triangle

(1.1) $$\boldsymbol{OB} = cT(-\boldsymbol{e}), \qquad \boldsymbol{Bb'} = at\boldsymbol{u}, \qquad \boldsymbol{Ob'} = act(-\boldsymbol{e}_L),$$

satisfy the vector relation

(1.2) $$at(-\boldsymbol{e}_L) = T(-\boldsymbol{e}) + at\boldsymbol{\beta},$$

where $\boldsymbol{\beta} = \boldsymbol{u}/c.$ Solving for t we obtain

(1.3) $$t = a^{-1}G(\beta, \pi - \theta)T,$$

where

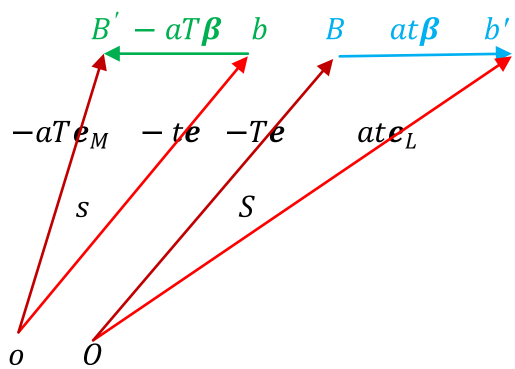
(1.4) $$G(\pi - \theta, \beta) = \frac{\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}}{1 - \beta^2}$$

is the Euclidean factor [1,2].

The scaling theory preserves the universal nature of time. Our goal therefore is to find the transformation from a universal frame to a frame co-moving with b such that the transformation and its inverse yield time durations as frame independent entities. The demand made on the transformation and its inverse is necessary so that either frame (but not both) can be started with as universal. Consider thus an inertial frame s that is moving in the universal frame S at velocity $\boldsymbol{u}$; the body b will be at rest in s. Let $o \in s$ be the s-observer that is contiguous to $O \in S$ at the instant $t = 0$. i.e., when the spherical wave emanates from $O \in S$. While b remains at rest in the moving frame s, the body $B \in S$ moves at velocity $(-\boldsymbol{u})$. In the frame s the wave hits b at the instant t and intercepts B at the instant T at a position $B' \in s$. We seek thus the transformation that maps the triangle OBb' in S to the triangle $oB'b$ in s, with the time lengths of OB *and* Ob' are equal to the time lengths of oB' *and* ob respectively, and vice versa. Since the role of the ordered pair (t, T) in s must be identical to the role of the pair (T, t) in S, the relation between t and T in s must results from that in S by interchanging t and T in (1.3) and replacing $\boldsymbol{u}$ by $(-\boldsymbol{u})$, or equivalently $\pi - \theta$ by θ. This yields

(1.5) $$T = a^{-1}G(\beta, \theta)t.$$

On the other hand, the quantities t and T already satisfy (1.3). Substituting one of the equations (1.3) or (1.5) in the other yields



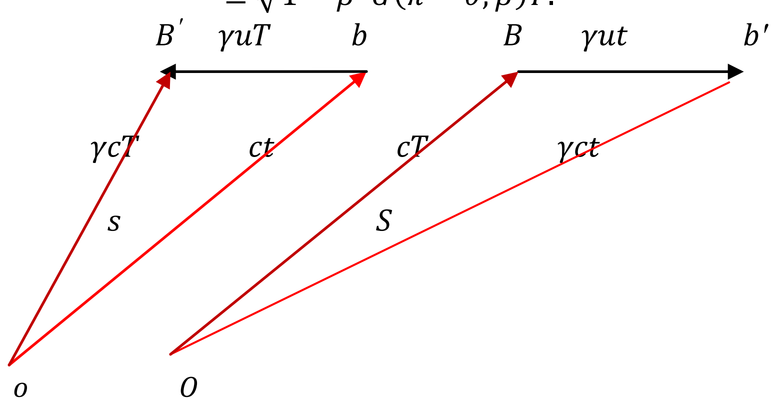
(1.6) $$1 = a^{-2}G(\pi - \theta, \beta)\, G(\theta, \beta) = \frac{1}{a^2(1 - \beta^2)}$$

or

(1.7) $$a = \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}.$$

The required transformation is therefore

(1.8a) $$t = \Gamma(\theta, \beta)T$$
$$\equiv \sqrt{1 - \beta^2}G(\pi - \theta, \beta)T.$$



Or

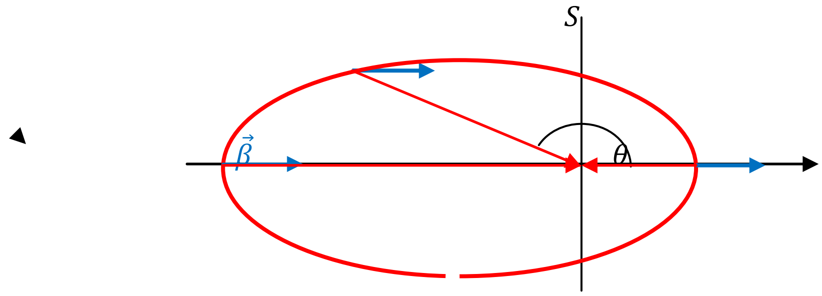
$$\frac{t}{\sqrt{1-\beta^2}} = G(\pi-\theta,\beta)T. \tag{1.8b}$$

We call the function $\Gamma(\theta,\beta)$ the scaling factor and the relation $t = \Gamma(\theta,\beta)T$ the scaling transformation. The relation (1.8b) is called the Euclidean form of the scaling transformation.

The scaling transformation holds within the same frame whether it was the stationary or the moving frame, as well as between the universal and moving frame, and allows for each frame to be the stationary frame. Note that the geometric lengths of the corresponding trips in S and s are different.

It is not difficult to convince oneself that if a light wave emanates from b when at B then the period it takes to arrive at O is given by (1.8), .i.e. if a light wave emanates from b simultaneously with a wave emanating from O then when the former arrives at O the latter arrives at b'. The latter fact was derived in earlier works by the authors.

The figure below illustrates that all s-pulses that emanate at $t = 0$ from geometric distances $T = t_0 \,/\, \Gamma(\theta,\beta)$ seconds from $O \in S$, where t_0 is fixed, arrive simultaneously in t_0 seconds at O. Indeed, and by (1.8a)



$$t = \Gamma(\theta,\beta)T = t_0.$$

In particular the pulse of the s-source which is heading directly towards O and initially at the distance $\Gamma(0,\beta) = \sqrt{(1+\beta)/(1-\beta)}$ seconds from O, arrive in one second at O. Indeed,

$$t = \Gamma(\pi,\beta)\mathrm{T} = \Gamma(\pi,\beta)\Gamma(0,\beta) = 1$$

Similarly, the pulse of the s-source which is receding from O and initially at the distance $\Gamma(\pi,\beta)$ seconds from O, arrives in one second at O.

With respect to s-observer that is contiguous to O at the instant t_0, all sources that are at a geometric distance T seconds from O are at equal geometric time distance t_0 from o.

2. On the Concept of Time in Scaling Theory

Time and geometric length in Newton's mechanics are independent absolute entities, and the velocity of an object in an inertial frame S is defined as the ratio $\beta_N = d/t$ of the travelled distance d and the corresponding time interval t. In measuring β_N light signals are redundant, for it is only necessary for point-wise observers to read the initial and final positions of the moving object together with the corresponding instants of time. Here distance is measured by a calibrated ruler, and time is read by local synchronized timers, or resorting to a universal timer (1.XII). Even if light signals are used to inform an observer O of such readings, these signals carry only information about the local measurements and can be sent at a later time.

In the scaling theory (ST), the frame independent entity, namely, "the time" is defined in terms of spatial displacements and can be measured by length units, which implies that velocity is a dimensionless quantity. Indeed, global timing in an inertial frame S is set up by synchronization with an arbitrary observer $O \in S$ employing light's signals. *The concept of time emerges through envisaging a "linear" correspondence between each instant of time t read by the timer O and the compound event: (the wave front of the pulse that was emitted from O at $t = t_0$ occupies at t, points at equal distances R from O).* Through this correspondence, time duration Δt, is essentially measured by distance R, i.e.

(2.1) $$\Delta t \equiv (t - t_0) = a\, R.$$

The proportionality constant defines a constant velocity c of light by $c = 1/a = R/\Delta t$. Because the space is geometrically homogeneous (delete: also physically, and in particular for the propagation of light), *time durations* defined in this way are independent of the master timer's position O. The homogeneity of time follows also from the homogeneity of space. The above correspondence is only one step short of synchronization, which is achieved by each S observer at $(R, \emptyset, \theta)$ taking note of his radial coordinate R and the instant of time t_0 at which the pulse emanated from O, and thus setting his timer at $t = t_0 + aR$ when he receives the pulse. By its way of construction, the global timing is unique up of course to an arbitrary choice of a time unit and of zeroing, i.e., up to a transformation of the form $t' = \alpha t + t'_0$, where $\alpha > 0$ and t'_0 is an arbitrary number. The global time in S prevails in any other inertial frame s, and timers therefore, have to reflect the facts stated in setting up the global time. Thus every moving timer, if needed, must read the same instant of time read by the S-clock that is touching it.

The 1-1 correspondence $R \to \Delta t = t - t_0$ is not linear in the strict meaning of linearity since it is defined only for $R > 0$, and hence for $t \geq t_0$. Past intervals, and time instants in the past, are defined by

$$\Delta t = t - t_0 = -aR \quad for\ t \leq t_0$$

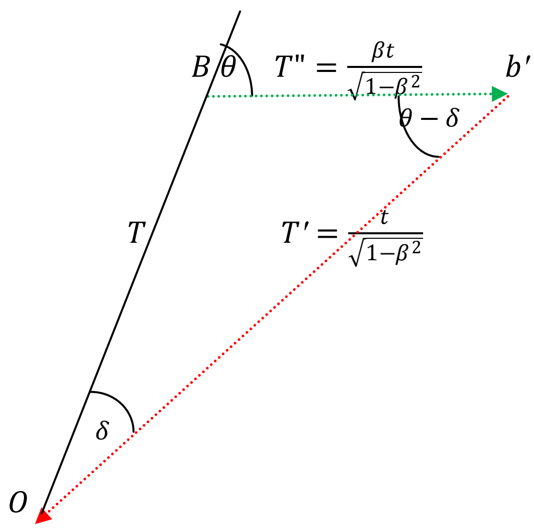
Instead of S, any other inertial frame s can be claimed universal, and a global time set up in s and prevailing in all other inertial frames must be the same as that constructed in S. The scaling transformation fulfills this quest by preserving time durations. In the frame S, the ratio between the duration of a given light's trip and its geometric time length ($i.e.,\ t/T = \Gamma(\beta, \theta)$) is determined by the velocity of the source and its direction, and time therefore, is set up mathematically in terms of equally primitive concepts, the geometric length (through T) and inertial frames (through β).

In reality, time durations and instants of time, become measurable as soon as an inertial frame is endowed with a global time. The instants of time corresponding, for example, to a moving body passing by some given points in S are determined by the time readings of hypothetical timers at these points. Thus, the primitive observables, the true time and geometric distances, exist together and are measurable in a timed inertial frame.

3.The Euclidean Body-Observer Triangle

The initial and final positions of the moving body together with the observer's position form a Euclidean triangle with sides length's

(3.1) $$T,\ \ T'' = \frac{\beta t}{\sqrt{1-\beta^2}},\ \ T' = \frac{t}{\sqrt{1-\beta^2}}.$$



The given lengths satisfy all triangle relations in Euclidean geometry, and yield a value *t* for the time duration as it is prescribed by the scaling transformation STI. An elementary fact in Euclidean geometry asserts that when three numbers are legitimate to form a triangle, this triangle is unique (up of course to arbitrary rotations, translations, or reflections). Thus the latter values determine a unique Euclidean triangle; it is the *body-observer triangle*.

By (3.1) we have

(3.2) $$\frac{T''}{\beta} = \frac{T'}{1} = \frac{t}{\sqrt{1-\beta^2}},$$

which are equivalent to the relations

(3.3) $$T'' = \beta T',\ \ T'^2 - T''^2 = t^2.$$

In terms of the initial geometric distance T we have

(3.4) $$\frac{T''}{\beta} = \frac{T'}{1} = \frac{\Gamma(\beta,\theta)T}{\sqrt{1-\beta^2}} \equiv G(\beta,\pi-\theta)T.$$

By the sinuses law in trigonometry,

(3.5) $$\frac{\gamma\beta t}{\sin\delta} = \frac{\gamma t}{\sin\theta} = \frac{T}{\sin(\theta-\delta)}\ ,$$

we have

(3.6) $$\sin\delta = \beta sin\theta,\ \ \sin(\theta-\delta) = sin\theta / G(\beta,\pi-\theta).$$

By (3.2) the pair of sides (T'', T') are in 1-1 correspondence with the pair (β, t), and the body-observer triangle is thus determined by (β, t, T). This expresses the obvious fact that the direction of the vector $\boldsymbol{\beta}$ relative to the observer, i.e. θ, is determinable by the quantities (β, t, T) through the scaling transformations $\Gamma(\beta,\theta) = t/T$, by which we can determine one out of the quantities $\{T, t, \beta, \theta\}$ in terms of the remaining three. In other words, the body-observer triangle is fully determined by three out of the four variables t, T, β, θ. This implies that in correspondence with each body-observer triangle there is one value of β, and hence the same value of β is obtained whether calculated from the expressions of $T'or\ T"$ or from the scaling transformations.

The Inertial Velocity

By (3.2), the displacement of the source, the distance travelled by the signal, and their duration *t* are in 1-1 correspondence. The quantity

(3.7) $$\beta = \frac{T"}{T'} ,$$

which is obtained from (3.2), will be called the *inertial velocity* of the body *b*. The definition (3.7) expresses the inertial speed in terms of geometric distances; *it is the quotient of the distance* $T"$ *travelled by the body b to the distance* T' *travelled by the light emanating from the observer O when they intercept each other*. The initial time for both motions is the instant (*b at B*). By Euclidation [1, section 13], T' is also the distance travelled by the pulse emitted from *b* when at *B* till arriving at *O*. The inertial velocity of a body $B \in S$ is nil, because *B* is not displaced whatever was the distance travelled by a pulse intercepting it and emitted from an *S*-observer.

The Abstract Right Space-Time Triangle

We revert to the body-observer triangle and set

-The initial geometric position of the body $\vec{R} = cT\vec{e} = R\vec{e}_R$

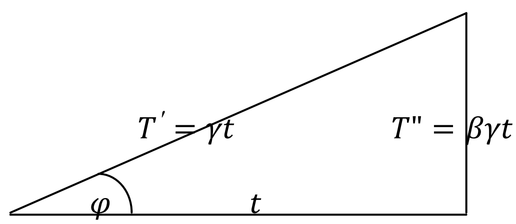
The body displacement: $\overrightarrow{Bb'} = cT"\vec{\imath} = D\vec{\imath}$

The "now" observer-body distance $\vec{r} = cT'\vec{e}_r = r\vec{e}_r$

The new notations are related to the old ones by $\vec{e}_R = -\vec{e}, \vec{e}_r = -\vec{e}_L$. By (3.2), the geometric lengths $D = cT"$, $r = cT'$ and true time t are related by the equations

$$r^2 - D^2 = c^2t^2, \qquad D = \beta r$$

The values ($T', T", t$) determine a unique abstract triangle, *the spatial-temporal right triangle,* in which T' is the hypotenuse.



The space-time right triangle is determined by β *and* t, which are independent variables. Assuming β is given then the triangle is determined on knowing t, which depends on the distance of the source from the observer. From the figure above

$$\beta = \frac{T"}{T'} = \sin\varphi, \ \ T" = T'sin\varphi, \quad t = T'cos\varphi$$

Or if we set

$$T' = t\ ch\sigma, \qquad T" = t\ sh\sigma, \qquad T"/T' = \beta = th\ \sigma$$

then

$$T' + T = te^{\sigma}, \ \ T' - T = te^{-\sigma}$$

4. The Universal Mechanics

The universal velocity of a body (or just velocity) refers to its velocity in a universal space; its definition is identical to that of velocity in classical mechanics. Thus the universal velocity of the body *b*, considered before, is the ratio of the distance it travels to the corresponding time interval. By (3.2) we have

(4.1) $$\beta_U \equiv \frac{T''}{t} = \frac{\beta}{\sqrt{1-\beta^2}}.$$

The universal velocity $\boldsymbol{U} = c\boldsymbol{\beta_U}$ (or $\boldsymbol{\beta_U}$ if we take $c = 1$) is measured locally by reading the instants t_1 and t_2 at which the particle occupies the positions $B \in S$ *and* $b' \in S$ respectively, and the length of the corresponding displacement vector $\boldsymbol{Bb'}$, which may of course be infinitesimal.

According to its expression, the magnitude of the universal velocity can assume any non-negative value with no upper bound (i.e. $0 \leq \beta_U < \infty$) and that β_U goes to zero with β.

The ***momentum*** of the particle *b* is defined by the product of its mass *m* and universal velocity $\boldsymbol{U} = c\boldsymbol{\beta_U}$:

(4.2) $$\boldsymbol{p} = m\boldsymbol{U} = \frac{mc\boldsymbol{\beta}}{\sqrt{1-\beta^2}} = \frac{m\boldsymbol{u}}{\sqrt{1-(u/c)^2}}.$$

Multiplying both sides of the identity

(4.3) $$\frac{\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} - 1,$$

by m^2 we obtain

(4.4) $$\frac{p^2}{c^2} = \frac{m^2}{1-\beta^2} - m^2.$$

According to the scaling theory time and distance have the same dimension, and the appropriate physical system of units results for the common systems (MKS for instance) through adopting the same unit for time and distance, or else, taking one unit as multiple of the other. In the reduced system of units (RSUI or RSUII, see appendix) mass and energy have the same dimension, and both are measured in RSUI by the same unit, *kilogram*. Thus, the *right hand-side of (4.4) can be envisaged as a difference between the squares of two values of the mass or energy of the moving body corresponding to the states of motion and rest respectively*. Denoting these values by E ($\equiv M$) and E_0($\equiv m$) respectively, i.e,

(4.5) $$E(kg) \equiv M(kg) = \frac{m}{\sqrt{1-\beta^2}}\ (kg), \quad E_0(kg) \equiv m\ (kg),$$

we write (4.4) in the form

(4.6) $$\frac{p^2}{c^2} = E^2 - E_0^2 = M^2 - m^2.$$

The latter relation reads: the state of motion of a body with rest mass *m* that is characterized by a momentum of magnitude *p* is accompanied by a ***total kinetic energy, or kinetic mass***,

(4.7) $$E = \frac{m}{\sqrt{1-\beta^2}} = \sqrt{m^2 + (p/c)^2}.$$

When p goes to zero, the total kinetic energy (or kinetic mass) tends to the rest energy (or rest mass) $E_0 = m$. The Hamiltonian of the particle coincides with its total kinetic energy:

(4.8) $$H = \sqrt{m^2 + (p/c)^2}.$$

For the time being we shall continue employing the reduced system of units

(4.9) $$RSUI \equiv \{LS = TS = meter, MS = kg\}$$
in which time and distance are measured by the same unit, say meter, and mass by kilogram. The velocity of light in $RSUI$ is 1, and hence c may be substituted by 1 in all relations that appeared so far in section 4. Making this substitution we get

(4.10) $$U = \beta_U \,, u = \beta, \boldsymbol{p} = m\boldsymbol{\beta_U},$$

(4.11) $$p^2 = E^2 - E_0^2 = M^2 - m^2, H = \sqrt{m^2 + p^2}$$

The force acting on a particle is defined as in Newtonian mechanics by the time rate of the change in its momentum:

(4.12) $$\boldsymbol{f} = \frac{d\boldsymbol{p}}{dt} = \frac{d}{dt}(m\boldsymbol{U}) = \frac{d}{dt}\frac{m\boldsymbol{u}}{\sqrt{1-\beta^2}}$$

If the magnitude of $\boldsymbol{U}$ remains constant while its direction changes,

(4.13a) $$\boldsymbol{f} = \frac{m}{\sqrt{1-\beta^2}}\frac{d\boldsymbol{u}}{dt} = M\frac{d\boldsymbol{u}}{dt} = m\frac{\beta_U}{\beta}\frac{d\boldsymbol{u}}{dt}$$

If only the magnitude of $\boldsymbol{U}$ changes while its direction remains constant,

(4.13b) $$\boldsymbol{f} = \frac{m\, d\boldsymbol{u}/dt}{(1-\beta^2)^{3/2}} = m\frac{\beta_U^3}{\beta^3}\frac{d\boldsymbol{u}}{dt} = \frac{M^3}{m^2}\frac{d\boldsymbol{u}}{dt}$$

Differentiating both sides of the equation $M^2 = m^2 + p^2$ with respect to time, we obtain

(4.14) $$\frac{dM}{dt} = \frac{1}{M}\boldsymbol{p}.\boldsymbol{f} = \boldsymbol{u}.\boldsymbol{f} = \frac{\boldsymbol{U}.\boldsymbol{f}}{\sqrt{1 + U^2/c^2}}$$

In the currently used units, $U = \beta_U \; and \; u = \beta$. The latter relation (4.14) determines the instantaneous rate at which the mass changes under the action of a force when moving at velocity $\boldsymbol{U}$. Looking on M as the kinetic energy of the particle, the equation (4.14) also determines the power of the force, i.e. the rate at which it does work. The work done by the force during a displacement $d\boldsymbol{r} = \boldsymbol{U}dt$ is given by

(4.15) $$dW = \frac{\boldsymbol{f}.d\boldsymbol{r}}{\sqrt{1 + U^2/c^2}}$$

It is customary to measure mass in kg, energy in $(c^{-2}kg = Joule)$, and momentum in $(c^{-1}kg = m.sec^{-1}.kg)$ (see appendix), which corresponds to using the reduced system of units

(4.16) $$RSUII \equiv \{LS = m, TS = sec \equiv c.m, MS = kg\}.$$

In RSUII, the expressions (4.5) become

(4.17) $$E = \frac{mc^2}{\sqrt{1-\beta^2}}\;(c^{-2}kg), \quad E_0 = mc^2\;(c^{-2}kg)$$

And equations (4.7) and (4.8) become

(4.18) $$E = \frac{mc^2}{\sqrt{1-\beta^2}} = \sqrt{m^2c^4 + p^2c^2}$$

(4.19) $$H = c\sqrt{m^2c^2 + p^2}$$

The equation (4.14) which gives the rate of kinetic energy change is written as follows

(4.20) $$\frac{dE}{dt} = \frac{c^2}{E}\boldsymbol{p}.\boldsymbol{f} = \boldsymbol{u}.\boldsymbol{f} = \frac{c\boldsymbol{U}.\boldsymbol{f}}{\sqrt{c^2 + U^2}}$$

And the work done by the force during a displacement $\boldsymbol{dr} = \boldsymbol{U}\, dt$ is written as

(4.21) $$dW = dE = \frac{c\boldsymbol{f}.\boldsymbol{dr}}{\sqrt{c^2 + U^2}}$$

From its definition, and by (4.9), the *momentum of a particle* is related to its velocity and rest energy by

(4.22) $$\boldsymbol{p} = \frac{\mathrm{E}_0}{\mathrm{c}^2}\boldsymbol{U} = \frac{E}{c^2}\boldsymbol{u}$$

Or equivalently

$$E_0 = \frac{c^2}{U}p, \;\; E = \frac{c^2}{u}p$$

For *particles* travelling at a universal velocity $\boldsymbol{U} = \boldsymbol{c}$, we have

(4.23) $$E_0 = cp.$$

We recall that $\boldsymbol{U}$ is unbounded in magnitude, and hence, the value c is attainable. For such particles

(4.24) $$E = \sqrt{2}E_0$$

The energy needed to give a stationary particle a universal velocity c is

$$E - E_0 = (\sqrt{2} - 1)mc^2 = 0.414mc^2$$

In the relations (4.22) and (4.23) the rest energy (or rest mass) is directly measurable, and the momentum of the particle is known if the body's velocity is known. For particles that can exist only in a state of motion, like photons in vacuum, it is meaningless to talk in S, which is the observer frame, about the photon's rest mass or rest energy. However, the relation (4.22), written in terms of inertial velocity

(4.25) $$\boldsymbol{p} = \frac{E}{c^2}\boldsymbol{u}$$

which is applicable to material particle, may be extended to comprise photons whose velocity is c in S. For material particles, both E and $\boldsymbol{p}$ are determined on knowing m and $\boldsymbol{u}$, which are immediately measurable. Setting $u = c$ in the latter relation we obtain for photons

$$E = \boldsymbol{c}.\boldsymbol{p} = cp$$

Unlike the relation (4.23) which connects directly measurable quantities, the latter relation which extends (4.25) to photons, requires in order to be fully meaningful a way by which either the energy or the momentum of a photon is prescribed. It is not satisfactory to merely deduce the energy and the momentum of the photon through its earlier or subsequent interaction with an external system. The contrary is required; the theory should be capable to quantify the photon's energy and momentum in terms of inherent characteristics, and thus capable of predicting the magnitudes of its interaction with an external system. Moreover, and whereas (4.23) assigns to a particle with a rest energy E_0 a definite momentum, the relation (4.25) does not distinguish between photons; it only states the relation between their energy and momentum. The additional information about the energy and momentum of a photon comes from Plank hypothesis which relate the energy (or momentum) of a photon to a macroscopic measurable quantity, the frequency ν of photon, by $E = h\nu$ (or $= h\nu/c = h/\lambda$).

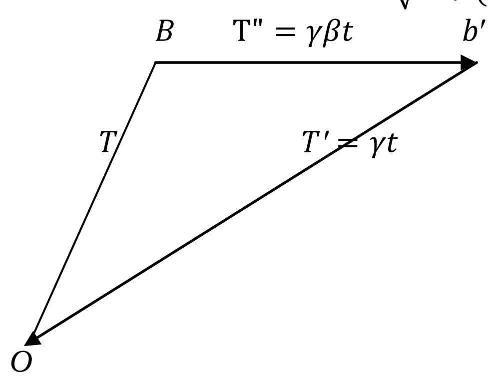
6. Point-wise Measurement of the Inertial Velocity

Suppose that the free source of light b is detected at a point $B \in S$ at $t = 0$ and at $b' \in S$ at an instant of time t. The pulse which was emitted from b when was at B arrives at the instant t at $O \in S$. i.e. the arrivals of b at b' and the pulse at O are simultaneous; they both occur at t. Let $T" = |\boldsymbol{Bb'}|$ be the geometric time length of the displacement $\boldsymbol{Bb'}$. The source b, at the time t, is at geometric distance

(6.1) $$\frac{\beta t}{\sqrt{1-\beta^2}} \equiv T"$$

from $B \in S$. This yields

(6.2) $$\beta = \frac{1}{\sqrt{1+(t/T")^2}}$$



The latter formula determines the inertial velocity of a moving body b in S in terms of the time t read at its final position $b' \in S$ and the geometric distance $T"$ of b' from the body's initial position $B \in S$. The formula (6.2) digresses from the Newtonian (or universal) definition of the velocity as the ratio of distance travelled to the corresponding time interval, which takes in our units the form $\beta_U = T"/t$. In terms of the universal velocity, the inertial velocity takes the form

(6.3) $$\beta = \frac{\beta_U}{\sqrt{\beta_U^2+1}} = \frac{U}{\sqrt{U^2+c^2}}.$$

The following comments illustrate some facts concerning the inertial velocity which applies, of course, only to material bodies, but not to light signals.

(i) Because the right hand side of (6.2) is less than 1, $0 \le \beta < 1$, and hence the inertial velocity β of any material object can not reach the value 1, i.e. can not reach the velocity of light.

(ii) If a body b is at rest at $B \in S$ which is distinct from $b' \in S$, then b will never be found at b'. Setting $t = \infty$ in (6.2) yields $\beta = 0$.

(iii) In spite of the fact that the inertial velocity of any object cannot reach the velocity of light, the object itself can overtake the pulse emanating from its starting position $B \in S$. If the moving body b and the light signal emitted from $B \in S$ when b passed by, arrive simultaneously at b', then $t = T"$, and the inertial velocity of the body is $\beta = \frac{1}{\sqrt{2}} \approx 0.707$. However, the universal velocity of the same body is $\beta_U = T"/t = 1$. While inertial velocity can not reach the velocity of light, universal velocity is unbounded and can exceed that of light. These facts demonstrate that there is nothing odd about the result of an experiment yielding a superluminal speed for an elementary particle.

(iv) For a fixed value of $T"$, β is a decreasing function of *t*; it tends to zero for *t* tending to infinity and to 1 for *t* tending to zero. This expresses the obvious fact, the faster the particle is the shorter time it takes to arrive at b'.

(v) For small velocities, $t/T" \gg 1$, and

$$\beta = \frac{T"}{t}\frac{1}{\sqrt{1+(T"/t)^2}} \approx \frac{T"}{t} - \frac{1}{2}(\frac{T"}{t})^3 = \beta_U - \frac{1}{2}\beta_U^3 \quad (6.4)$$

If the inertial velocity is sufficiently small we can neglect the third order term in comparison with the first order term and write

$$\beta \approx \frac{T"}{t} = \beta_U. \quad (6.5)$$

We also obtain the same result simply by neglecting 1 in the dominator on the right hand-side of (6.2) in comparison with the much larger term $(t/T")^2$. Therefore, for small values, the classical expression is an approximation of the inertial velocity formula (6.2).

For high velocities, $t/T" \ll 1$, and the formula (6.2) can be approximated by

$$\beta \approx 1 - \frac{1}{2}(\frac{t}{T"})^2 + \frac{3}{8}(\frac{t}{T"})^4 = 1 - \frac{1}{2}\beta_U^{-2} + \frac{3}{8}\beta_U^{-4} \quad (6.6)$$

The inertial velocity can also be deduced in terms of T'. Indeed, on solving the expression of T' for β we obtain the following equivalent expression of the inertial velocity,

$$\beta = \sqrt{1-(t/T')^2}. \quad (6.7)$$

The μ −meson particles lifetime once more

The $\mu -$ meson particles whose mean lifetime is $t = 2 \times 10^{-6} sec$, are generated at $60km$ above the earth surface, that is, $T" = 60km/(300{,}000km/\text{sec}) = 2 \times 10^{-4} sec$. Inserting $t/\text{T}" = 10^{-2}$ in formula (6.2) yields the inertial velocity of the particles that just arrive at the earth surface as

$$\beta = \frac{1}{\sqrt{1+10^{-4}}} \approx 1 - \tfrac{1}{2}10^{-4} = 0.99995 \quad (6.8)$$

The particles with inertial velocities not less than the latter value can cover *60 km* in the earth's frame *S* in spite of the fact that a pulse of light emitted from an *S*-observer can travel during the period of the $\mu - meson$ lifetime only

$$t \times c = 2 \times 10^{-6} \times 3 \times 10^5 km = 0.6\ km \quad (6.9)$$

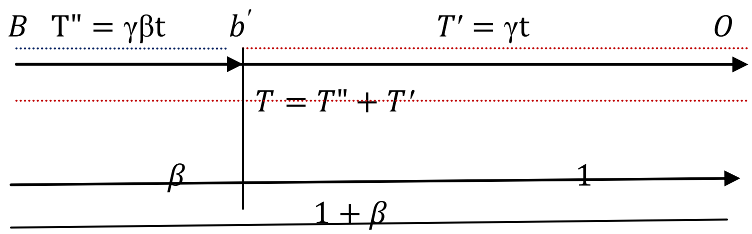
Relative to the Earth frame the universal velocity of the mesons particles that reach the earth surface is

$$\beta_U = \frac{T"}{t} = \frac{2 \times 10^{-4}}{2 \times 10^{-6}} = 100, \quad (6.10)$$

one hundred times of the velocity of light!.

7. The Simultaneous Positions of a Particle and its Emitted Signal.

Suppose that the body b heads towards O with inertial velocity β. A pulse emitted from b when at $B \in S$, say at $t = 0$, arrives at $O \in S$ simultaneously with the body arriving at $b' \in S$. The Galilean picture in which light is envisaged to emanate from the body's current position, corresponds to body-triangle (3.1), which reduces in the current head-on motion to a 3 straight segments, with



(7.1) $$\frac{T''}{\beta}=\frac{T'}{1}=\frac{T}{1+\beta}=\gamma t.$$

The third ratio is obtained from the properties of the proportion. Equivalently,

(7.2) $$T''=\gamma\beta t,\;\; T'=\gamma t \qquad T=T''+T'=\gamma(\beta+1)t$$

The first and third ratios show that when the body is at a distance T'' from B, the pulse it emits is at a distance

(7.3) $$T=\frac{1+\beta}{\beta}T''$$

from *B*. Thus, a particle always lacks behind the pulse it emits in its direction. As an example, a *hypothetical* light signal emitted from the meson particles when generated, travels by the time during which a meson particle arrives at the earth surface the distance

$$T=\frac{1+0.99995}{0.99995}\times 2\times 10^{-4}sec\approx 4\times 10^{-4}sec=120\ km$$

which (assuming not absorbed) is almost twice as much the distance travelled by the particle itself.

By (7.1), the particle and the light, arrive at b', at t_p and t_l respectively, where

(7.4) $$t_p=\sqrt{1-\beta^2}\frac{T''}{\beta}.$$

(7.5) $$t_l=\sqrt{\frac{1-\beta}{1+\beta}}T''=\sqrt{1-\beta^2}\frac{T''}{1+\beta}=\frac{\beta}{1+\beta}t_p.$$

Thus light arrives first at b' advancing b by

(7.6) $$t_p-t_l=t_p\left(1-\frac{\beta}{1+\beta}\right)=\frac{t_p}{1+\beta}=\frac{t_l}{\beta}$$

Employing the STI to determine the position of the light front when b arrives at b'; i.e. at the instant of time t_p, we get

(7.7) $$T=\sqrt{\frac{1+\beta}{1-\beta}}t_p=\sqrt{\frac{1+\beta}{1-\beta}}\sqrt{1-\beta^2}\frac{T''}{\beta}=\frac{1+\beta}{\beta}T''$$

which coincides with (7.3). The same relation has been obtained by the Galilean picture, which on scaling T'' *and* $T'=T-T''$, yields the particle travelling the distance $\sqrt{1-\beta^2}T''$ with velocity β and light travelling the distance $\sqrt{1-\beta^2}T'$ with velocity 1. i.e.

$$\sqrt{1-\beta^2}\left(T-T''\right)=t_p$$

Substituting for t_p from (7.4) we obtain T as given by (7.3). On changing to distances instead of geometric time distances we write (7.3) in the form

$$\frac{X}{c+u}=\frac{X"}{u}$$

which is the same as the classical picture, apart from the fact that the quotients are γt, but not t, and hence u refers to the inertial velocity. For small velocities, $X"/u \approx t$, and $X \approx (c+u)t$.

Appendix: The Reduced System of Units

Time in scaling theory is defined in terms of spatial displacements. Geometric length and time durations have accordingly the same dimension; both are measured by the same unit. If the unit of time TS in S is defined as the duration required by light to cross the unit of distance LS (a given rod stationary in S) from one end to another, say $LS = 1meter$, we may designate the unit of time also by "meter", to mean the time required by a light's signal to cross this distance. In terms of a system of units of time, length, and mass $\{TS = LS = m, MS = kg\}$, the dimensions of some mechanical observables are listed in the table:

(1i) $$[velocity] = LS.TS^{-1} = 1$$

$$(1ii)$$

(1iii) $$[momentum] = kg.LS.TS^{-1} = kg$$

(1iv) $$[force] = kg.LS.TS^{-2} = kg.m^{-1}$$

(1v) $$[energy] = [work] = kg.LS^2.TS^{-2} = kg = [mass]$$

(1vi) $$[angular\ momentum] = m.kg$$

(1vii) $$[torque] = kg$$

As seen from (1i), the velocity $\vec{v} = \Delta\vec{R}/\Delta t$ in this system of units is a dimensionless 3-vector, and the speed of light in vacuum is 1 regardless of the chosen unit of length LS, provided we choose $TS = LS$. Mass and energy have the same unit, "kilogram". In the reduced system of units (I) $\{TS = LS, MS = kg\}$, LS and MS are arbitrarily chosen, once and for all. In practical applications it is convenient to take $LS = 1\ meter \equiv m$, and adopt a multiple of the unit $TS = 1\ meter$, namely, "$second$". The latter is defined by the period taken by light to travel a distance of $c\ meters = 3 \times 10^8 m$. Thus $1\ second = c\ meters$, or $1\ meter = \frac{1}{c} seconds$. In the reduced system of units (II)

(2) $$\{LS = meter, second \equiv c.meter, kg\},$$

(3i) $$[velocity] = m.(c.meter)^{-1} = \frac{1}{c},$$

(3ii) $$[acceleration] = c^{-2}m^{-1} = (c\ sec)^{-1},$$

(3iii) $$[momentum] = \frac{1}{c}kg$$

(3iv) $$[force] = kg.m.sec^{-2} = kg.c^{-2}m^{-1} = kg.(c\ \mathrm{sec})^{-1} \equiv Newton$$

(3v) $$[energy] = [work] = kg.m^2 sec^{-2} = kg.c^{-2} \equiv Joule$$

(3vi) $$[angular\ momentum] = c^{-1}\ m.kg = c^{-2}sec.kg = Joule.sec = [action]$$

(3vii) $$[torque] = c^{-2}\ kg \equiv Joule$$

The reduced systems of units (RSU), I or II, suggest that observables which are measurable by the same unit are of the same nature, although they may manifest themselves in different facets. Mass and energy for instance are both scalar quantities and both measurable in RSUI by kg. This means that 1 kg of mass is equal to 1 kg of energy, and that under suitable circumstances either quantity can be transformed to the other. In the RSUII, $1kg = c^2(c^{-2}kg) = c^2 Joule,$ and $m(kg) = mc^2 Joule$. The latter relation hold for any type of energy.

For a vector observable $\vec{A}$ that has the same dimension as a scalar observable B, the squares of these observables, A^2 and B^2 are of the same nature and in principle are transformable to each other.

It is noted that the reduced system of units I and II are the system MKS with the unit of time is taken as the unit of length itself in RSUI, or defined in terms of the unit of length, with $c^{-1}.sec \equiv m$ for RSUI, and by $sec = c.m$ for RSUII. Symbolically, $RSUI \equiv MKM \text{ and } RSUII \equiv MKcM$

References

1.C P Viazminsky and P K Vizminiska, The Scaling Theory (I- XIII). General Science Journal, www.gsjournal.net

2. C P Viazminsky and PK Vizminiska, Motion's Observation by Light's Signals. General Science Journal.

3. C P Viazminsky and P K Vizminiska, The Reduced System of Units in the Scaling Theory, General Science Journal.