Exact Calculation of the Fine Structure Constant

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Abstract

This paper presents an analytical method used to calculate the exact value of the Fine Structure Constant. The method is based on an interpretation of integer based derivations of Einstein's relativistic mass-energy equation and deBroglie's wave equation that suggest the Compton and deBroglie wavelengths are governed by a Pythagorean triple whose multiples provide orbital stability by maximizing the overlap between the interlaced waves. A systematic search across the uncertainty range of the Fine Structure Constant reciprocal (137.035999084(51)) results in a single primitive Pythagorean Triple that forms 9X the number of n^2 multiples above any other primitive triple suggesting it is the exact value of this constant:

$$\frac{1}{\propto} = 137.035999065849$$

1. Integer based derivations used in the analysis

Einstein equated energy with mass and momentum in his famous relativistic energy momentum equation shown in Equation (1).

$$(1) \qquad E^2 = (mc^2)^2 + (pc)^2$$

Energy (E)	in Joules
Mass (m)	in Kg
Speed of Light (c)	2.99792456E08 m/s
Velocity (v)	in m/s
Momentum (p)	mv in Kgm/s

The relativistic energy momentum equation can be quantized using dimensionless integers by including a rest mass (n_m) and a kinetic energy (n_v) term. The integers can be converted to energy units by the use of Plank's Law where the wavelength is represented by a constant unit wavelength λ_u divided by the integer terms of mass (n_m) and kinetic energy (n_v). The total energy is simply the sum of these two unit energy units as shown in Equation (2).

$$(2) \qquad E = \frac{hc(n_m + n_v)}{\lambda_u}$$

Energy (E)	in Joules
Plank's Constant (h)	6.62606896E-34 Js
Speed of Light (c)	2.99792456E08 m/s
Unit Energy Wavelength (λ_u)	constant, in meters
Number of Rest Mass Energy Units (n_m)	dimensionless integer

Number of Kinetic Energy Units (n_v) dimensionless integer

Equation (3) equates the number of rest mass energy units (n_m) with rest mass.

$$(3) \qquad m = \frac{hn_m}{\lambda_u c}$$

Einstein's relativistic energy momentum equation, Equation (1), is in the form $C^2=A^2+B^2$ which is Pythagorean's Theorem equating the sides of a right triangle. The two energy Equations (1) and (2) are equated by matching the sides of this right triangle as shown in Figure (1). In this case, the hypotenuse C is the total unit energy count ($n_m + n_v$), the adjacent side A is the rest mass energy count (n_m), and the opposite side B can be calculated using Pythagorean's Theorem to be $\sqrt{n_v^2 + 2n_m n_v}$ which is inversely proportional to the deBroglie Wavelength.

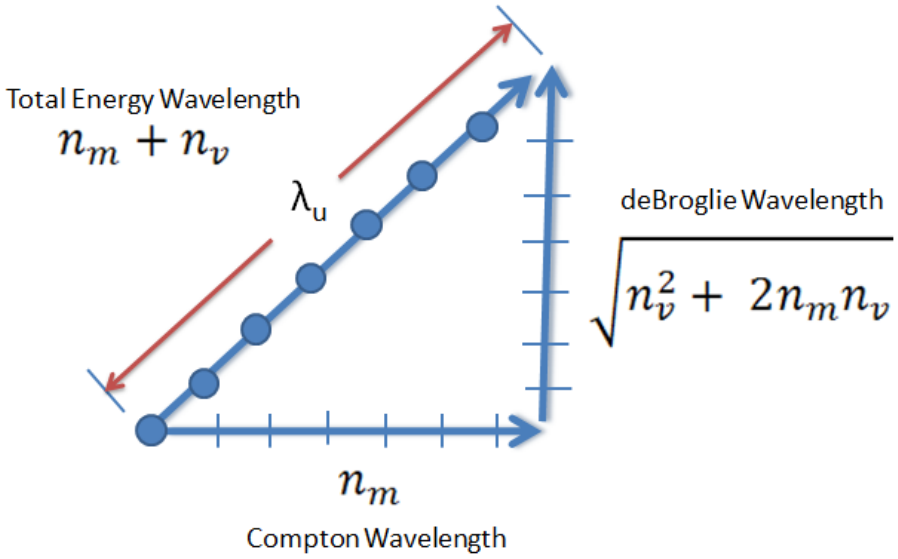

Figure (1)

Equation (4) equates velocity with the energy relationship shown in Figure (1). The velocity equation is an energy relationship multiplier of the speed of light (c), with the maximum speed being light speed when $n_m = 0$, or, the rest mass equals zero.

$$(4) \qquad v = \frac{\sqrt{n_v^2 + 2n_m n_v}}{n_m + n_v} c$$

The quantized Lorentz factor Equation (6) is a much simpler form than the classic Lorentz version Equation (5). The relativistic mass increases as the sum of n_m and n_v divided by n_m, only when $n_v=0$ (no kinetic energy), does the relativistic mass equal rest mass.

$$(5) \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$(6) \qquad \gamma = \frac{n_m + n_v}{n_m}$$

Louis de Broglie introduced the matter wave shown in his famous Equation (7). The Lorentz factor has been added in this version to account for relativity.

$$(7) \qquad \lambda_{dB} = \frac{h}{\gamma m v}$$

By inserting the quantized Lorentz Equation (6), the quantized mass Equation (3), and the quantized velocity Equation (4) into Equation (7) and simplifying, the quantized deBroglie Equation (8) can be calculated. Singularities are eliminated in this quantized version since the maximum value of the deBroglie wavelength can obtain is λ_u when $n_v = 1$ and $n_m = 0$.

$$(8) \quad \lambda_{dB} = \frac{\lambda_u}{\sqrt{n_v^2 + 2n_v n_m}}$$

Although the quantized versions of Einstein's relativistic mass-energy equation and deBroglie's wave equation are very different in form, they are mathematically equivalent as shown in Table (1). The first two columns of Table (1) provide various ratios of kinetic energy units (n_v) and rest mass units (n_m). Setting the unit wavelength λ_u to 1, the quantized mass can be calculated using Equation (3). Since the number of rest mass units (n_m) is held constant at 100000000, the resulting mass is also a constant 2.2102E-34 kg. Column 4 is the velocity of the inertial system from Equation (4) displayed as a decimal multiple of the speed of light (c). The next two columns show the classical Equation (5) and quantized Equation (6) Lorentz factors which show identical values over all velocities including those nearing light speed. The next columns show the classical Equation (1) and quantized Equation (2) energy momentum equation results which also show identical values over all velocities. The deBroglie wavelengths, Equations (7) and (8), show slight variations (in the sixth decimal place) at high velocities.

Nm (integer)	Nv (integer)	Mass (xE-34 kg)	Velocity (x c m/s)	Equation (5) Classic Lorentz Factor	Equation (6) Quantized Lorentz Factor	Equation (1) Classic Energy Momentum (xE-17 J)	Equation (2) Quantized Energy Momentum (xE-17 J)	Equation (7) Classical De Broglie Wavelength (m)	Equation (8) Quantized De Broglie Wavelength (m)
100000000	1	2.2102	0.0001414	1.0000000	1.0000000	1.9864	1.9864	7.07113E-05	7.07107E-05
100000000	10	2.2102	0.0004472	1.0000001	1.0000001	1.9864	1.9864	2.23609E-05	2.23607E-05
100000000	100	2.2102	0.0014142	1.0000010	1.0000010	1.9864	1.9864	7.07113E-06	7.07107E-06
100000000	1000	2.2102	0.0044721	1.0000100	1.0000100	1.9864	1.9865	2.23608E-06	2.23606E-06
100000000	10000	2.2102	0.0141411	1.0001000	1.0001000	1.9866	1.9866	7.07095E-07	7.07089E-07
100000000	100000	2.2102	0.0446879	1.0010000	1.0010000	1.9884	1.9884	2.23553E-07	2.23551E-07
100000000	1000000	2.2102	0.1403708	1.0100000	1.0100000	2.0063	2.0063	7.05352E-08	7.05346E-08
100000000	10000000	2.2102	0.4165978	1.1000000	1.1000000	2.1851	2.1851	2.18220E-08	2.18218E-08
100000000	100000000	2.2102	0.8660254	2.0000000	2.0000000	3.9729	3.9729	5.77355E-09	5.77350E-09
100000000	1000000000	2.2102	0.9958592	11.0000000	11.0000000	21.8507	21.8507	9.12879E-10	9.12871E-10

Table (1): Classical versus Quantized Equation Results

2. Method used to calculate the Fine Structure Constant

Einstein's relationship between the deBroglie wavelength (opposite side B), and the total energy wavelength (hypotenuse side C) must follow Pythagorean's theorem. By definition, the hypotenuse ($n_m + n_v$) and the adjacent Side A (n_m) are integers. In the case of an orbiting electron, the deBroglie wavelength (opposite side B) must also be an integer because, to form an overlapping standing wave, the deBroglie nodes must also be equally spaced (integer divisor). When all three sides of a right triangle are integers, they form a Pythagorean Triple.

Pythagorean Triples can be generated using Euclid's formula that generates triples from an arbitrary set of positive integers m and n where m > n. The equations that calculate the sides of the Pythagorean Triple from these integers are shown in Figure (2). Euclid's formula for deriving Pythagorean Triples generates both primitive (no common factors between sides) and non-primitive Triples. While there are an infinite number of Pythagorean Triples that provide Fine Structure constant values that fall in the measured range of 137.035999084(51), the simplest (smallest integer values) primitive triples that are of most interest because they form the most multiples in cyclic orbits.

The right triangle relationship in Figure (1) also represents the lead of a helix suggesting deBroglie's matter waves may be helical waves overlaid on top of a sinusoidal total energy wavelength $(\lambda_u/(n_m + n_v))$. Every time $n_m + n_v$ cycles elapse, the sinusoidal wave intersects (overlaps) with the deBroglie helical wave and the cycle repeats itself. The shorter the cycle (simplest primitive triple), the more stable the orbit.

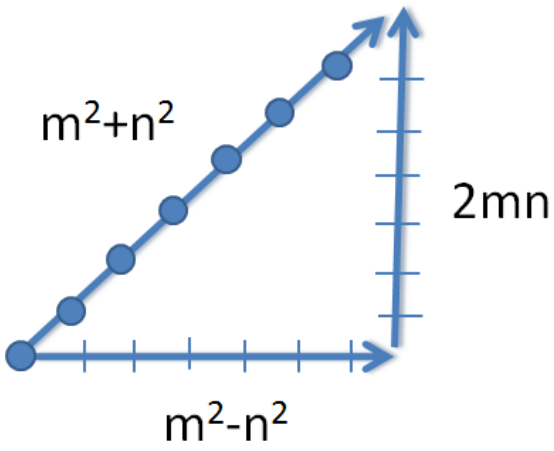

Figure (2) Euclid's Formula

Mapping Euclid's equations with the equations in Figure (1), the following relations are formed;

$$(9) \quad n_m + n_v = m^2 + n^2$$

$$(10) \quad n_m = m^2 - n^2$$

$$(11) \quad n_v = 2n^2$$

The Rydberg Constant relates the shared photon energy in the lowest orbital with the Fine Structure Constant in the following Equation (12):

$$(12) \quad R_\infty = \frac{1}{\lambda_{photon}} = \frac{\propto^2}{2\lambda_{electron}}$$

Relating the ratio of photon energy to the electron energy with the Fine Structure Constant and taking the reciprocal of each, this equation takes the form of Equation (13):

$$(13) \qquad \frac{1}{\propto} = \frac{1}{\sqrt{\frac{2\lambda_{electron}}{\lambda_{photon}}}}$$

The integer constants n_v and n_m are related to the electron and photon wavelengths in Equations (14) and (15):

$$(14) \qquad \lambda_{electron} = \frac{\lambda_u}{n_m}$$

$$(15) \qquad \lambda_{photon} = \frac{\lambda_u}{n_v}$$

Inserting Equations (14) and (15) into Equation (13) and simplifying, the result is Equation (16) relating the reciprocal of the Fine Structure Constant to n_v and n_m.

$$(16) \qquad \frac{1}{\propto} = \frac{1}{\sqrt{\frac{2n_v}{n_m}}}$$

3. Discussion of Results

A computer program was written to systematically test the Pythagorean Triples that lie within the measured range of the reciprocal of the Fine Structure Constant (137.035999084(51)). Euclid's m and n values (from Figure (2)) were sequentially tested up to an n value of 258604 and an m value of 70876587. For each of the Triples calculated, integer values for n_v, n_m, the total energy wavelength, and the deBroglie wavelength were calculated using Equations 8-11. Then the real value for the reciprocal of the Fine Structure Constant was calculated using Equation 16.

For non-primitive Pythagorean Triples the highest common factor was also calculated to detect if the Triple was a multiple of a previously calculated primitive. Because Euclid's formula uses the squares of the m and n, the factors (or multiples) are also squares of a more fundamental multiplier. Of particular interest is the n^2 multiple (where n=1,2,3,4,5, …. ∞) because we know that electron orbitals are n^2 multiples of the deBroglie wavelength.

Figure (3) shows a scatter plot of Pythagorean Triple factors (multiples) versus the Fine Structure Constant reciprocal that yield n^2 multiples calculated up to an m value of 70876587. There is no need to test higher m values, because the pattern is clear. The highest multiple peak is at a Fine Structure Constant reciprocal of 137.035999065849. The primitive triple that yields this value generates 117^2 multiples, it is clearly the most primitive triple with n^2 multiples within the measured range of the Fine Structure Constant.

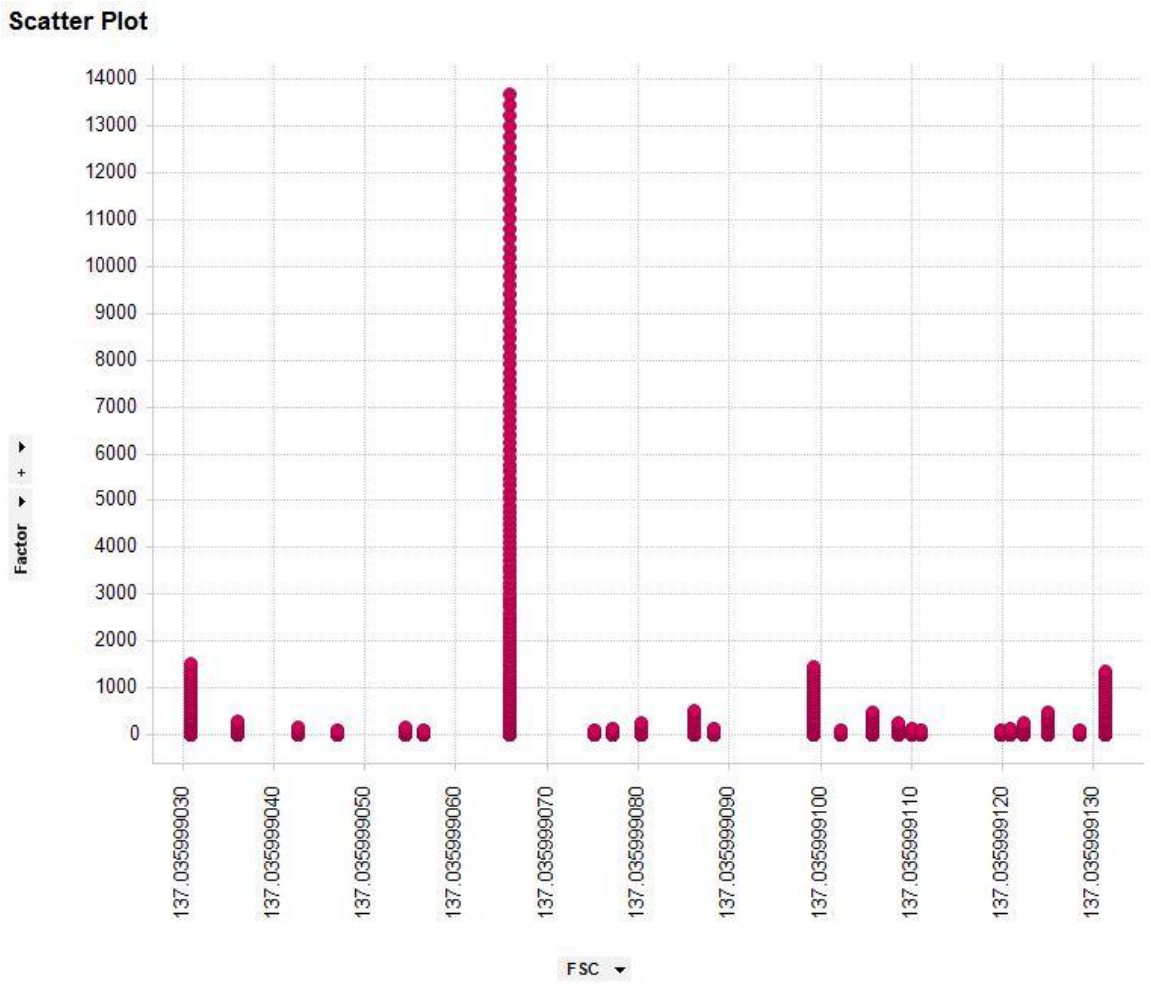

Figure (3) Electron Triple n^2multiples versus the Fine Structure Constant Reciprocal

Figure (4) shows integer values for the sides of the primitive Pythagorean Triple at the peak shown in Figure (3).

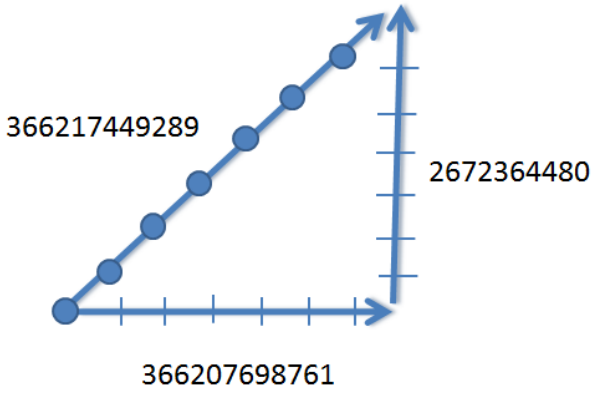

Figure (4): Electron Pythagorean Triple Primitive

Additional parameters of this primitive Electron Triple include:

Eulcid's n value	2208
Euclid's m value	605155
Number of Electron Rest Mass Energy Units (n_m)	366207698761
Number of Kinetic Energy Units (n_v)	9750528
deBroglie Wavelength Integer	2672364480

Plugging n_m and n_v into Equation (16), provides the exact calculation of the Fine Structure Constant reciprocal:

$$(17) \qquad \frac{1}{\propto} = \frac{1}{\sqrt{\frac{2(9750528)}{366207698761}}} = 137.035999065849$$

It is very likely the proton also forms a primitive Pythagorean Triple in its own orbital as it shares a photon with an energy proportional to $2n_v$. In searching for Pythagorean Triples in the range of the measured m_p/m_e mass ratio of 1836.15267245(75) and the measured range of the Fine Structure Constant reciprocal, there are several that form n^2 multiples as shown in Figure (5).

In Figure (5), the green markers signify the Fine Structure Constant reciprocal of 137.035999065849. This multiple peak provides further evidence that this exact photon energy (n_v =9750528) provides n^2 multiple orbital stabilities for both the electron and the proton.

Additional parameters of this primitive Proton Triple include:

Eulcid's n value	37536
Euclid's m value	440825849
Number of Electron Rest Mass Energy Units (n_{me})	105834024941929
Number of Proton Rest Mass Energy Units (n_{mp})	194327427737619505
Number of Kinetic Energy Units (n_v)	2817902592
deBroglie Wavelength Integer	33093678136128

This is the 17^2 multiple of the Electron primitive Triple, this Proton primitive did not form until Euclid's n & m values increased significantly. The $m_{proton}/m_{electron}$ mass ratio can be exactly calculated to be:

$$(18) \qquad \frac{m_{proton}}{m_{electron}} = \frac{194327427737619505}{105834024941929} = 1836.152672491165$$

The results presented in this paper may not be validated, or invalidated until further refinements can be made in measuring these constants. The methods presented in this paper may also be used to calculate other particle mass ratios and coupling constants.

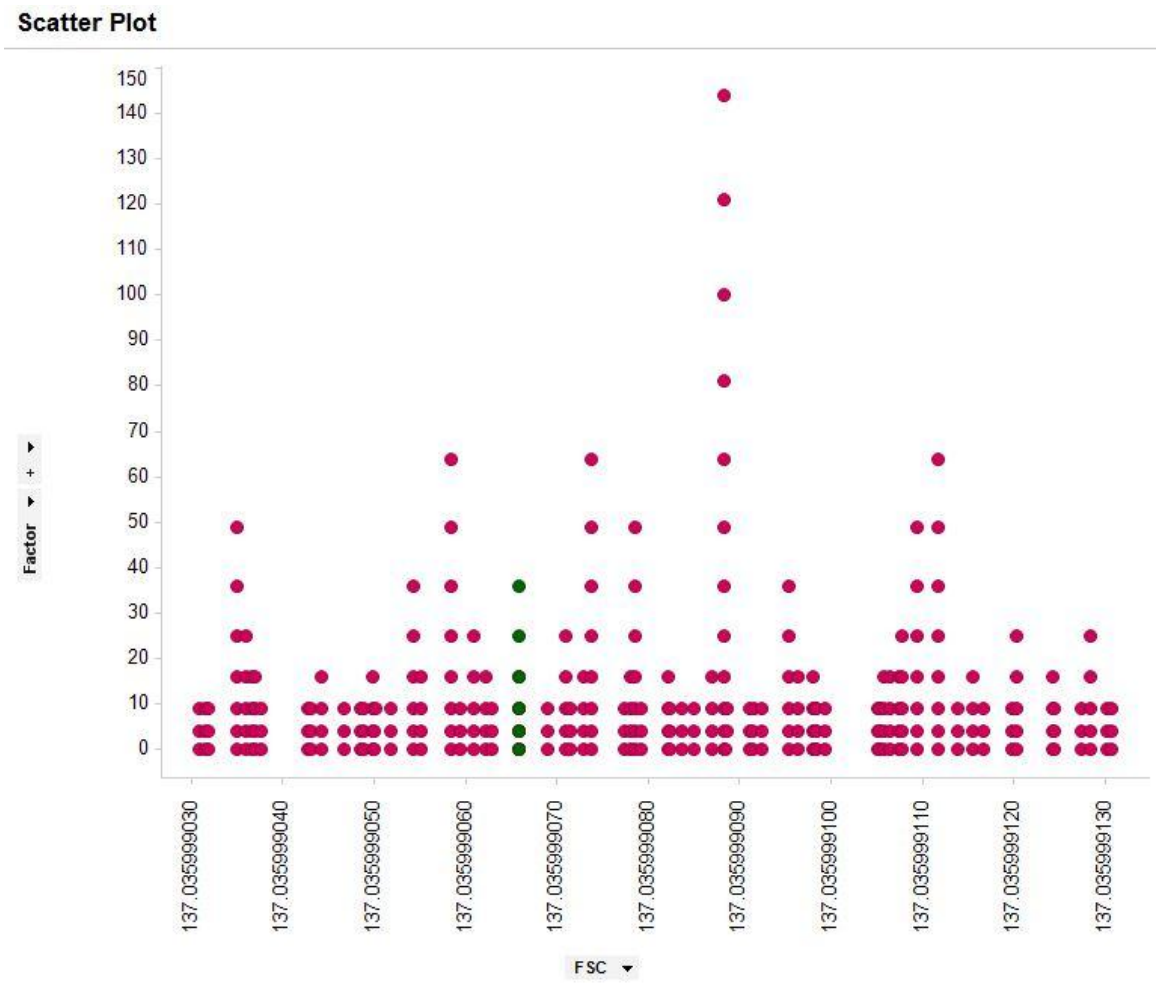

Figure (5) Proton Triple n^2multiples versus the Fine Structure Constant Reciprocal