

# Why there is no Gravitomagnetic Force

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## Abstract

Using a simple model this paper explains that there is no gravitomagnetic force contrary to a widely spread belief that such a force must exist. This is usually supported by the analogy with the Maxwell theory of Electromagnetic fields. The gravitomagnetic force, analogous to the force described by the Lorentz force equation, and the accompanied gravitoelectromagnetic field equations are derived from Einstein field equations by linearization for the weak gravitational fields. The nonexistence of the gravitomagnetic field thus questions the validity and correctness of Einstein field equations.

**Key words:** gravitomagnetic force, Einstein field equations, Maxwell field equations, Lorentz force equation, field energy of charged parallel plates, field energy of massive parallel plates, potential field energy, kinetic field energy

## 1. Introduction

There have been many papers published that derive the gravitational field equations for the weak fields by linearization of Einstein field equations. This activity is so popular that it almost seems to be a new branch of physics called the gravitoelectromagnetic (GEM) field theory. An example of already a decade old publication on this topic is the paper authored by Ruggiero et al. (2002). Another review of the concept is published by Mashhoon et al. (1999) and by NASA (2004) that has dedicated a significant effort to detect the gravitomagnetic force.

The result of linearization of Einstein field equations resembles the Maxwell's equations of the Electro-Magnetic (EM) field theory and from this derivation also follows the analogous equation for the Lorentz force acting on a massive body in a gravitational field. The Maxwell-like equations are the vector equations and are thus easier to handle than the tensor equations of the general relativity theory.

In this paper a simple model of the two charged parallel plates will be investigated and the field energy resulting from the force acting between the plates will be analyzed. The study will start first with the derivation of the EM force acting between these two plates when they are at rest relative to the laboratory coordinate system and will continue with the case when the plates are moving. Similarly, the field energy resulting from the force acting between the massive plates will be derived using the gravitoelectric and the gravitomagnetic field approach. It will be shown that this leads to an unreasonable result and that there can be no gravitomagnetic analogy to the magnetic force and the magnetic field energy of the Maxwell EM field theory.

## 2. Lorentz force acting between the two parallel moving plates and the corresponding field energy

The attractive force observed between the two charged nonconductive moving plates that have an uniformly distributed embedded charge throughout their thickness can be calculated by considering that in addition to the electrostatic force attracting the plates there is also a force based on the Biot-Savart law acting between the currents, which the moving plates also represent. To calculate this force it is useful to first find the magnetic field  $H$  existing in the space between the plates. For the selected configuration the simplest way is to use the integral form of Maxwell's equation:

$$\oint \vec{H} \cdot d\vec{s} = I, \quad (1)$$

where the integration path and the current flow are illustrated in a drawing in Figure 1. The magnetic field intensity between the plates up to their internal surfaces is then equal to:

$$H = \frac{Q}{A} v. \quad (2)$$

Similarly for the electric field intensity from the Gauss law it is:

$$\oiint \vec{D} \cdot d\vec{S} = Q, \quad (3)$$

where the integrating surface  $S$  encloses one of the plates. The result is:

$$E = \frac{Q}{\epsilon_0 A}. \quad (4)$$

Both, the magnetic field as well as the electric field intensities are, of course, zero on the external surfaces of the plates, but throughout the plate's thickness increase linearly from zero to the full value found between the plates. This is the consequence of the original assumption that the embedded charge distribution within the plates' volume is uniform. The formula for the force is obtained from the well known Lorentz force equation:

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}), \quad (5)$$

which must be integrated over the plate's thickness  $z_p$ .

$$|\vec{F}| = \frac{Q}{\epsilon_0 A} \left(1 - \frac{v^2}{c^2}\right) \int_0^{z_p} \frac{z}{z_p} \frac{Q}{z_p} dz. \quad (6)$$

After completion of integration in Eq.6 where the substitutions for the parameters:  $\vec{D} = \epsilon_0 \vec{E}$ ,  $\vec{B} = \mu_0 \vec{H}$ , and  $\epsilon_0 \mu_0 = 1/c^2$  was made, the result becomes:

$$F_q = \frac{Q^2}{2\epsilon_0 A} \left(1 - \frac{v^2}{c^2}\right). \quad (7)$$

When the plates are stationary it is  $v = 0$  and the force equation simplifies as follows:

$$F_{q0} = \frac{Q^2}{2\epsilon_0 A_0}. \quad (8)$$

The parameters with the subscript zero indicate the rest reference frame values. However, for the moving plates it is necessary to also consider the Lorentz length contraction of the plate's area  $A$  which results in the final formula for the force between the moving plates as follows:

$$F_q = \frac{Q^2}{2\epsilon_0 A_0} \sqrt{1 - \frac{v^2}{c^2}}. \quad (9)$$

In the next step it will be important to find the energy stored in the field when the moving plates are displaced apart to a distance  $d$ . This is obtained by integrating the force over the distance between the plates with the result:

$$W_e = \frac{Q^2 d_0}{2\epsilon_0 A_0} \sqrt{1 - \frac{v^2}{c^2}} = m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}}, \quad (10)$$

where the following simplifying substitution was made:

$$\frac{Q^2 d_0}{2\epsilon_0 A_0} = m_0 c^2. \quad (11)$$

The parameter  $m_0$  can be thought of as an equivalent rest mass of the electric field energy. However, this is not all the energy of the setup. This is only the potential energy. There is also an energy stored in the magnetic field that was not taken into account even though the force on the plates resulting from the currents has been included. The magnetic field energy is as follows:

$$W_m = \frac{1}{2} \mu_0 H^2 d_0 A = \frac{Q^2 d_0}{2\epsilon_0 A_0} \frac{v^2 / c^2}{\sqrt{1 - v^2 / c^2}} = \frac{m_0 v^2}{\sqrt{1 - v^2 / c^2}}. \quad (12)$$

By combining these two results the total EM field energy stored by the moving plates that move in the parallel direction to the plates' surfaces is thus as follows:

$$W_{em\parallel} = V_{\parallel} + T_{\parallel} = m_0 c^2 \sqrt{1 - v^2 / c^2} + \frac{m_0 v^2}{\sqrt{1 - v^2 / c^2}} = \frac{m_0 c^2}{\sqrt{1 - v^2 / c^2}}. \quad (13)$$

The total EM field energy thus consists of the two components, the potential energy:

$$V_{\parallel} = m_0 c^2 \sqrt{1 - v^2 / c^2}, \quad (14)$$

and the kinetic energy:

$$T_{\parallel} = m_0 v^2 / \sqrt{1 - v^2 / c^2}. \quad (15)$$

For the plates moving in the perpendicular direction to the plates' surfaces the situation is similar. The potential energy will be the same, since the distance between the plates now undergoes the Lorentz contraction and the area  $A$  is unchanged:

$$V_{\perp} = m_0 c^2 \sqrt{1 - v^2 / c^2} = V_{\parallel}. \quad (16)$$

There is no force from the currents and there is no magnetic field in the plates' vicinity now. The kinetic energy thus cannot be related to the magnetic field in this case, since there is no magnetic field present anywhere. The electric field terminates on charges of the plates, so when the plates move in the perpendicular direction the electric field must follow, but with a delay equal to the distance between the plates divided by the field propagation velocity, which is the speed of light  $c$ . The electric field changes linearly within the plates thickens, so the work the top plate has performed in the field is equal to:

$$W_{i\perp} = \frac{1}{2} \frac{Q^2 v}{2 \epsilon_0 A_0} \frac{d}{c - v}, \quad (17)$$

calculated similarly as in Eq.6. The bottom plate returns some of the work back, so the difference is the kinetic energy of the setup assuming that the plates are externally held together at a distance  $d$ :

$$T_{\perp} = \frac{1}{2} \frac{Q^2}{2 \epsilon_0 A_0} \frac{v d}{c - v} - \frac{1}{2} \frac{Q^2}{2 \epsilon_0 A_0} \frac{v d}{c + v} = \frac{Q^2 d}{2 \epsilon_0 A_0} \frac{v^2}{c^2 - v^2} = \frac{m_0 v^2}{\sqrt{1 - v^2 / c^2}} = T_{\parallel}. \quad (18)$$

The EM field energy for the plates moving in either direction is thus the same as expected. This also follows from the fact that, as is well known, the field energy is a scalar quantity.

$$W_{em\parallel} = W_{em\perp}. \quad (19)$$

### 3. The gravitomagnetic analogy

The simplest way to derive the gravitomagnetic analogy for the above described experiment is to compare the Maxwell's field equations with the field equations for the GEM approach.

$$\nabla \cdot \vec{E}_g = -4\pi\kappa\rho_g, \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad (20)$$

$$\nabla \cdot \vec{B}_g = 0, \quad \nabla \cdot \vec{B} = 0, \quad (21)$$

$$\nabla \times \vec{E}_g = -\frac{1}{2c} \frac{\partial \vec{B}_g}{\partial t}, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (22)$$

$$\nabla \times \frac{1}{2c} \vec{B}_g = \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t} - \frac{4\pi\kappa}{c^2} \vec{j}_g, \quad \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}, \quad (23)$$

$$\vec{F}_g = M \left( \vec{E}_g + \vec{v} \times \frac{1}{2c} \vec{B}_g \right), \quad \vec{F} = Q \left( \vec{E} + \vec{v} \times \vec{B} \right). \quad (24)$$

The test setup will be the same as before except that the plates will be massive and not charged. The plates will have the rest mass  $M_0$ . The gravitoelectric and the gravitomagnetic forces between the plates will be obtained similarly as for the electric and magnetic fields from the integral formulas. First, for the stationary plates it is:

$$F_{g0} = \frac{2\pi\kappa M_0^2}{A_0}, \quad (25)$$

and then for the moving plates in the parallel direction to the plates' surfaces, considering again the Lorentz contraction and also the inertial mass increase with the velocity:

$$F_g = \frac{2\pi\kappa M_0^2}{A_0} \frac{(1 - 2v^2 / c^2)}{\sqrt{1 - v^2 / c^2} (1 - v^2 / c^2)}. \quad (26)$$

The equivalent expressions for the potential energy is:

$$V_{g\parallel} = \frac{m_{g0} c^2 (1 - 2v^2 / c^2)}{(1 - v^2 / c^2)^{3/2}}. \quad (27)$$

The factor of two in the numerator parenthesis results from the factor of one half standing by the gravitomagnetic induction vector  $\vec{B}_g$  of the Lorentz force equation. Similarly, as derived in Eq.12, the equivalent expression for the kinetic energy is:

$$T_{g\parallel} = \frac{1}{2} \frac{c^2}{4\pi\kappa} \left( \frac{1}{2c} B_g \right)^2 = \frac{2\pi\kappa M^2 d_0 v^2}{c^2 A} = \frac{m_{g0} v^2}{(1-v^2/c^2)^{3/2}}, \quad (28)$$

The expressions for the field energies were again simplified using the abbreviation:

$$m_{g0} c^2 = \frac{2\pi\kappa M_0^2 d_0}{A_0}. \quad (29)$$

The total GEM energy is thus equal to:

$$W_{gem\parallel} = V_{g\parallel} + T_{g\parallel} = \frac{m_{g0} c^2}{\sqrt{1-v^2/c^2}}, \quad (30)$$

which is again a reasonable result that is expected.

When the plate's motion is in the perpendicular direction to the plates' surfaces the potential field energy is:

$$V_{g\perp} = \frac{2\pi\kappa M_0^2 d_0 \sqrt{1-v^2/c^2}}{A_0 (1-v^2/c^2)} = \frac{m_{g0} c^2}{\sqrt{1-v^2/c^2}}. \quad (31)$$

Following the same reasoning as for the EM case the kinetic field energy for the plate's motion in the perpendicular direction is:

$$T_{g\perp} = \frac{2\pi\kappa M_0^2 d_0 \sqrt{1-v^2/c^2}}{A_0 (1-v^2/c^2)} \frac{v^2}{c^2 - v^2} = \frac{m_{g0} v^2}{(1-v^2/c^2)^{3/2}} = T_{g\parallel}. \quad (32)$$

The total GEM energy for the plate's motion in the perpendicular direction is thus equal to:

$$W_{gem\perp} = V_{g\perp} + T_{g\perp} = \frac{m_{g0} c^2}{\sqrt{1-v^2/c^2}} + \frac{m_{g0} v^2}{(1-v^2/c^2)^{3/2}} = \frac{m_{g0} c^2}{(1-v^2/c^2)^{3/2}}. \quad (33)$$

The total GEM energy now depends on the motion direction of the plates, which is not acceptable.

$$W_{gem\parallel} \neq W_{gem\perp}. \quad (34)$$

This is not reasonable and it is a fatal problem for the theory that can be traced back to the relativistic mass dependence on velocity and to the Einstein weak equivalence principle (WEP) where the gravitational mass and the inertial mass are always identical. In the Maxwell EM field theory charge is an absolute invariant independent of velocity while in the Einstein's relativity the mass is variable. This problem thus questions the correctness of the whole concept of the gravitomagnetic force and consequently the Einstein field equations from which the GEM field equations are uniquely derived.

#### 4. Solution of the problem

This problem can be resolved by considering that there is no gravitomagnetic force and that the inertial mass and the gravitational mass of bodies depend on velocity differently as derived previously by Hynccek (2005).

$$M_i = M_0 / \sqrt{1-v^2/c^2}, \quad (35)$$

$$M_g = M_0 \sqrt{1-v^2/c^2}. \quad (36)$$

The gravitational mass dependency on velocity is consistent with the fact that photons, that move at the velocity  $c$ , have only the inertial mass and do not have any gravitational mass, Okun' at al. (1999).

For the potential field energy of the plates moving in the parallel direction to their surface it is thus:

$$V_{g\parallel} = \frac{2\pi\kappa M_0^2 d_0 (1-v^2/c^2)}{A_0 \sqrt{1-v^2/c^2}} = m_{g0} c^2 \sqrt{1-v^2/c^2} = W_{g\parallel}. \quad (37)$$

Since there is no gravitomagnetic field and the corresponding field energy for this motion direction, this is the total field energy. For the plates moving in the perpendicular direction to their surface the potential field energy is:

$$V_{g\perp} = \frac{2\pi\kappa M_0^2 d_0 (1-v^2/c^2)^{3/2}}{A_0} = m_{g0} c^2 (1-v^2/c^2)^{3/2}. \quad (38)$$

The kinetic field energy, following the similar derivation as shown in Eq32, is equal to:

$$T_{g\perp} = \frac{2\pi\kappa M_0^2 d_0 (1-v^2/c^2) \sqrt{1-v^2/c^2}}{A_0} \frac{v^2}{c^2 - v^2} = m_{g0} v^2 \sqrt{1-v^2/c^2}. \quad (39)$$

The total field energy for this motion direction is thus:

$$W_{g\perp} = m_{g0}c^2(1-v^2/c^2)^{3/2} + m_{g0}v^2\sqrt{1-v^2/c^2} = m_{g0}c^2\sqrt{1-v^2/c^2} = W_{g\parallel}. \quad (40)$$

The energy stored in the field is again independent of the direction of motion as is expected and as it should be. This fact confirms again the gravitation mass dependency on velocity as derived by Hynecek (2005). Of course there is also the energy associated with the inertial mass of the plates themselves, which is equal to:

$$W_i = \frac{2M_0c^2}{\sqrt{1-v^2/c^2}}. \quad (41)$$

The total energy of the moving massive parallel plates in any direction is thus equal to:

$$W_{gt} = \frac{2M_0c^2}{\sqrt{1-v^2/c^2}} + m_{g0}c^2\sqrt{1-v^2/c^2}. \quad (42)$$

In the previous case of the EM field the mass of the plates and the mass of charge were neglected.

## 5. Conclusions

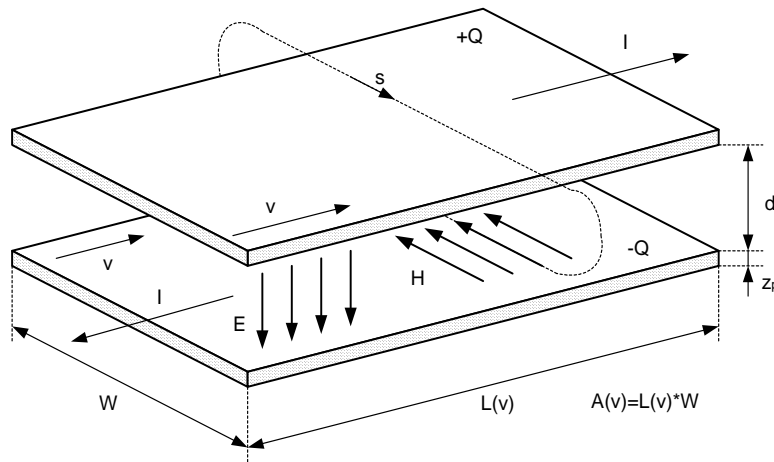
In this article it was clearly shown, based on simple principles, that there is no gravitomagnetic force and no gravitomagnetic field. The fundamental reason for this fact is the nature of the gravitational force, which has the unipolar character with only attractive forces between the massive bodies. The carriers of this force are most likely the spin zero gravitons. The gravitational waves thus seem to be the longitudinal aether compression and dilation waves. The second reason for the nonexistence of the gravitomagnetic force are the different dependencies of the inertial and gravitational masses on velocity.

This clearly contrasts with the EM field that is bipolar generated by both the positively and negatively charged particles with the unity spin photons as the force carriers. The photons propagate in the aether as the transversal waves. It is, therefore, misleading to draw the analogy for the gravitational field from the Maxwell EM field theory.

Finally, the paper also concludes that the Einstein field equations are most likely not correct, since the GEM field equations and the equation for the gravitomagnetic force are uniquely derived from them.

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**Figure 1.** Orientation of current  $I$  generated by the moving charged plates and of the resulting electric and magnetic fields. The magnetic field integration path  $s$  used in Eq.1 is also indicated.