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An Explanation of Inertia and Derivation of Mass-energy Law

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Abstract

A particle of charge Q moving at time t with velocity $\mathbf{v}$ and acceleration $\frac{d\mathbf{v}}{dt}$ generates an electrodynamic field of intensity $\mathbf{X}$. This field acts on the self-same charge, to produce the inertial force $XQ = -m\frac{d\mathbf{v}}{dt}$, equal and opposite to the accelerating force. A mass-energy formula, $E = \frac{1}{2}m\left(c^2 + v^2\right)$, is obtained for a particle of constant mass m moving with speed v.

Keyword: Acceleration, electric charge, field, force, inertia, mass, velocity.

1. Introduction

Newton's second law of motion defines force $\mathbf{F}$ in terms of acceleration $\frac{d\mathbf{v}}{dt}$ imparted to a particle of constant mass m, moving at time t with velocity $\mathbf{v}$, and acceleration $d\mathbf{v}/dt$ as:

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} \tag{1}$$

In this case, inertia, the tendency of a body to resist being accelerated or decelerated, becomes the reverse effective force, equal and opposed to the accelerating force. A moving charged particle carries along its electrostatic field $\mathbf{E}_o$ and is associated with a magnetic field $\mathbf{H}$ and, if accelerated, it generates an electrodynamic field.

2. Magnetic field due a moving electric charge:

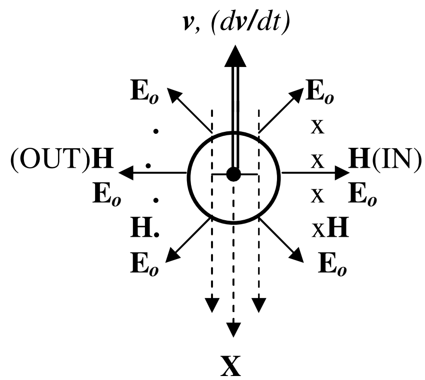
A moving charged particle is associated with a magnetic field $\mathbf{H}$, as shown in the Figure below. The magnetic field intensity $\mathbf{H}$ and magnetic flux intensity $\mathbf{B}$ are given by Biot and Savart law [1] of electromagnetism as the vector (cross) product;

$$\mathbf{B} = \mu_o\mathbf{H} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o \tag{2}$$

where μ_o is the permeability and ε_o the permittivity of a vacuum and $\mathbf{E}_o$ is the electrostatic field carried by the charge. Equation (2) becomes:

$$\mathbf{B} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o = -\mu_o\varepsilon_o\mathbf{v}\times\nabla\varphi \tag{3}$$

where $\mathbf{E}_o = -\nabla\varphi$ and φ is the electric potential at a point due to the charge as given by Coulomb's law and ∇ denotes the 'gradient' of a scalar quantity.



An electric charge Q and its electrostatic field $\boldsymbol{E_o}$ moving in a straight line with velocity $\boldsymbol{v}$ and acceleration *(d**v**/dt)* at time *t*, generating a magnetic field $\boldsymbol{H}$ (out of and into the page) and electrodynamic field of intensity $\boldsymbol{X}$ in the opposite direction of acceleration.

Vector transformation of equation (3) gives:

$$\mathbf{B} = -\mu_o \varepsilon_o \mathbf{v} \times \nabla \varphi = \mu_o \varepsilon_o \nabla \times \varphi \mathbf{v} \tag{4}$$

where $\nabla \times$ denotes the 'curl' of a vector quantity.

3. Electrodynamic field generated by an accelerated electric charge

If the charge undergoes acceleration, an electrodynamic fielde of intensity $\boldsymbol{X}$ is generated as given by Faraday's law of electromagnetic induction [2]:

$$\nabla \times \mathbf{X} = -\frac{d\mathbf{B}}{dt} \tag{5}$$

Equation (4) and (5) give:

$$\nabla \times \mathbf{X} = -\frac{d\mathbf{B}}{dt} = -\mu_o \varepsilon_o \nabla \times \varphi \frac{d\mathbf{v}}{dt}$$

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$$\mathbf{X} = -\mu_o \varepsilon_o \varphi \frac{d\mathbf{v}}{dt} = -\frac{\varphi}{c^2} \frac{d\mathbf{v}}{dt} \tag{6}$$

where $c = \sqrt{\dfrac{1}{\mu_o \varepsilon_o}}$ is the speed of light in a vacuum, a constant as determined by James Clerk Maxwell [3].

4. Inertial force on a charged particle

The fundamental assumption here is that the electrodynamic field $\boldsymbol{X}$ acts on the self-same charge Q to produce the inertial force $\boldsymbol{F}$, a reactive force which is equal and opposite to the accelerating force. Thus equation (6) and Newton's second law of motion give the inertial force $\boldsymbol{F}$ as:

$$\mathbf{F} = \mathbf{X}Q = -\frac{\varphi Q}{c^2} \frac{d\mathbf{v}}{dt} = -m \frac{d\mathbf{v}}{dt} \tag{7}$$

where mass of a particle m is considered as independent of its speed v.

5. Mass-energy equivalence law

Equation (7) gives mass m as:

$$m = \frac{\varphi Q}{c^2} = \frac{2w}{c^2} \tag{8}$$

$$w = \frac{1}{2} m c^2 \tag{9}$$

where $w = \varphi Q/2$ is the well-known formula for the electrostatic energy of a charge Q in an electrostatic (its own) potential φ. For a particle of mass m moving with speed v, the sum of the electrostatic and kinetic energies is:

$$E = \frac{1}{2} m \left(c^2 + v^2\right) \tag{10}$$

6. Inertial force on a body

A body of mass M is composed of an equal number or equal quantities of $N/2$ positive (+Q) and $N/2$ negative (–Q) electric charges whose electrostatic fields cancel out at any point in space. The electrodynamic fields, as expressed in equation (6), act on the respective charges of total mass $M = Nm$ so that equation (7) becomes:

$$\mathbf{X} N Q = -\frac{\varphi N Q}{c^2} \frac{d\mathbf{v}}{dt} = -Nm \frac{d\mathbf{v}}{dt} \tag{11}$$

Equation (9) becomes:

$$W = \frac{1}{2} M c^2 \tag{12}$$

where W is the total electrostatic energy of the charges constituting the body of mass M.

7. Conclusion

Newton's second law of motion, Coulomb's law of electrostatic force and basic electrostatic, electromagnetic and electrodynamic principles are employed to explain the origin of inertia (equation 7) and to derive equation (10) and (12), without recourse to the theories of relativity or any other principle. This explanation makes inertia a property residing in a body and electrical in origin. The derivation of the mass-energy law is quite straightforward but differs from the relativistic formula ($E = mc^2$) by a factor of one half.

7. References

[1] I.S. Grant & W.R. Phillips; *Electromagnetism*, John Wiley & Sons, New York (2000), p. 137-8.

[2] D.J. Griffith; *Introduction to Electrodynamics*, Prentice-Hall, Englewood Cliff, New Jersey (1981), pp. 257 – 260.

[3] J.C. Maxwell; *A Treatise on Electricity and Magnetism,* 3rd Ed. (1892), Part IV, Chap.2.