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On the Energy and Mass of Electric Charges in a Body

– A Unification of Electrostatic and Gravitational Forces –

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It is shown that the mass M of a distribution of N positive and negative electric charges $(\pm Q_i)$, constituting a body, is given by the sum:

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i U_i = 2\mu_o \varepsilon_o W = \frac{2W}{c^2}$$

where μ_o is the permeability, ε_o the permittivity and c the speed of light in a vacuum, U_i is the electrostatic potential at the position of Q_i the ith charge and W the electrostatic energy of the mass. The total energy E of mass M moving with speed v, is:

$$E = W + \frac{1}{2}Mv^2 = \frac{1}{2}M\left(c^2 + v^2\right)$$

This is in contrast to the relativity theory which makes $E = Mc^2$. The derivation of mass in terms of the electric charges in a body leads to an explanation of the origin of inertia and a unification of electrostatic and gravitational forces outside general relativity.

Keywords: Electric charge, energy, gravitation, inertia, mass, velocity, relativity.

Introduction

Einstein's [1, 2] most famous formula of special relativity, the mass-energy equivalence law, gives the energy content E of a particle of mass m and rest mass m_o moving with speed v, as:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

In this equation, which also expresses the mass-speed formula, the mass m of the particle and energy content E becoming infinitely large at the speed of light c is a difficulty. In spite of this problem, equation (1) is the most celebrated formula in the world.

Equation (1) is used by the proponents of special relativity to explain why an electron, the lightest particle known in nature, cannot be accelerated beyond the speed of light c. In fact, electrons are easily accelerated and have been accelerated to the speed of light, through a potential energy of 15 MeV or higher, as demonstrated by Bertozzi using a linear accelerator [3]. Cyclic electron accelerators (betatrons and electron synchrotrons) of over 100 BeV [4] have been built and operated, with electron speed equal to that of light c for all practical purposes. Such "massive" electrons would have increased the weight of the accelerator and, on impact, should have crushed through the target to cause a catastrophe. It is most likely that something else, other than increase of mass with speed, is responsible for restraining the electron, accelerated by an electrostatic field, from going beyond the speed of light c.

The author [5] shows that the mass m of a moving electron remains constant at the rest mass m_o and that it is the accelerating force exerted by an electrostatic field, on a moving charged particle, which decreases with the speed of the particle, becoming zero at the speed of light c. In this respect we have the ultimate speed without infinite mass.

The author [5] also shows that, for an electron of mass m and charge $-e$, revolving with constant speed v in a circle of radius r, under a central (radial) electrostatic field of magnitude E, the quantity "m" in equation (1) is the ratio $(eE/v^2)r$ of the magnitude of the force (eE) on a stationary electron, to the centripetal acceleration (v^2/r). The centripetal acceleration reduces to zero at the speed of light c. The radius r and the quantity $(eE/v^2)r$ may then become infinitely large, for motion in a circle of infinite radius (a straight line is an arc of a circle of infinite radius), without any problem of infinitely large masses at the speed of light.

The purpose of this paper is to derive expressions for the electrostatic energy and physical mass of a distribution of equal number of positive and negative electric charges constituting a neutral body of mass M. The total energy of the body of mass M moving with speed v, relative to an observer, is then deduced as the electrostatic energy of the mass plus its kinetic energy. The total energy content is then compared with the mass-energy equivalence law of the theory of special relativity [1, 2].

The value of mass M in terms of electrical quantities is then inserted in Newton's universal law of gravitation to obtain a unification of electrostatic and gravitational forces. This explains gravitation, as electrical in nature, without resorting to the four dimensional space-time continuum of the theory of general relativity.

2 Energy content of an electric charge distribution

If an isolated electric charge Q assumed any shape or configuration, it is most likely to be a spherical shell of radius a, with all the charges on the surface at the same potential, as in Figure 1(a). Such a figure has "force of explosion" as well as self energy or intrinsic energy due to the charge being situated in its own potential. The intrinsic energy of the charge Q in Figure 1(a) is the work w done by an external force in assembling the magnitude of the charge from 0 to Q as a spherical shell of fixed radius a. The intrinsic energy is always positive.

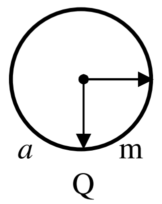

Figure 1(a). Uniformly charged spherical shell of fixed radius a, total charge Q and mass m

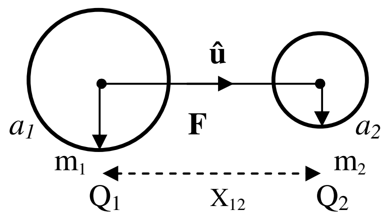

Figure 1(b). Two charges Q_1 and Q_2 distance X_{12} apart with force of repulsion $\boldsymbol{F}$ in the $\hat{\boldsymbol{u}}$ direction

For points outside a uniformly charged spherical surface, it is as if the whole charge is concentrated at the centre. The potential inside and on the spherical surface of radius a carrying a charge q, is $q/4\pi\varepsilon_o a$ and the work done in increasing the potential from 0 to U and building the charge from 0 to Q at a fixed radius a, by equal infinitesimal amounts (dq), is the intrinsic (self) energy w, given by the definite integral:

$$w=\int_0^Q \frac{q}{4\pi\varepsilon_o a}(dq)=\frac{Q^2}{8\pi\varepsilon_o a}=\frac{QU}{2} \tag{2}$$

Equation (2) shows that the intrinsic or self energy w is *½Q times the electrostatic potential U in which the charge Q is located*. The energy content is always positive and proportional to the square of the (positive or negative) charge.

Figure 1(b) shows two positive electric charges Q_1 and Q_2 of fixed internal radii a_1 and a_2 respectively, separated by a distance X_{12} in a body. The electrostatic energy w_2 of the charges is the sum of the intrinsic (self) energy of each charge being in its own potential and the extrinsic (mutual) energy due to one charge being in the electrostatic potential of the other, which is expressed in the equation:

$$w_2=\frac{Q_1^2}{8\pi\varepsilon_o a_1}+\frac{Q_2^2}{8\pi\varepsilon_o a_2}+\frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}}$$

$$w_2=\frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1}+\frac{Q_2}{4\pi\varepsilon_0 X_{12}}\right)+\frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2}+\frac{Q_1}{4\pi\varepsilon_o X_{12}}\right) \tag{3}$$

For 3 electric charges, Q_1, Q_2 and Q_3, of respective radii a_1, a_2, and a_3, separated by distances X_{12}, X_{13}, and X_{23} in a body, the total of the intrinsic (self) energies and extrinsic (mutual) energies is obtained as:

$$w_3=\frac{Q_1^2}{8\pi\varepsilon_o a_1}+\frac{Q_2^2}{8\pi\varepsilon_o a_2}+\frac{Q_3}{8\pi\varepsilon_o a_3}+\frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}}+\frac{Q_1 Q_3}{4\pi\varepsilon_o X_{13}}+\frac{Q_2 Q_3}{4\pi\varepsilon_o X_{23}}$$

$$w_3=\frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1}+\frac{Q_2}{4\pi\varepsilon_0 X_{12}}+\frac{Q_3}{4\pi\varepsilon_o X_{13}}\right)+$$
$$\frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2}+\frac{Q_1}{4\pi\varepsilon_o X_{12}}+\frac{Q_3}{4\pi\varepsilon_o X_{23}}\right)+$$
$$\frac{Q_3}{2}\left(\frac{Q_3}{4\pi\varepsilon_o a_3}+\frac{Q_1}{4\pi\varepsilon_o X_{13}}+\frac{Q_2}{4\pi\varepsilon_o X_{23}}\right)$$

$$w_3=\frac{1}{2}\{Q_1(U_1+\Lambda_1)+Q_2(U_2+\Lambda_2)+Q_3(U_3+\Lambda_3)\}$$

$$w_3=\frac{1}{2}(Q_1V_1+Q_2V_2+Q_3V_3)$$

Where the charges are positive and negative, the extrinsic potentials are positive and negative and may cancel out. It should be noted that the product *(Q_iV_i)* at any point, outside a charge *$(Q_i = 0)$*, is zero.

For a number N of charges in a body, the total electrostatic energy W is the sum:

$$W=\frac{1}{2}\sum_{i=1}^{N}Q_i\left(U_i+\Lambda_i\right)=\frac{1}{2}\sum_{i=1}^{N}Q_iV_i \tag{4}$$

where U_i is the intrinsic potential due Q_i the ith charge at the point of location of the ith charge, Λ_i is the total of the extrinsic potentials due to all the other charges (excluding the ith charge) at the position of the ith charge and V_i is the total electrostatic potential due to all the charges (including the ith charge) at the point of location of the ith charge. In a neutral body, containing an equal number of positive and negative electric charges, the products, Q_iU_i, are all positive and they add up. The extrinsic potentials Λ_i are positive and negative and their sum at a point may be zero, so that equation (4) becomes:

$$W=\frac{1}{2}\sum_{i=1}^{N}Q_i\left(U_i+\Lambda_i\right)=\frac{1}{2}\sum_{i=1}^{N}Q_iU_i \tag{5}$$

where U_i is the self potential of the charge Q_i. Equations (5) will be used to derive an expression for the energy content of a body of mass M.

3 Mass of an isolated electric charge

An isolated positive electric charge of magnitude Q moving in a straight line with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$ at time t, is associated with a circular magnetic field of intensity $\boldsymbol{H}$ and an electrodynamic field of intensity $\boldsymbol{E}_a$, as shown in Figure 2.

The magnetic flux intensity, $\boldsymbol{B} = \mu_o\boldsymbol{H}$, due to an electric charge of magnitude Q, with its electrostatic field of intensity $\boldsymbol{E}$, (Figure 2) moving at velocity $\boldsymbol{v}$ in free space or vacuum, is given as a vector (cross) product, by Biot and Savart law of electromagnetism [6], as a vector equation, thus:

$$\mathbf{B}=\mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}=-\mu_o\varepsilon_o\mathbf{v}\times\nabla\phi \tag{6}$$

where μ_o is the permeability and ε_o the permittivity of free space or vacuum, ϕ (a scalar) is the instantaneous electric potential at any point due to the charge and $\boldsymbol{E} = -\nabla\phi$, is the electrostatic field intensity, as given by Coulomb's law of electrostatics:

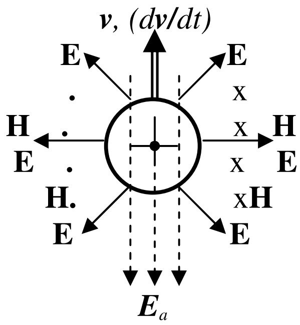

Figure 2 An isolated electric charge Q and its electrostatic field $\boldsymbol{E}$ moving in a straight line with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$ at time t, generating magnetic field of intensity $\boldsymbol{H}$ and electrodynamic field of intensity $\boldsymbol{E}_a$.

In Figure 2, the magnetic field ***H*** is out of the page on the left and into the page on the right. The electrodynamic field $\boldsymbol{E}_a$ points downwards, opposite to the direction of acceleration.

Vector transformation of equation (6) gives:

$$\mathbf{B} = -\mu_o\varepsilon_o\mathbf{v}\times\nabla\phi = \mu_o\varepsilon_o\nabla\times(\phi\mathbf{v})$$

Here ∇ denotes the "gradient" of a scalar quantity, $\nabla\times$ depicts the "curl" of a vector quantity and the curl of velocity $\boldsymbol{v}$, $(\nabla\times \boldsymbol{v}) = 0$. Faraday's law of electromagnetic induction [7] gives:

$$\nabla\times\mathbf{E}_a = -\frac{d\mathbf{B}}{dt} = -\mu_o\varepsilon_o\nabla\times\left(\phi\frac{d\mathbf{v}}{dt}\right)$$

$$\mathbf{E}_a = -\mu_o\varepsilon_o\phi\frac{d\mathbf{v}}{dt} \tag{7}$$

It is assumed that the electrodynamic field $\boldsymbol{E}_a$ (equation 7) acts on the self-same charge Q to produce the reactive force, reverse effective force or inertial force, equal and opposite to the accelerating force. For a particle of mass m, equation (7) and Newton's second law of motion give the reverse effective force or inertial force as:

$$\mathbf{E}_aQ = -\mu_o\varepsilon_oQU\frac{d\mathbf{v}}{dt} = -m\frac{d\mathbf{v}}{dt} \tag{8}$$

$$m = \mu_o\varepsilon_oQU \tag{9}$$

where U is the electrostatic potential due the charge Q at the location of the same charge.

For a uniform spherical shell of charge Q, radius a and mass m, equations (2) for intrinsic energy and equation (9) for mass, give:

$$w = \frac{Q^2}{8\pi\varepsilon_oa} = \frac{QU}{2} = \frac{m}{2\mu_o\varepsilon_o} = \frac{1}{2}mc^2 \tag{10}$$

$$m = \frac{\mu_oQ^2}{4\pi a} \tag{11}$$

where μ_o is the permeability, ε_o the permittivity and $c = (\mu_o\varepsilon_o)^{-1/2}$ is the speed of light in a vacuum, as discovered in 1873 by Maxwell [8].

5 Mass of a distribution of electric charges constituting a body

For a distribution of N electric charges (Q_1, Q_2, Q_3, …, Q_i… Q_N) composing a body, the total electrodynamic field $\boldsymbol{E}_i$ at the location of Q_i the ith charge is:

$$\mathbf{E}_i = -\mu_o\varepsilon_o(U_i + \Lambda_i)\frac{d\mathbf{v}}{dt} = -\mu_o\varepsilon_oU_i\frac{d\mathbf{v}}{dt} \tag{12}$$

where U_i is the intrinsic potential due the ith charge at the point of location of the ith charge, Λ_i is the total of the extrinsic potentials due to all the other charges (excluding the ith charge) at the position of the ith charge, The extrinsic potentials are positive and negative cancelling out at a point to give equation (8).

The idea put forward here is that the total electrodynamic field $\boldsymbol{E}_i$ (equation 12) acts internally on the self-same charge Q_i to produce the reactive force, reverse effective force or inertial force, equal and opposite to the accelerating force. For a distribution of N electric charges composing a body of mass M, equation (12) and Newton's second law of motion give the reverse effective force or inertial force as:

$$\sum_{i=1}^{N} \mathbf{E}_i Q_i = -\mu_o \varepsilon_o \sum_{i=1}^{N} Q_i U_i \frac{d\mathbf{v}}{dt} = -M \frac{d\mathbf{v}}{dt}$$

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i U_i \tag{13}$$

For a rigid neutral body of mass M containing $N/2$ positive electric charges and N/2 negative electric charges moving with the same acceleration, *(d**v**/dt)*, the total electrodynamic field generated at an external point, comes to zero. This is obvious as the constituent electric charges generate equal and opposite fields. Combining equation (5) with equation (13) gives:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i U_i = \frac{1}{2}\frac{M}{\mu_o \varepsilon_o} = \frac{1}{2} M c^2 \tag{14}$$

According to equation (14), the work done W in creating a distribution of charges constituting a body of mass M, is $W = ½Mc^2$, equal to the electrostatic energy of the mass. Where mass is independent of speed, the kinetic energy of a body of mass M, moving with speed v, in accordance with classical mechanics, is $K = ½Mv^2$. The total energy content E, is:

$$E = W + K = \frac{M}{2}\left(c^2 + v^2\right) \tag{15}$$

Equations (13), (14) and (15) are what this paper set out to derive..Equation (15) is in contrast to the mass-energy equivalence law of special relativity [1, 2] as expressed in equation (1).

5 A unification of Electrostatic and Gravitational Forces

The law of force of attraction between masses in space was discovered by the English physicist Sir Isaac Newton around 1687 [1]. Newton's universal law of gravitation states:

> *"Every object in the universe attracts every other object with a force that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects".*

Mathematically, the gravitational force of attraction $\boldsymbol{F}_G$ between two objects of masses M_1 and M_2 in space separated by a distance Z, between their centres of mass, is expressed as:

$$\mathbf{F}_G = -G\frac{M_1M_2}{Z^2}\hat{\mathbf{u}} \tag{16}$$

where $\hat{\boldsymbol{u}}$ is a unit vector pointing in the direction of the force of repulsion, opposite to the force $\boldsymbol{F}_G$ and G is the gravitational constant.

Substituting for the masses M_1 and M_2 from equation (13) into equation (16), gives the the gravitational force of attraction as product of sums, thus:

$$:\mathbf{F}_G = -\frac{G}{Z^2}M_1M_2\hat{\mathbf{u}} = -\frac{G}{Z^2}\left(\mu_o\varepsilon_o\sum_{i=1}^{N_1}Q_iV_i\right)\left(\mu_o\varepsilon_o\sum_{i=1}^{N_2}K_iP_i\right)\hat{\mathbf{u}} \tag{17}$$

where U_i and P_i are the intrinsic potentials at the respective locations of the charges Q_i and K_i

5 Conclusion

The derivation of equations (9) and (13) gives a clarification for the origin of inertia as electrical and internal to an accelerated or decelerated body. This is a new and important description in physics, particularly electrodynamics.

Equation (11), expressing the mass of a charged particle, in the form of a spherical shell of radius *a*, is instructive. If the electric charge remains constant with speed of the particle, it is reasonable to conclude that the mass should similarly remain constant, contrary to special relativity as expressed in equation (1).

Equations (1) and (15), for a stationary particle, differ by a factor of *2*. However, each equation gives a body of rest mass M_o as the source of a tremendous amount of energy locked up in the particles. As to which equation is correct, remains to be decided. If the decision is in favour of equation (15), then it would have a tremendous impact in redirecting the course of modern physics.

In equation (1) the kinetic energy is contained in the increase of mass of the particle, which becomes infinitely large at the speed of light *c*. In equation (15), mass *M* remains constant at the rest mass M_o and the kinetic energy reaches a maximum value equal to $½M_oc^2$ at the speed of light *c*. Thus, bodies may be accelerated to the speed of light, without mass becoming infinitely large.

The mass-energy equivalence formula, as given by equations (10) and (15), is more realistic than equation (1) for a particle, such as an electron, that can easily be accelerated to the speed of light *c*. According to equation (15), the maximum total energy content of an electron of mass *m*, moving with the speed of light *c*, is $E_m = mc^2$. Such an electron is easily brought to rest, losing kinetic energy equal to $½mc^2$ but retaining it electrostatic energy $½mc^2$. The electron can impinge on a target and may recoil without causing any damage. If the mass were infinite, its impact on a target would have been destructive.

In equation (17) a unification of electrostatic and gravitational forces is achieved. Gravitational forces are shown to be electrical in nature and the forces manifest in space by virtue of presence of bodies as centres of energy. This should put to rest the idea of warping of four dimensional space-time continuum, in the presence of matter, to make for gravitation.

We conclude that mass (equations 11 and 13) is not a fundamental quantity; electric charge is more significant..The four fundamental quantities of measurements are better put as *Length (L)*, *Time (T)*, *Electric Charge (Flux) (Q)* and *Electric Potential (V)*. This system, the *(Metre–Second–Coulomb–Volt)* system, gives the dimension of *Area (A)* as *[L^2]*, Volume

($\forall$) is *[L^3]*, *Mass (M)* as *[$L^{-2}T^2QV$]*, that of *Permittivity (ε)* becomes *[$L^{-1}QV^{-1}$]*, *Permeability (μ)* is *[$L^{-1}T^2Q^{-1}V$]*, M*agnetic Field (H)* is *[$L^{-1}T^{-1}Q$]*, *Magnetic Flux (Ψ)* is *[TV]*, *Resistance (R)* is *[$TQ^{-1}V$]*, *Resistivity (ρ)* is *[$LTQ^{-1}V$]*, *Conductivity (σ)* is *[$L^{-1}T^{-1}QV^{-1}$]*, *Capacitance (C)* is *[QV^{-1}]*, *Inductance (L)* is *[$T^2Q^{-1}V$]*, *Force (F) is [$L^{-1}QV$]*, *Energy (E)* is *[QV]*, *Power (P)* is *[$T^{-1}QV$]*,, *Momentum (p)* is *[$L^{-1}TQV$]* and *Angular Momentum (L)* is *[TQV]*. *Gravitational Constant (G)* is *[$L^5T^4Q^{-1}V^{-1}$]*, *Gravitational Field (Γ)* is *[LT^{-2}]*. There is no fractional exponent of a fundamental quantity in the dimension of any derived quantity.

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