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# Note

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# Brief History of Bohr Magneton

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Content:



11Note.Hist.Bohr mag. II

# Timeline

**The Bohr magneton ($\mu_B$) is the magnitude of the elementary magnetic dipole moment of an orbiting electron with an orbital angular momentum of one $\hbar = h/2\pi$.**

● 1789 Quantitative magnetic moments were introduced by Charles-Augustin de Coulomb for compass needles [1].

● 1820 Hans Christian Orsted discovered the relationship between electricity (electric current) and magnetism [2].

● 1821 Andre-Marie Ampere. The hypothesis of molecular currents (Amperiancurrents) [3] as a link between electricity and magnetism (microscopic scale). By default, emerged the concept of magnetic moment of area,

$$\mu \propto IA \qquad (1)$$

● 1832-1895 Develop systems of measurement units in electromagnetism (Gauss,[4], Weber,[5], Maxwell,[6]).
Hertz [7] combines the electrostatic CGS system of units (esu) with electromagnetic CGS system of units (emu) into a single system, related by the speed of light c, called the "Gaussian system of units". In the system of units, used mainly in theoretical physics, magnetic moment has been classified in the category of magnetic quantities (by a factor of units 1/c, unrelated to the expression of the magnetic moment),

$$\mu = \frac{1}{c}\, IA \quad \text{(emu).} \qquad (2)$$

The factor of units $1/c = 1/3\times 10^{10}$ *s/cm*, experimentally determined, made the transition from the electric current expressed in electrostatic units (esu) to the electric current expressed in electromagnetic units (emu).

$$\frac{1}{c} I_{esu} \rightarrow I_{emu}\,. \qquad (3)$$

● 1903 J.J. Thomson. Magnetic properties of corpuscles describing circular motion hypothesis [8].

● 1907 W. Ritz: The idea of elementary magnets [9].

● 1907 P. Weiss explains ferromagnetism by way of small domains of magnetic polarization within a material (Weiss experimental magneton, $\mu_W$), [10], [11],

$$\mu_W = 1123.5 \text{ emu/atom-gram.} \qquad (4)$$

The term “magneton” is due to P. Weiss.

● 1911 P. Langevin obtained submultiple of experimental magneton [12].

● 1911 R. Gans computed a value that was twice as large as the Bohr magneton [13].

● 1912 S. Procopiu obtained for the first time the relationship and default, value magneton [14], [15]. This relationship and the numerical value is referred to “Bohr-Procopiu magneton” in Romanian scientific literature,

$$h = 4\pi\mu \frac{m_e}{e} \quad . \qquad (5)$$

Using the numerical values of fundamental physical constants of work [13], ($h = 6.55\times10^{-27}$ erg.s ; $e = 4.7\times10^{-10}$ esu; $m_e = 0.84\times10^{-27}$g; $c = 3\times10^{10}$ cm/s), result (implicit), in emu units,

$$\mu = \frac{1}{c}\,\frac{eh}{4\pi m_e} = 9.7\times10^{-21} \text{ emu} \qquad (6)$$

from $9.27\times10^{-21}$ emu, the current value.

● 1913 N. Bohr [16], on the basis of Rutherford’s atomic model, developed quantified theory of the atom, creating the basis for the interpretation of magnetism at atomic scale.

● 1915 A.L. Parson [17] suggests that the electron is not only electric charge but is also a small magnet (or „magneton” as he called it). In the literature is known as „Parson magneton” or „magnetic electron”. This led to the Parson’s model of atom.

● 1918 H.S. Allen [18]. The arguments in favour of an electron in the form of a current circuit capable of producing magnetic effect.

● 1920 W. Pauli introduced the elementary unit of magnetic moment, interpretation in terms of the Bohr’s atom [16], [19]. From the classical expression for magnetic moment $\mu = (1/c)IA$, and considering the effective current $I = - e/T = - ev/2\pi r$, which can be rewritten as $I = - em_e vr/2\pi m_e r$, the magnetic moment result, $\mu = -(e/2m_e)L$, where angular momentum $L$, is quantified, conformable Bohr-Sommerfeld theory, $L = n\,(h/2\pi)$,

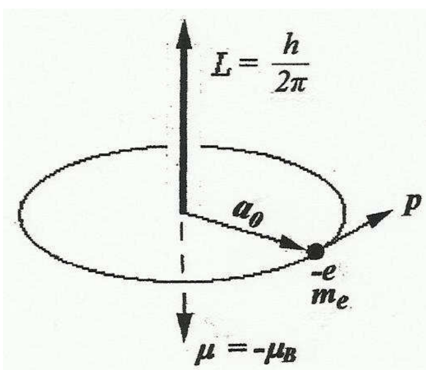
$n = 1, 2, 3, \ldots, n$. A elementary unit of magnetic moment (for $n = 1$), called the "Bohr magneton" is

$$\mu_B = \frac{1}{c}\frac{eh}{4\pi m_e} = 9.21\times10^{-21} \text{ emu}, \quad (\text{erg.gauss}^{-1}) \tag{7}$$

or related to the mol,

$$\mu_B = \frac{1}{c}\frac{eh}{4\pi m_e} N_0 = 5548 \text{ Gauss.cm}, \tag{8}$$

$N_0$ being the number of atoms or molecules per gram-atom, ( $N_0 = 6.06\times10^{23}\ mol^{-1}$) and the *1/c* factor being for conversion *esu→emu*, (Fig.1)




**Notation**
Bohr radius ($a_0$) = $r_1$ = $0.529\times10^{-8}$cm

**Fig.1** The vector model of Bohr's magneton (implicit) in connection with the orbital motion of electron

● 1924-1928 On the basis of quantum theory developed by W. Pauli [20], S. Goudsmit & G.E. Uhlenbeck, [21], W. Heisenberg & P. Jordan [22] (the quantum theory of angular momentum and spin 1/2) and PAM. Dirac [23], Bohr magneton came in the expression of spin magnetic moment ($\boldsymbol{\mu}_\mathbf{S}$), and its rotational component ($\boldsymbol{\mu}_\mathbf{S}^\mathbf{z}$) Fig.2

$$\mathbf{S} = \sqrt{s(s+1)}\ \frac{h}{2\pi} = \frac{\sqrt{3}}{2}\frac{h}{2\pi}, \quad (s=1/2), \tag{9}$$

$$\mathbf{S_z} = \pm\frac{1}{2}\frac{h}{2\pi}, \tag{10}$$

$$\boldsymbol{\mu}_\mathbf{S} = -\,g\frac{e}{2m_e}\mathbf{S} \approx -\sqrt{3}\,\mu_B \quad (g \approx 2, \text{Dirac}), \tag{11}$$

$$\boldsymbol{\mu}_\mathbf{S}^\mathbf{z} = \pm\frac{1}{2}g\mu_B, \quad (\approx \pm\,\mu_B). \tag{12}$$

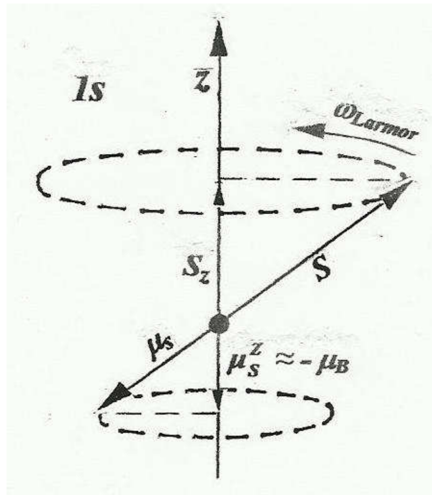



**Fig.2** The entity spin angular momentum – spin magnetic moment

● 1929 R.T. Birge includes the Bohr's magneton in the physical constants table [24]:

Magnetic moment of 1 Bohr magneton (spectroscopic).......$\mu_1 = (h/4\pi)(e/m)_{sp} = (0.9174_{70} \pm 0.0013) \times 10^{-20}$ erg.gauss$^{-1}$
Magnetic moment of 1 Bohr magneton (deflection).......$\mu_1 = (h/4\pi)(e/m)_{defl} = (0.9216_{38} \pm 0.0016) \times 10^{-20}$ erg.gauss$^{-1}$

●1940-1955 On the basis of quantum mechanics, magneton theory gets a new interpretation, more elaborate (Fig.3a and 3b):

The current density (or flux probability) in atom, if electron is in stationary state $(L_z = m\, h/2\pi)$,

$$J = -\frac{ieh}{4\pi m_e} (\psi \nabla \Psi^* - \Psi^* \nabla \Psi). \tag{13}$$

In polar coordinates we have (Fig.3a),

$$J_r = -\frac{ieh}{4\pi m_e} \left(\psi \frac{\partial \psi^*}{\partial r} - \Psi^* \frac{\partial \psi}{\partial t}\right) = 0\,, \tag{14}$$

$$J_\theta = -\frac{ieh}{4\pi m_e r} \left(\psi \frac{\partial \psi^*}{\partial \theta} - \Psi^* \frac{\partial \psi}{\partial \theta}\right) = 0\,, \tag{15}$$

$$J_\varphi = -\frac{ieh}{4\pi m_e r \sin\theta} \left(\psi \frac{\partial \psi^*}{\partial \varphi} - \Psi^* \frac{\partial \psi}{\partial \varphi}\right) = m \frac{eh}{2\pi m_e r \sin\theta} |\Psi|^2 . \tag{16}$$

The intensity of electric current $dI$ through $d\sigma$ section is

$$dI = J_\varphi \, d\sigma . \tag{17}$$

The magnetic moment of this electric current is

$$d\mu_z = \frac{1}{c}\, dI\, A = \frac{1}{c}\, J_\varphi\ d\sigma\ A\,. \qquad (18)$$

Because $A$ is equal to $\pi r^2 \sin^2\theta$ (Fig.3a), we have,

$$d\mu_z = \frac{1}{c}\ \pi r^2 \sin^2\theta\ J_\varphi\ d\sigma = -\ m\ \frac{1}{c}\ \pi r^2 \sin^2\theta\ \frac{eh}{2\pi m_e r \sin\theta}\ |\Psi|^2\ d\sigma\,. \qquad (19)$$

The magnetic moment is obtained by adding the elementary tubes,

$$-\ m \frac{1}{c}\ \frac{eh}{4\pi m_e} \int\ 2\pi r \sin\theta\, d\sigma\ |\Psi|^2 = -\ m\ \frac{1}{c}\ \frac{eh}{4\pi m_e} = -\ m\ \mu_B\,, \qquad (20)$$

where,

$$\mu_B = \frac{1}{c}\ \frac{eh}{4\pi m_e} = 9.274 \times 10^{-21} emu \text{ (and same time Gaussian units).} \qquad (21)$$

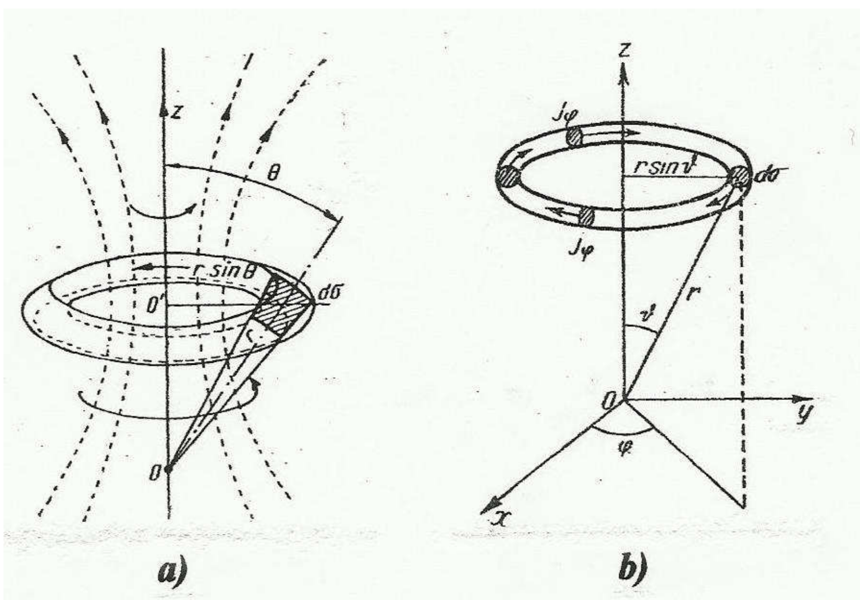



**Fig.3** Two epoch sketch of interpretation of Bohr's magneton in quantum mechanics: a) after D.I. Blohintev [25]; b) after E.V. Spolschi [26]

● 1970-2006 In textbooks and papers, recommendations on mandatory use of International System of Units (SI) [27] are noticed more often. So,

$$\mu_B = \frac{eh}{4\pi m_e} = 927.400\ 915(23)\times 10^{-26}\ JT^{-1} \quad (22)$$

were:

$e = 1.602\ 176\ 487(40) \times 10^{-19}$ C ,
$h = 6.626\ 068\ 96(33) \times 10^{-34}$ J.s ,
$m_e = 9.109\ 382\ 15(45) \times 10^{-31}$ kg ,

the 1/c factor is included in the amount of electric charge.
Bohr magneton is expressed in units derived from SI, such:

$\mu_B /\{e\} = 5.788\ 381\ 75555(79) \times 10^{-5}$eV $T^{-1}$ ,
$\mu_B / h = 13.996\ 246\ 04(35) \times 10^{9}$ Hz $T^{-1}$ ,
$\mu_B / hc = 46.686\ 4515(12)$ $m^{-1}$ $T^{-1}$
$\mu_B / k = 0.671\ 7131(12)$ $KT^{-1}$.

● 1978-2010 D.P. Dănescu [28], [29], shows that the only system of units without contradictions is the Gaussian system. It is indicated for use in quantum theory and relativity, or, more generally, in theoretical physics.
An amendment related to the expression of the magnetic moment (IdA) was made to the Gaussian system of units. The „current element" (Idℓ) is of electrokinetic nature and therefore should not contain the factor 1/c, disposal changing the measure unit of magneton: *emu* $\rightarrow$ *esu*. (See Appendix 1)
In other words, the relationship of Bohr magneton, recommended to be expressed in Gaussian units, must be,

$$\mu_B = \frac{eh}{4\pi m_e} = 2{,}780\ 278\ 10(24)\times 10^{-10}\ \text{esu} \quad (23)$$

where:
$e = 4.803\ 204\ 41(41) \times 10^{-10}$ esu,
$m_e = 9.109\ 382\ 6(16) \times 10^{-28}$ g,
$h = 6.626\ 069\ 3(11) \times 10^{-27}$ erg.s.

Factor 1/c does not disappear. He is transferred to the torque's relationship, where just like the forces, it highlights the relativistic nature of magnetism. This amendment is denoted schematically as follows:

$$\mu = \frac{1}{\text{¢}} \, I\mathbf{A} \qquad\qquad \tau = \frac{1}{c} \, I\mathbf{A} \times \mathbf{B} \tag{24}$$

$$\downarrow \underline{\qquad\qquad\qquad\qquad} \uparrow$$

Under these conditions, calculating the spin magnetic moment of electron , we get:

$$|\boldsymbol{\mu}_{\mathbf{s}}| = g\frac{e}{2m_e}\,\mathbf{S} \approx \sqrt{3}\,\frac{\text{eh}}{4\pi m_e} = \sqrt{3}\,\mu_B = 4{,}81\times 10^{-10} \text{ ues}, \tag{25}$$

where $g \approx 2$ and $\boldsymbol{S} = \sqrt{3}\,\hbar/2$, the measure unit can be written in the form $Fr \,.\, cm^2/s$. Besides electric charge value (e) and double Compton wavelength $(2\lambda_C)$, [31], these constants are in an equality value (approximate) of exceptional importance, equal to a one-dimensional system (by transition LTM Gaussian $\rightarrow$L) and can be geometrically interpreted [32]:

$$|\boldsymbol{\mu}_{\mathbf{s}}| = 4.81\times 10^{-10} \text{ esu},$$
$$|\mathbf{e}^{\pm}| = 4.80\times 10^{-10} \text{ esu},$$
$$2\lambda_C = 4.85\times 10^{-10} \text{ cm}.$$
$$\downarrow$$
$$2\lambda_C \approx |e^{\pm}| \approx |\boldsymbol{\mu}_{\mathbf{s}}| \approx 4.8\times 10^{-10} \text{ cm. [L]}, \tag{26}$$

$$\downarrow$$

$$|e^{\pm}| \approx \frac{8\pi}{\sqrt{3}c} \quad [L]. \tag{27}$$

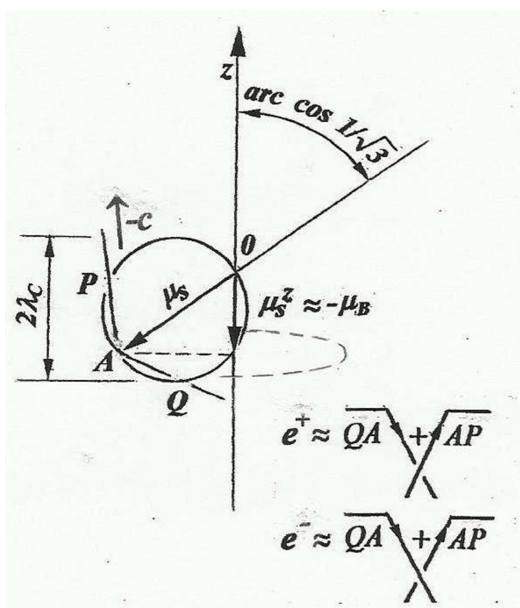



**Fig.4** Interpretation of equalities $2\lambda_C \approx |e^{\pm}| \approx |\mu_s| \approx 4.8 \times 10^{-10}$ *cm. [L]*, in connection with the spin motion, after D.P.Danescu (see Appendix 2)

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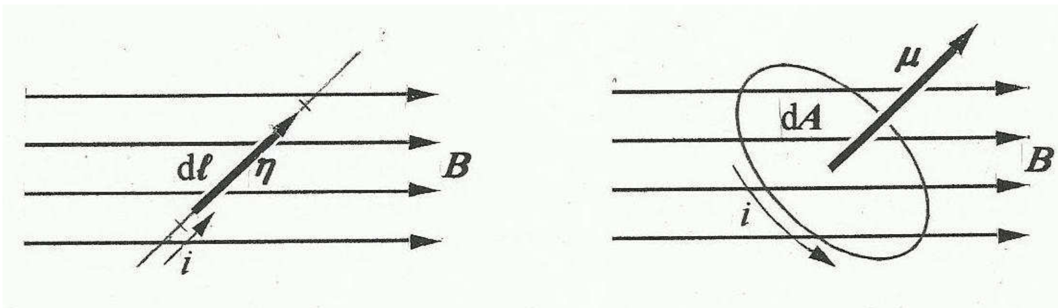
# An amendment to the Bohr magneton unit (in Gaussian system of units)

*Comparison of the current element and the magnetic moment follows*




| $\boldsymbol{\eta} = i\mathbf{d}\boldsymbol{\ell}$ *(The current element)* | $\boldsymbol{\mu} = i\mathbf{d}\mathbf{A}$ *(The magnetic moment)* |
|---|---|
| $d\mathbf{F} = \frac{1}{c}\, i\mathbf{d}\boldsymbol{\ell}\times\mathbf{B}$ *(The Laplace force)* | $d\boldsymbol{\tau} = \frac{1}{c}\, i\mathbf{d}\mathbf{A}\times\mathbf{B}$ *(The torque)* |

*The quantities $\boldsymbol{\eta} = i\mathbf{d}\boldsymbol{\ell}$ and $\boldsymbol{\mu} = i\mathbf{d}\mathbf{A}$, introduced by definition, have geometric and electrokinetic nature not electrodynamic nature. They are without 1/c factor of units, transferred in the force respectively in the torque expression .*

*Consequence:*

*I. The Bohr magneton is* $\mu_B = \frac{e\hbar}{2m_e} = 2.780277 \times 10^{-10}$ *esu, and not*

$$\mu_B = \frac{1}{c}\frac{e\hbar}{2m_e} = 9.274009 \times 10^{-21} \text{ emu;}$$

*II. The spin magnetic moment is* $|\boldsymbol{\mu}_s| = g\frac{e}{2m_e}\mathbf{S} = 4.821167 \times 10^{-10}$ *esu,*

*and not* $|\boldsymbol{\mu}_s| = g\frac{1}{c}\frac{e}{2m_e}\mathbf{S} = 1.608168 \times 10^{-20}$ *emu.*

# The spin motion and Bohr magneton

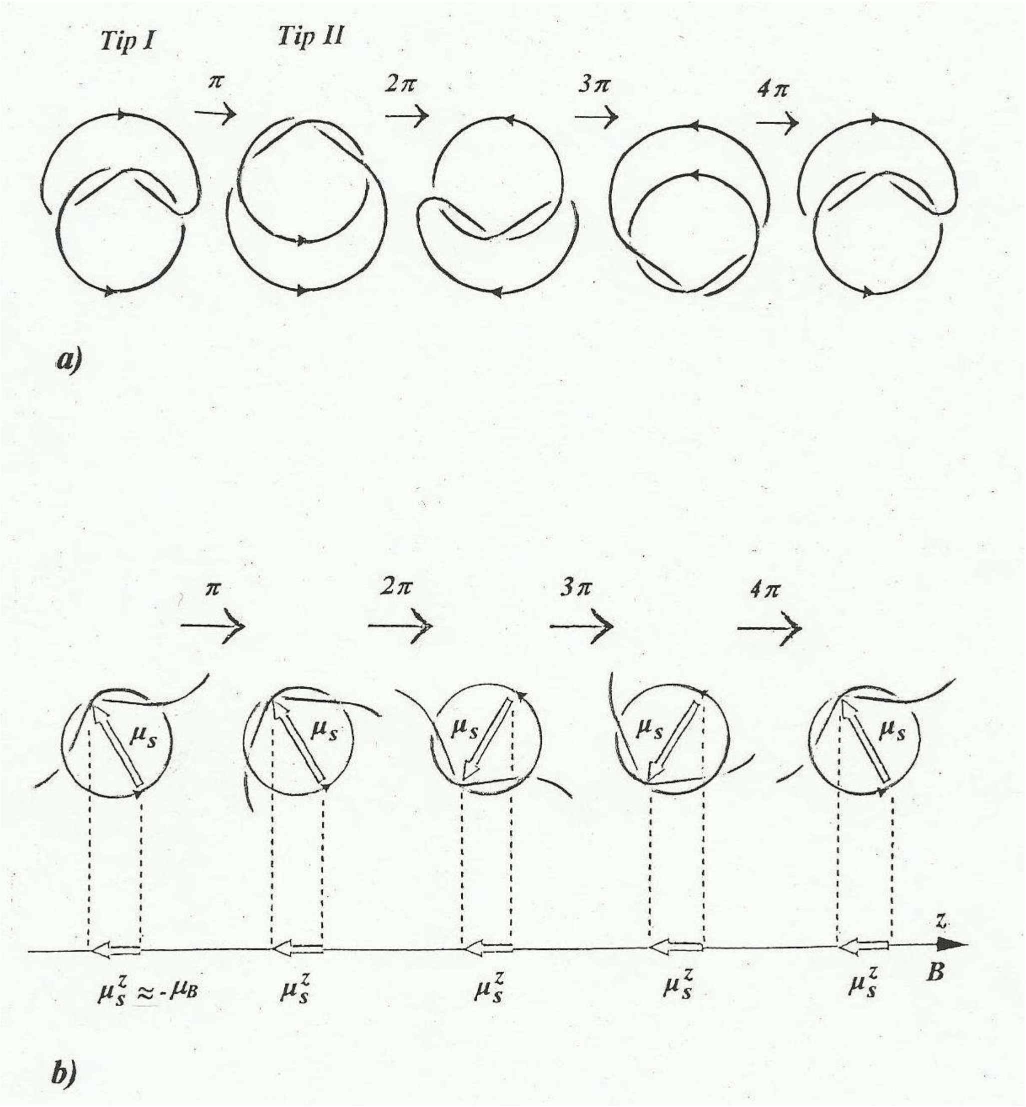



Illustration of the radial spin motion, as an internal double twist, starting from trefoil knot:

a) The spin motion with $4\pi$ internal rotations in the case of trefoil knot.

b) The spin motion with $4\pi$ rotations in the case of elastic overhand knot (open trefoil knot) identified with extended quantum electron (charge). Vector $\boldsymbol{\mu_s}$ projection on the $0z$ axis (rotational component) is permanently $\boldsymbol{\mu_s^z} \approx -\mu_B$ .