Motion's Observation Through Light's Signals (I)

C. P. Viazminsky[1] and P. K. Vizminiska[2]
[1]Department of Physics, University of Aleppo, Syria
[2]Department of Computer Engineering, University of Detroit Mercy, Michigan

Abstract

A novel view of space, time, and inertial frames that maintains the absolute nature of time is presented. One but arbitrary inertial frame say *S*, is considered stationary and identified with the absolute space, while all other inertial frames are moving relative to *S*. The constancy of the light's velocity in free space within each inertial frame is postulated and employed to link time durations measurements to geometric distance. The geometric distance in the chosen stationary frame plays the decisive role in the determination of time and distance in all inertial frames. A unique time prevails in all inertial frames, but distance between a moving object in *S* and a stationary observer in *S* is identified by the optical length of a light trip from the object to the observer; this distance functions as the geometric distance in the frame in which the object is at rest when the latter frame is considered stationary. The arbitrariness of the chosen stationary frame guarantees that all inertial frames are equivalent, and according the physical laws are the same in all. The so-called scaling transformations which relate the geometric distances in *S* and in a moving frame are derived and applied to explain the Doppler's effect and the lifetime of meta-stable particles phenomenon. The quantitative predicted Doppler's effect, which is in a striking agreement with the Ives-Stilwell experimental results, coincides with the relativistic prediction for longitudinal motion, but yet predicts complete absence of a traverse effect. The direction of the light trip is observed from a moving frame to be tilted from its direction in the stationary frame by the aberration angle; a fact which is employed to explain the phenomenon of stellar aberration. The true status of the Lorentz transformations as an equivalent form of the scaling transformation is illuminated. In a forthcoming part of this work, a second type of scaling transformations corresponding to given beginning and end of a light's trip in a stationary frame is derived and employed to explain the Michelson and Morley experiment, the Michelson and Gale experiment, and the Sagnac effect. The translative nature of the latter effect is explored and studied in detail. Moreover, the numerical results predicted by the scaling theory, in regards of the pioneer anomaly, are within the range of the observed data. A fuller account illuminating that there is nothing anomalous in this effect will be discussed separately.

Key words: contiguity, universal space, scaling transformations, illusive Lorentz transformations.

1.Introduction

The Newtonian conceptions of space and time are modified to incorporate observations through light's signals. The absolute Newtonian nature of distance and time is carried over to what we call universal (or optical or true) time and distance. In contrast with the geometric distance (and geometric time) which are frame dependent, the universal time and distance are absolute. The universal distance between an object and an observer, which depends on their relative velocity, is determined in terms of the familiar geometric distance by means of what we call the scaling transformations. Our novel conception of space and time is characterized by the following:

(i)Time measurements are linked to spatial measurements through a constant light's velocity, which is postulated to hold within every inertial frame.
(ii) One arbitrary inertial frame S is considered stationary and identified with the absolute space, while all other inertial frames are moving relative to S. The absolute space may be corresponded with any other inertial frame, say s, without any bearing on the transformations that relate geometric distances in S and s regardless of which frame we choose to be stationary.

Based on the above requirements and the implicit assumption regarding the Euclidean nature of the absolute space we employ the geometric distance in the stationary frame to endow any other inertial frame s, through light's signals, with distance and time intervals such that the speed of light is also c within s.

The chart of logic that leads to the transformations between the geometric distances in the two frames can be summarized as follows:

- We start by identifying one arbitrary inertial frame S, which is initially coordinated using a given unit of geometric length, with the absolute space. Utilizing the postulate that light propagates rectilinearly in all directions within (inside) S at a constant speed c we set up a global time in S using the familiar procedure of clock synchronization [1]. A frame such as has been described will be called a *timed or universal frame*.
- If b is source of light moving at a constant speed $\vec{u}$ in S then the scaling theory determines the *universal (or optical or true) duration t* of a light's trip $(b \ \ at\ B \in S \rightarrow O \in S)$ in terms of its *geometric length* $|\overrightarrow{BO}| = R$. It is t what is measured for the latter trip in the universal frame S and in any other inertial frame. We shall see that the optical duration t is generally different from the geometric duration of the trip in S, which is by definition $T \equiv R/c$; they are equal however when the source of light is at rest in S.
- A basic concept of the scaling theory is that, time flows equably in all inertial frames, and in particular in S and in the frame s in which the source of light b is stationary. If o is an s-observer which is contiguous to the S-observer O when light arrives at O, then the light's trip $(b \in s\ at\ B \in S \rightarrow O \in S)$ in S is the same as the light's trip $(b \in s\ at\ B \in S \rightarrow O \in S\ and\ o \in s)$, which occurs conclusively within s, and hence its duration is the same in S and s. When looking at this trip from the stationary S, the Galilean law of velocity addition is employed to calculate its universal (or optical) duration t which must be the same in s. In order to get velocity of light also c in s, we should associate with this trip the universal length $r = ct$ in s. Because the source of light b is at rest in s, the geometric length of the latter trip in s, when s is considered stationary, is identical to its optical length r.
- In the same way by which the geometric length R in S of the above light trip gives rise to the optical length r, which is identified by the geometric length of the trip in s, the geometric length r of the virtual trip $(B \ \ at\ b \rightarrow o)$ in s, with B is the source of light, should induce when s is considered the stationary frame an optical length of the trip in s that is equal to its geometric length R in S. It useful to stress that geometric duration and length of a light's trip are simply two equivalent measures of the same quantity which is either length or duration.
- Since every inertial frame can be identified with the absolute space, the transformations we seek should be the same whether we identify S or s by the absolute space.

The implementation of the above view of space and time yields an anisotropic transformations, called the scaling transformation of first type (STI), between the geometric and optical characters of a light trip in *S,* or equivalently, between the geometric distances in S and *s.* The STI will be employed to explain the meta-stable particles' lifetime phenomenon, Doppler's effect, drag effect and aberration. The scaling transformations of second type (STII) are concerned with the case in which the beginning and the end of the trip are known in S and in s from start. Although neat explanations, based on the scaling theory, of the drag effect, Sagnac effect, Michelson and Morley experiment, Michelson and Gale experiment, Doppler's effect, and the stellar aberration had already appeared in earlier works [2V,VI,VIII,IX,X], more profound explanations of these effects and experiments will be presented here and in forthcoming parts of this work. The pioneer effect and the possibility of non-expanding universe will be discussed separately.

2. Global Time

Consider an arbitrary inertial frame $S \equiv OXYZ$. The coordinates system is assumed to be already calibrated using a given unit of length, say *LS*. The existence of the Cartesian system of coordinates *OXYZ* in *S* requires an implicit assumption that the geometry of the space is Euclidean [3]. The geometric distance between two points $A \in S$ and $B \in S$, or geometric length of a rod *AB* stationary in *S*, refer to the result L_{AB} obtained by laying the unit of length *LS* in *S* along the rod repeatedly from one end till reaching the other by multiples and fractions of *LS*. This can be done rationally (or it had already been done) and takes no time, i.e., *a coordinate system, and accordingly geometric distance in S are already given data*. If $R = R_g.LS$ is the geometric distance of the point *B* from *O,* then the dimensionless quantity R_g is the radial coordinate of *B*.

The unit of time, though arbitrary, is chosen as the duration, say "second", between two consecutive ticks (or readings) of identical clocks that run at synchrony with each other. Contemplating in the last statement, we may be astounded by the fact that we really have defined nothing concerns the world outside the clocks [4]. In order that a unit of time, say a second, bears a meaning as far as motion in *S* is concerned, *it should be correlated to what can happen during "a second" in the world outside the clocks* [4], and more precisely, it should quantify the amount of the spatial displacement intrinsic to some *reference physical phenomenon*, such as the propagation of light from an arbitrary point in *S*, or be related to a free reference (spherical) body that is not translating in *S* but rotating about its axis [2XII]. *A "second" must thus be corresponded with (and actually could be measured by) the distance traveled by light within S during the period of a second in the former case, and with the angle at which the reference body rotates relative to the remote universe in the latter* [2XII]. *Time measurements therefore must be reducible to specific types of spatial displacement's measurements.*

Employing the postulate that light propagates rectilinearly within the inertial frame S in all directions at a constant velocity *c,* synchronization of the clocks in S can be materialized in a measurable meaning. Indeed, we can now proceed with the Newtonian view and imagine that as soon as *S* is furnished with a system of coordinates through geometrical means, a system of synchronized timing is immediately established with respect to one timer, say $O \in S$. This means that, in the same way we envisage rationally the assignment of a triplet $(R, \emptyset, \theta)$ to each point B in *S*, we can also imagine that a timer can be placed at each point $B \in S$ which is synchronized with $O \in S$ and runs uniformly at the same rate as the master timer, and accordingly with all other timers. Indeed, due to the latter postulate a *global timing* in S can be practically established, with the notion of an "instant T_0" has a global meaning in S, in the sense that if an event takes place at $B_0(R_0, \emptyset_0, \theta_0)$ at T_0 then it will be detected at $B(R, \emptyset, \theta)$ through a light signal emanating from $B_0$and arriving at B at the instant $T = T_0 + |\vec{R}_0 - \vec{R}|/c$. Thus every S observer B assigns to the event of light's emission the same instant $T_0 = T - r/c$, where r is his spatial separation from B_0 and T is the time read at the clock B when light is received. It follows that the concept of time arrow -past, present, and future- has a global meaning in S, and any two or more S observers have the same temporal ordering of the events monitored by them. In particular, the notions of simultaneity and non-simultaneity are well-defined global concepts in S.

It is emphasized that Newton's global time was assumed to be readable at each point of space [2XII]. The synchrony of all point-wise timers was partially circumvented through appealing to a *universal timer* formed by the fixed stars in the firmament. This seems to be a generalization of the approximately uniform global time set up in the region from which almost all our observations are conducted, namely the earth surface. The earth's global time is induced by the configuration of the firmament relative to the earth. It is also stressed that no synchronization in the real sense is to be done in order an inertial frame becomes timed. Indeed and for any observer, say O, the existence of a coordinate system as well as a timer at O is sufficient to determine the duration of any light's trip with given ends, of which one end is O. Also, the duration Δt of any event at any point in S, say $B \in S$, is observed form O to have the same duration. The observer O, if he wishes, can replace his timer by a universal one since his timer should be equivalent to the universal one [2XII].

Synchronization in a uniformly rotating frame, or more accurately, in the part from which observations are conducted, can be achieved without appealing to light's signals [2XII]. An approximate example of this is the earth's surface. The existence of a global time in non-inertial frames motivates the following definition:

Synchronous frames: A frame s, not necessarily inertial, is said to be synchronous if it is endowed with a global time. In other words, the frame s can be furnished by a system of clocks that remain synchronous according to a specific criterion not requiring necessarily light signals.

3. Distance and Simultaneity by Contiguity

We proceed here to closely model the Newtonian conceptions of absolute space and time in a measurable way, but with observation through light signals is still discarded.

Assume that the inertial frame $S \equiv OXYZ$ is synchronous. The following discussion which will be confined to the X-axis is valid allover S. Suppose that $S \equiv X'OX$ is furnished with a lattice of points $\{X_n.LS = \pm n.LS,\ n = 0,1,2, \dots\}$ with LS is the length of a bar which we choose a unit of distance in S. Let $s \equiv oxyz$ be an inertial frame in standard configuration with S and translating uniformly relative to S. According to the Newtonian concepts, the length of a rigid rod is the same when measured from any inertial frame. This applies in particular to the unit length rod LS, which accordingly enjoys the same *identity* in all inertial frames. Any two points in another inertial frame s can be thought of as the ends of a rigid rod in s, and the distance between them, say $u.LS$, will be the same in s and S.

But how can we judge practically that two copies of *LS*, the first is stationary in *S* and the second is moving, say stationary in *s*, have the same length? In the synchronous frame *S* the answer is simple: if the ends of the moving rod occupies at an instant of time $T_0 = 0$ the points $A \in S$ *and* $B \in S$ then the distance between the latter points in *S* should be *LS*. Because of the absence of synchronized clocks, or global time, in *s*, the reasoning we have just applied in *S*, seems to break down in *s* when a rod *LS* that is stationary in *S* is considered. This is because *s* is not yet endowed with a global time. We shall show however that this reservation is not necessary and that the same clocks employed to read time in *S* are also qualified to indicate the same instants of time in *s*. In fact we shall demonstrate that *the absoluteness of time follows from the absoluteness of length.*

Consider a rod *ob* of length *u.LS* stationary in *s*. Because length is absolute, the length of this rod is also *u.LS* in *S*. Suppose that at an instant $T_0 = 0$ in *S* the rod occupies the interval $[O, B] \equiv [0, u] \subset X'OX$, with the points $o \in s$ and $b \in s$ are contiguous to $O \in S$ and $B \in S$ respectively. The latter points can be imagined to be the ends of a rod *OB* stationary in *S*. The frame *s* admits that the contiguity of $o \in s$ and $O \in S$ [or instead, *b* and *B*] signifies the same instant of time in both frames, and he has no objection to denote this instant, as *S* did, by $T_0 = 0$, but he may doubt that the contiguity of *b* and *B* [the contiguity of *o* and *O*] took place at the instant of contiguity of *o* and *O* [*b* and *B*]. To eliminate this doubt we assume the contrary: (the contiguity of *o and O*) took place before (after) (the contiguity of *b and B*). In the first (second) case, *s* will find the length of *OB* less (greater) than *u.LS*, which is a contradiction, since length is absolute. It follows therefore that if it was found at $T_0 = 0$ in *S* that ($o \in s$ is contiguous to $O \in S$) and ($b \in s$ is contiguous to $B \in S$) then the same compound event takes place at the same instant in *s*, which we denote by $t = T_0 = 0$. Since at the instant $T_0 = 0$ in the synchronous frame *S* there corresponds to *every* point $B \in S$ a contiguous point $b \in s$, all clocks in *s* must read when *o* is contiguous to *O* the same instant of time $t = 0$. Therefore, when *o* and *O* are contiguous, we have

(3.1) $\quad X = x, Y = y, Z = z, T = t = 0 \; everywhere \; in \; S \; and \; s,$

with (X, Y, Z) and (x, y, z) are the coordinates of an arbitrary contiguous points $B \in S$ and $b \in s$ respectively.

Suppose that the frame $s \equiv x'ox$ is also furnished by a lattice of points $\{x_n.LS = n.LS: n = 0, \pm 1, \pm 2, \dots\}$, and let's redefine the unit of time *TS* in *S* by the period during which a point x_n of *s* which is at the instant $T_0 = 0$ contiguous to X_n moves to become contiguous to the point $X_{n+1} = x_n + 1$. By the concept of inertial frames, and because length is absolute, the last relation applies to every lattice point *n* of *s* which moves to the lattice point $n + 1$ of *S*. The new state of contiguity corresponds to the displacement $LS = TS$ of each point of *s*. Note that the unit of time in *S* has been defined now through spatial displacement of *s* relative to *S;* it can be specified either by the period *TS* during which an *s*-object is displaced by *LS* in *S*, or by the difference in readings of two clocks in *S*, separated by the distance *LS*, when the same *s*-object passes by. Thus time and distance have the same dimension. In a similar way to what was proven earlier, there corresponds to the new instant of time $T.TS \equiv \Delta T.TS = 1.TS$ in S an instant of time $t.TS \equiv \Delta t.TS$ in *s* at which the new state of contiguity is also realizable in *s*, and which results from displacing each lattice point *n* of *S*, initially contiguous to the lattice point *n* in *s*, by the same magnitude *LS* but in the opposite direction to become contiguous to the point $n - 1$ in *s*. But as determined in *s*, $LS = \Delta t.TS$. Comparing the last two expressions of the equal displacements we get $\Delta t.TS = 1.TS$. Since LS, and accordingly $\Delta T.TS \equiv TS$ can be chosen arbitrarily, we permanently have $\Delta T = \Delta t$, It follows therefore that *accepting length as absolute results in time flowing equably in S and s.*

The following remarks help to illuminate the concept of absolute time by contiguity:

-To each instant of time T_0 in S there corresponds a unique state of contiguity between S and s which is characterized by the following: all events of the form $(b \in s\ is\ contiguous\ to\ B \in S)$ are simultaneous in s and in S. This defines a unique instant of time in s which is conveniently denoted by T_0 (though it may be denoted by another number). A new state of contiguity corresponding to relative displacement

$$|\Delta X|_1.LS = |\Delta x|_1.LS = LS$$

defines a unit of time TS in both frames, and corresponds to the instant of time $(T_0 + 1)TS$ in S and in s. If T is any real number, then the relative displacement

$$|\Delta X|.LS \equiv T|\Delta X|_1.LS = T|\Delta x|_1.LS = T.LS = T.TS$$

corresponds to the period of time $T.TS$ elapsing in both frames.

Thus there corresponds to each given arbitrary instant of time T in the synchronous frame S a unique instant of time T in s, which signifies the same instant of time in both frames. This implies in particular that simultaneity is absolute, in the sense that it is frame independent. An instant of time T in both frames is fully meaningful and may be identified by a unique state of simultaneous contiguity of the points of s and S as realized in both frames.

- Since the frame s yields itself to synchrony by means of contiguity to the synchronous frame S, we may consider both frames as equivalent in terms of which is a hypothesis and which is a conclusion. In other words, it makes no difference to the result whether we start from S or from s as being synchronized by hypothesis (or in practical arrangements) and then conclude that other frame is also synchronized by means of contiguity. It follows that one system of synchronized clocks in one frame will be sufficient to determine time in both frames. Thus and regardless of his state of motion, any observer registers the time shown by the S-clock (or the s-clock) which is just contiguous to him. Of course, it makes no harm to imagine an additional s-system of clocks with each clock is always at synchrony with the S-clock that is contiguous to it, or each registering $T + T_0$ where T_0 is constant. What matters really is that the S- and s-clocks register the same period ΔT of time. We shall see later how to endow the heliocentric frame with time from that set up on Earth, in spite that the geocentric frame is only inertial for short periods of time.

- In practical applications it is convenient to take $LS = 1\,meter,$ and define the unit of time "$TS = a\ second$" by the period taken by light to travel the distance $c\ meters = 3 \times 10^8 m$. Thus $1\,meter = second/c$. The numerical value of light's velocity in these units is $c = second/meter.$ Now if the frame s is displaced u meters in a second then $\Delta X\ meters = u\ meters = \frac{u}{c} seconds$. In T seconds the frame s is displaced by $\Delta X\,[m] = u\left[\frac{m}{s}\right]T[s]$, which is the familiar expression of displacement using the familiar units.

- The induction of time and distance in an inertial frame s through its state of contiguity with the synchronous frame S amounts operationally to the following: Assume that at T_0 as determined in S, the points $a \in s$ and $b \in s$ are contiguous to $A \in S$ and $B \in S$ respectively. Now

-We define the time reading at an every point $b \in s$ in s by the reading of the contiguous clock at $B \in S$.

-We define geometric distance $d(a, b)$ between a and b in s by $D(A, B)$, where D is the geometric distance in S.

The scaling theory retains equal time readings for contiguous clocks but modifies the second Newtonian requirement to incorporate a constant speed of light. This results in a universal time length of any given light's trip, and consequently a universal length, prevailing in all frames, but in different geometric lengths of the given trip in different frames.

4. Timed Inertial Frame- Universal Space

Let us consider the set of all inertial frames. It is clear what motion of a frame with respect to another means, but what needs elaboration is that the concept of a frame being at rest. The latter concept requires the existence of an *independent entity* with respect to which the state of being at rest is referred. This entity is reminiscent of Newton's absolute, or physical, space; it corresponds to *the physical space when referred to a frame set up by a force-free (i.e. far from matter) observer and not rotating relative to the fixed stars. Any given frame S defined by the latter statement is an inertial frame that can be identified by Newton's absolute space, and thus considered stationary, while all other inertial frames are then moving relative to S, and accordingly relative to the fixed stars.* Recalling that light propagates within the stationary frame *S* at a constant velocity *c*, the frame *S* which is already furnished by a coordinate system through geometric measurement can be endowed with a global time, with synchronization is carried out in the familiar way [1]. All other frames which are moving with respect to *S* derive their global time from *S* by contiguity. We thus define a *timed frame S by a stationary inertial frame in which a global time has been set up.* Since the state of being stationary can be assigned to any inertial frame, the absolute, or physical, space in its Newtonian sense as the *unique standard of rest* has to be abandoned, or else, modified to admit identification with any inertial frame of fixed stars, *but one at a time*. However, *the role of the absolute space as a unique standard of orientations is retained. The latter requirement is essential to define rotation and to single out inertial frames from rotating ones.* The physical space which can then be corresponded with one arbitrary stationary frame *S* will be referred to as "the *universal space*"; it is universal because every observer participating in any observation has agreed to consider it as the standard of absolute rest, and has yielded to project the global timing in *S*, by contiguity on his own frame.

Starting from a timed inertial frame *S* a global time can be set up by contiguity in any other inertial frame *s*. Consequently *one system of clocks* in *S* is sufficient to determine time in *S* and in any other inertial frame. The last statement implies that simultaneous events in *S* are also so in any other inertial frame. It is important however, to note that the frame *S* is an arbitrary inertial frame, in the sense that one should be able at any stage to view the other frame *s*, if inertial, as the timed inertial frame, and thus identifiable with the universal or physical space, while the frame *S* is moving. In realistic terms it is more convenient, and probably the only measurably possible way, to set up a global time in an inertial frame from time employed in the observers' frame, although the latter may be only approximately inertial. A very important example of this procedure, as it will be seen in a forthcoming part, is the *time set up in the heliocentric frame from that in the geocentric frame.*

Global time in a timed frame S is compatible with geometric measurements. Indeed, when we say that the length of a rod that is stationary in S, or the geometric distance between its two ends A and B in S, is L, we mean that had we measured this length by a calibrated ruler, or by a light signal and two synchronized clocks situated at $A \in S$ and $B \in S$, the two results will be L. In the second type of measurement, the length of the rod is $L = cT$, where T is the period taken by light to cross this rode from one end to another regardless of which end we choose as the initial point of the light signal. We will thus refer to L and T appearing in the latter equation as *geometric length and geometric duration* of either of the light trips $(A \in S \to B \in S)$, or $(B \in S \to A \in S)$. Benefiting from the compatibility of global time with geometric measurements in S, geometric distance in S, and in particular coordination of S, can be carried out either by: (i) employing geometric measurements to determine the length of a baseline through the origin O, and then measuring the distance between any point $B \in S$ and O, by triangulation, or (ii) using a clock at O to determine half the period of the return light trip $(O \to B \to O)$, say T; the sought distance will be $R = |OB| = cT$. It is clear that L and T are simply two equivalent measures of the distance $d(A, B)$ using different units. Whether obtained through geometric, optical measurements or other means, *geometric distances should be looked at as available data prior to any other kinematical, or physical, measurement*.

We may think ideally of a timed frame as any laboratory S, sufficiently far from all matter, and not rotating relative to the remote universe. The unit of length - a meter - which serves to set up coordinates $OXYZ$ in the laboratory, serves also, when combined with the constancy of the speed of light within S, to define a unit of time and to synchronize all timers in S. The system of coordinates and synchronized timing in the laboratory can be extended indefinitely. With all other frames employing the S timing and admitting S as the standard of rest, S becomes a timed inertial frame.

5. Universality of Physical Laws

Since any inertial frame can be identified with the physical space, the description of the physical world from any inertial frame, with measurements are conducted within this frame, should be the same. For geometric length within any inertial frame, copies of a rod in one frame can be transported to all frames and act as a unit of geometric length. *Or instead, the equivalent of this rod's length in wavelengths of the stationary emission of a specific spectral line can be used as a unit of length within any frame.* The unit of time is then the same in all inertial frames. Under this arrangement, any physical experiment in a frame S yields the same results of a similar experiment conducted within another frame s. The sameness of physical laws when formulated within any inertial frame will be referred to as the *universality of physical laws*.

The above paragraph does not contradict what was asserted that there is only one stationary frame at a time. In fact as long as the configuration of any physical system in an inertial frame S is determined through measurements within S (i.e. by S-observers), then as far as S is concerned any other inertial frame adopted by another set of observers is illusionary; the frame S have a direct access for measurements pertaining to any physical system, and the observers in any other inertial frame s, are as if not existing. The same thing is true for the s observers for whom the frame S can be dismantled, with their measurements are not affected. The frames which we have described with measurements of spatial and time intervals are carried out within each frame, are called *independent* *inertial frames*. If for instance two spaceships S ad s, employing the same unit of length, are employed as inertial frames then any experiment conducted within each will yield the same results.

It is only when the same physical phenomenon is observed through light (or electromagnetic) signals from two different frames, then either frame, but not both, can be considered timed and identifiable with the universal space while the other is moving in the universal space.

6. Absolute Light's Trips in the Universal Space

If a source of light b has an arbitrary vector velocity $\vec{u}$ relative to the inertial frame $S \equiv OXYZ$, we may choose without loss of generality the velocity vector in the direction of the X-axis, for we may always rotate the S-axes so that the X-axis is in the direction of $\vec{u} = u\vec{\imath}$, where $\vec{\imath}$ is the unit vector of the X-axis. Let $s \equiv oxyz$ be an inertial frame whose axes are parallel to those of S, and moving with respect to S at a constant velocity $\vec{u} = u\vec{\imath}$ $(u > 0)$, so that the source b is stationary in s. We endow the frames S and s with systems of spherical coordinates $(R, \theta, \emptyset)$ and $(r, \theta', \emptyset')$ respectively, with θ (θ') is the azimuth angle between the X-axis (x-axis) and the radius vector $\vec{R}$ $(\vec{r})$. The latitude angles $\emptyset$ and $\emptyset'$ will be suppressed because of the axial symmetry of the motion about the X-axis (Fig.(6.1)).

Assume that the source of light b which is stationary in s emits, when at $B(R, \theta, \emptyset)$ in S, a spherical pulse of light. When light arrives at O, it reaches also an s-observer whom we choose the origin o of s. Two S and s observers who are contiguous when hit by the pulse are called *conjugate observers*. Similarly, two sources, each emitting a pulse of light when contiguous, are called *conjugate sources*.

The situation we have displayed has the following features:

(i) In a given frame S, a source of light b is moving. Or equivalently, in the inertial frame s in which the source is stationary, an inertial observer O, attached to a frame S, is moving at velocity $(-\vec{u})$.

(ii) Light is emitted from b when at $B \in S$.

(iii) On arriving at $O \in S$ light arrives at the conjugate observer $o \in s$. While O is already given, o emerges at the instant light arrives at O; it is the s-observer that is contiguous to O when light is received at O and hence by o. But we may equally imagine that when light arrives at $o \in s$ it also arrives at $O \in S$, and thus $o \in s$ is already known while $O \in S$ is known when light arrives at o.

The S frame can be considered at rest throughout the light's trip which starts from (*B when occupied by the source b*) and ends up at (*O and o*), while b is moving at velocity $\vec{u} = u\vec{\imath}$ in S. Also the s frame can be claimed the stationary frame during the duration of the trip which starts from the point (*b when was at B*) and ends up at (*o and O*), while S is moving at velocity $-\vec{u} = -u\vec{\imath}$. Since each frame is entitled to claim itself stationary, and thus identifiable with the universal space, while the other is moving, all observers (the S and s observers) accept that light emanated "at the same time" from one and the "same point" in the universal space, whether identified by S or s, and ended at the same time at the same point. To elaborate, the phrase ($b \in s$ *when at* $B \in S$) defines in each frame a pair composed of a location and an instant of time, or what we shall call a *universal point*, and denote by $(b\ at\ B)$. In particular, *a true source of light gives rise to a universal point, which is a frame independent entity that embodies the same instant of time in both frames together with an S- and s-locations that are* *coincident in the universal space at the instant of contiguity of b and B.* Similarly the end point of the pulse in one frame determines a conjugate end in the other frame, and accordingly, another universal point. It follows that *all observers concede to the fact that there is one and the same trajectory in the universal space associated with a given light's trip*, which starts from $(b\ at\ B)$ and ends at $(O\ and\ o) \equiv (O, o)$. In other words, the single pulse traces a universal straight path connecting the universal points (*b at B*) and (*O,o*). The last fact is valid whether S or s is considered stationary and thus identified by the universal space.

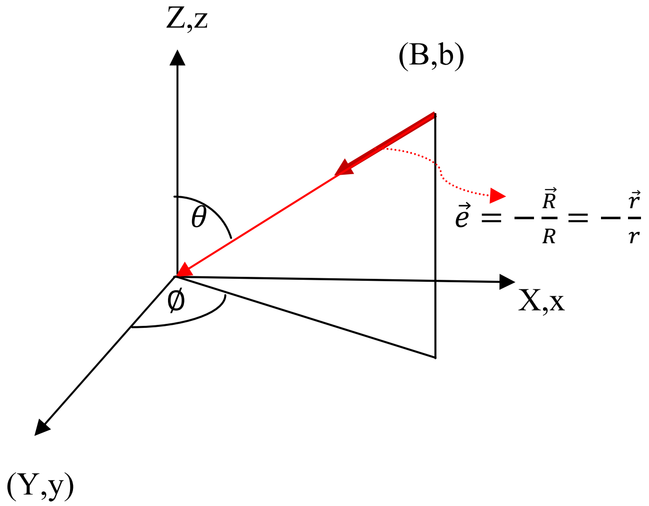


Fig.(6.1). The path of the trip $(b\ at\ B \rightarrow O\ and\ o)$ in the universal space whether identified by S or by s .

Direct consequences of the latter statements are the following:

(i) *If* $(\theta, \emptyset)$ *are the directional angles of the path in S when considered stationary and* $(\theta', \emptyset')$ *are its directional angles in s when s is considered stationary, then*

(6.1) $$\theta = \theta', \quad \emptyset = \emptyset'.$$

The velocity of the source, or equivalently the relative velocity of S and s, does not appear in the last relations. This implies that, had the frame S been replaced by another frame S_2 in standard configuration with the former, the directional angles of the path would not change: $\theta_2 = \theta' = \theta, \quad \emptyset_2 = \emptyset' = \emptyset$.

(ii)The latter result is contained in a more general consequence: *The light's trip possesses universal characters concerning its length and direction.* This means that associated with a given trip, whose beginning and end are known (specified by universal points), there are one universal length and one universal direction in the universal space, which could be identified by S or s. The universal time (and accordingly length) persists whether S or s is specified as the universal space, but the direction of the trip is determined inside the moving frame in terms of its direction in the stationary one. Thus each frame assigns the same optical (or universal) length r (equivalently the same time length t), but different geometric lengths.

(iii) To obtain the universal time length of a trip, it is sufficient to measure it in any inertial frame; it is identical to geometric time length of the trip in the frame in which the source is stationary, and it is calculable in terms of the geometric length of the trip in the frame in which it is moving. If S was the universal frame then a global time exists in S and prevails in all other inertial frames. If the beginning and end of a light trip corresponds to $(\vec{R}_1, t_1)$ and $(\vec{R}_2, t_2)$ in S, then the universal length of this trip is $c(t_2 - t_1)$ in S and in every other inertial frame. Indeed, an another frame S' assigns $(\vec{R'}_1, t_1) and\ (\vec{R'}_2, t_2)$ to the beginning and end of the same trip.

7. The Anisotropic Scaling Transformations of the First Type

Let b be a source of light moving in an inertial frame $S \equiv OXYZ$ at a constant velocity $\vec{u}$, with the X-axis of S is taken along $\vec{u} = u\vec{\imath}$ $(u > 0)$. Let s be an inertial frame which is moving relative to S at a constant velocity $\vec{u} = u\vec{\imath}$, and hence the light's source b is stationary in s. Now, we set out to determine the transformations which allows for each frame, S or s, to be considered stationary while the other is moving.

Assume that when at $B \in S$ the source b emits a pulse of light. When the pulse arrives at the point (or observer) $O \in S$, it arrives also at its s-conjugate point (or observer) $o \in s$, which is contiguous to $O \in S$ at the moment the pulse hits O (or when the pulse arrives at $o \in s$ it also arrives at its S-conjugate $O \in S$). We choose now the axes of s such that $s \equiv oxyz$ are in standard configuration with $S \equiv OXYZ$. Each of the conjugate observers O and o is entitled to consider his frame stationary relative to the fixed stars and thus identifiable with the universal space while the other frame is moving relative to his own frame. Each observer, O and o, assigns to the pulse path the same beginning (B, b) and the same end (O, o). In other words the pulse follows a universal path connecting the universal points (B, b) and (O, o), and the direction of the path is the same when looked at from the stationary frame whether it was S or s. In each frame, when considered stationary, the pulse propagates along a direction determined by a unit vector $\vec{e}$, with

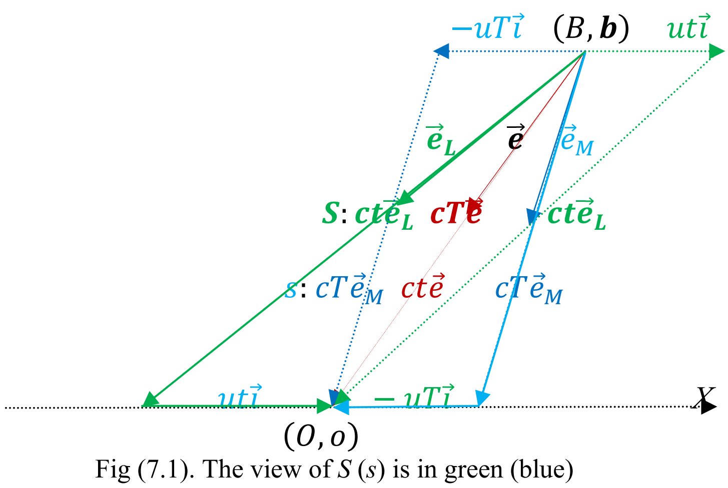


Fig (7.1). The view of S (s) is in green (blue)

$$\vec{e} = \frac{\overrightarrow{BO}}{\|\overrightarrow{BO}\|}\ (in\ S) \equiv \frac{\overrightarrow{bo}}{\|\overrightarrow{bo}\|}\ (in\ s) \tag{7.1}$$

If S is chosen the stationary frame, then the *geometric length* $\|\overrightarrow{BO}\| = R$ of the trip $(b\ at\ B \to O\ and\ o)$ in S is employed to induce corresponding length $r = \|\overrightarrow{bo}\|$ and duration $t = r/c$ in s such that r and t have the status of geometric length and time in s, when s is the stationary frame, in the same way R and T have in S.

By the Galilean law of velocity addition, the velocity of the pulse in S is the vector sum of its velocity in s and the velocity $\vec{u}$ of its emitter. However the pulse emanating from $(b\ at\ B)$ and arriving at O should had been ejected in a direction $\vec{e}_L$ in S such that the resultant velocity $c\vec{e}_L + u\vec{\imath}$ is along the unit vector $\vec{e}$. The duration $t = r/c$ taken by the pulse to arrive at O is given by the quotient of its displacement vector $\overrightarrow{BO} = -\vec{R} = R\vec{e}$ and its velocity $c\vec{e}_L + u\vec{\imath}$ in S, i.e.

(7.2) $$R\vec{e} = (c\vec{e}_L + u\vec{\imath})\, r/c = (\vec{e}_L + \beta\vec{\imath})r,$$

where $\beta = u/c$.Thus the *geometric length of the trip in S*, which is R, has given rise to *the optical length* $r = ct$ *of the trip in S*. The length r can be looked on as the *geometric length of the trip in s* because b is stationary in s, and r therefore must induce for a source B an optical distance $R = cT$ in s which should be identical to its geometric distance R in S. Since B is moving in s at velocity $(-\vec{u})$, the velocity in s of the pulse that emanates from the virtual source B is the sum of its velocity in S and the velocity $(-\vec{u})$ of its emitter. The pulse emanating from B should have then been ejected in a direction $\vec{e}_M$ in s such that the resultant velocity $c\vec{e}_M - u\vec{\imath}$ is along $\vec{e}$. The duration $T = R/c$ taken by the pulse to arrive at o is given by the quotient of the displacement $\overrightarrow{bo} = -\vec{r} = r\vec{e}$ of the pulse as seen in s and its velocity $c\vec{e}_M - u\vec{\imath}$ in s, i.e.

(7.3) $$r\vec{e} = (c\vec{e}_M - u\vec{\imath})\, R/c = (\vec{e}_M - \beta\vec{\imath})R.$$

Whether b or B was the source, we start only with one quantity, R if S is the stationary frame or r if s is the stationary frame (but not both), which is already geometrically measured whereas the other quantity is induced in the other frame by the relations (7.2) and (7.3). It follows therefore that it is sufficient to know the ratio $r/R = \Gamma(\beta,\theta)$ to determine both quantities, regardless of which had been measured geometrically, or equivalently, which frame was considered stationary.

Dividing the equations (7.2) and (7.3) side to side we obtain

(7.4) $$\frac{R}{r} = \frac{\vec{e}_L + \beta\vec{\imath}}{\vec{e}_M - \beta\vec{\imath}}\frac{r}{R}\ .$$

Or

(7.5) $$\Gamma(\beta,\theta)^2 = (\frac{r}{R})^2 = \frac{\vec{e}_M - \beta\vec{\imath}}{\vec{e}_L + \beta\vec{\imath}}.$$

By equations (7.2) and (7.3) the vectors appearing in the numerator and dominator on the right hand-side of the last equation are both along $\vec{e}$. Setting

(7.6) $$\vec{e}_L + \beta\vec{\imath} = k\vec{e}, \quad \vec{e}_M - \beta\vec{\imath} = k'\vec{e}\ ,$$

we get

(7.7) $$\vec{e}_L = k\vec{e} - \beta\vec{\imath}, \quad \vec{e}_M = k'\vec{e} + \beta\vec{\imath}.$$

Taking the norms of both sides in each equation (7.7) we get

$$1 = k^2 + \beta^2 - 2\beta k(\vec{\imath}.\vec{e}) = k^2 + 2\beta cos\theta\ k + \beta^2,$$
$$1 = k'^2 + \beta^2 + 2\beta k'(\vec{\imath}.\vec{e}) = k'^2 - 2\beta cos\theta\ k' + \beta^2.$$

Solving for k and k' we obtain

$$k = -\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta},$$
$$k' = \beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}.$$

Dividing the latter equations side to side gives

$$\Gamma(\beta,\theta)^2 = \left(\frac{r}{R}\right)^2 = \frac{k'}{k} = \frac{\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}}{-\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}}$$
$$= \frac{(\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta})^2}{1 - \beta^2},$$

which yields the *scaling factor* $\Gamma(\beta,\theta)$ given by

(7.8) $$\Gamma(\beta,\theta)=\frac{r}{R}=\frac{\beta cos\theta+\sqrt{1-\beta^2 sin^2\theta}}{\sqrt{1-\beta^2}}.$$

The scaling transformations are therefore

(7.10a) $$\mathrm{r}=\Gamma(\beta,\theta)R,\qquad t=\Gamma(\beta,\theta)T,\qquad \emptyset=\emptyset',\qquad \theta=\theta'.$$

These can be written in terms of the angle $\theta=\measuredangle(\vec{u},\vec{R})$ between the velocity of the source *b* in *S* and the radius vector as follows:

(7.10b) $$\vec{r}=\Gamma(\beta,\theta)\vec{R},\ t=\Gamma(\beta,\theta)T.$$

Another explicit forms that hold for arbitrary relative orientations of axes of *S* and *s* are the following:

(7.10c) $$\frac{\vec{r}}{\vec{R}}=\frac{t}{T}=\Gamma(\beta,\theta),$$

(7.10d) $$r\vec{e}=\frac{\vec{\beta}.\vec{R}+\sqrt{R^2-(\vec{\beta}\times\vec{R})^2}}{\sqrt{1-\beta^2}}\vec{\mathrm{e}},$$

supplemented by $R=cT$ and $r=ct$.

Basic Properties of the Scaling Factor and the Scaling Transformations:

It is easily seen that

(7.9a) $$\Gamma(0,\theta)=\Gamma(\beta,\pi/2)=1,$$

(7.9b) $$\Gamma^{-1}(\beta,\theta)=\Gamma(-\beta,\theta)=\Gamma(\beta,\pi-\theta).$$

Moreover, and since

$$\frac{\partial\Gamma(\beta,\theta)}{\partial\theta}=-\beta sin\theta.\Gamma(\beta,\theta)<0,$$

the function $\Gamma(\beta,\theta)$, with β fixed and positive, is a monotonically decreasing with $\theta\in[0,\pi]$, and

(7.9c) $$\Gamma(\beta,0)=\sqrt{\frac{1+\beta}{1-\beta}}>\Gamma(\beta,\theta)>\sqrt{\frac{1-\beta}{1+\beta}}=\Gamma(\beta,\pi)\qquad 0<\theta<\pi.$$

Also, and since

$$\frac{\partial\Gamma(\beta,\theta)}{\partial\beta}=\frac{cos\theta.\Gamma(\beta,\theta)}{(1-\beta^2)\sqrt{1-\beta^2 sin^2\theta}},$$

the function $\Gamma(\beta,\theta)$, for a fixed value of θ, increases with β if $\theta\in[0,\pi/2]$ and decreases with β if $\theta\in[\pi/2,\pi]$.

Intercepting a Moving Body by a Light's Signal

Assume that an instant $t=0$ which corresponds to $(b\in s\ is\ at\ B\in S)$ and $(o\in s\ is\ at\ O\in S)$, a spherical pulse emanates from the observer $O\in S$ that intercepts the moving object *b* at a position $b'\in S$ at an instant *t*. If the geometric distance $d(O,B)=R=cT$, then the pulse arrives at *B* at an instant *T*, and at b' at a instant *t*. The trips $(O\ at\ o\to b'\in S)$ and $(O\ at\ o\to B\in S)$ are viewed in *s* as $(o\to b)$ and $(o\to B'\in s)$. In the frame *S*, the vector sum $\overrightarrow{Ob'}=\overrightarrow{OB}+\overrightarrow{Bb'}$, or $ct(-\vec{e}_L)=cT(-\vec{e})+ut\vec{\imath}$, can be written as

(7.11) $$T\vec{e}=t(\vec{e}_L+\beta\vec{\imath}).$$

When the frame *s* is the stationary frame, the vector sum $\overrightarrow{oB'}=\overrightarrow{ob}+\overrightarrow{bB'}$ gives

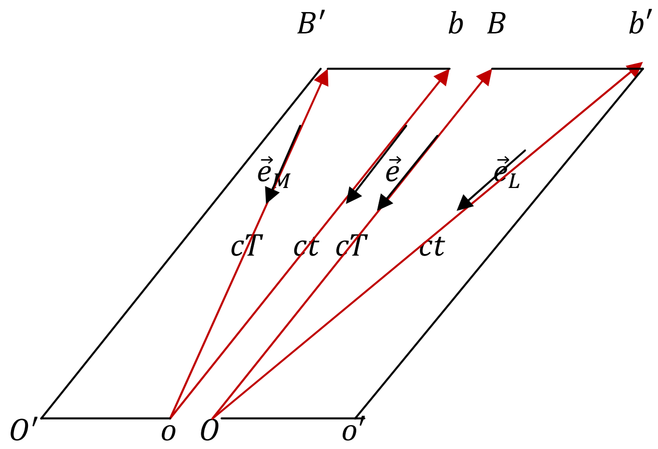
(7.12) $$t\vec{e}=T(\vec{e}_M-\beta\vec{\imath}).$$

From the latter two equations we have

(7.13) $$\frac{t}{T}=\frac{T}{t}\frac{\vec{e}_M-\beta\vec{\imath}}{\vec{e}_L+\beta\vec{\imath}},$$

which is the same equation (7.5), and the relation between *t* and *T* is given by (7.10).

Note that t is the geometric (and optical) time length of the trip $(O \in S \rightarrow b' \in S)$, and it is the optical length of this trip in s. Indeed, when light is received by $b' \in S$, it is also received by the contiguous observer $b \in s$. Thus the optical length of this trip is universal. The same applies to the trip $(O\ at\ o \rightarrow B\ at\ b)$, which has the same optical length T in S and s.



Let S be a timed inertial frame in which an object b is moving at velocity $\vec{u}$, and assume that at the same instant of time $t = 0$, which corresponds to $(b\ is\ at\ B)$ two pulses of light are emitted from b and O respectively. It follows from the latter identical scaling transformations that the pulse emitted from O arrives at b at the time of arrival of the pulse emitted from b at O.

The latter result is a direct consequence of the symmetry of the relationships between the source and observer in the universal space. The symmetries of the scaling transformations will have a systematic study in future works.

8. The Active View in Interpreting the Scaling Transformations

The active view corresponds to the frame S taken from start as the timed frame and thus identifiable with the universal space with its global timing is valid in every other inertial frame. Any non S-observer yields to the fact that his frame is moving in the universal space S. Consider the light trip

$$(b(r,\theta,\emptyset)\ at\ B(R,\theta,\emptyset)) \rightarrow (O\ and\ o)$$

in which b is a true source of light. The transformations (7.9) can be understood in the timed inertial frame S in either of the following ways:

(i)It determines in *the timed frame S the ratio between the characters (length and duration) of the true light trip* $(b\ at\ B \rightarrow O)$ *and the corresponding characters of the light's trip* $(B \rightarrow O)$, *whether B was a true or a virtual source*.

(ii) It determines *the optical (or universal or true) distance r from O of the moving body b in terms of the geometric (which is also optical) distance R of a conjugate body B that is stationary in S.*

(iii) It determines *the optical distance r between the moving body b when at B and an observer* $O \in S$ *in terms of its geometric distance* $|\overrightarrow{BO}| = R$. Note that if two pulses are emitted simultaneously from (*b at B*) and from O, then when the former arrives at o the latter intercepts b at some point in S, say $b' \in S$. It follows that the optical distance r is the present geometric distance $|\overrightarrow{b'O}|$ in S of the source from O, and hence the STI can be interpreted *as transformations between the initial geometric distance* $|\overrightarrow{BO}|$ *of the source from O (i.e. when light was emitted) and its present geometric distance (i.e when light was received at O)*. In other words, the

STI gives the geometric distance of the source from the observer at the instant of observation in terms of its geometric distance when was at the point at which the received signal was emitted.

In all above interpretations, the geometric distance $|\overrightarrow{BO}| = R$ is already known, whereas the values *r* and *t* are what *S* measures for the length and duration of the true light trip $(b\ at\ B \rightarrow O)$, or equivalently, for the distance $|\overrightarrow{b'O}| = r = ct$. If the theory is correct, these measured values must be related to the known geometric data *R* and *T* by the transformations

(8.1) $$r = \Gamma(u,\theta)R, \quad t = \Gamma(u,\theta)T = \Gamma(u,\theta)R/c.$$

If $T_0 = 0$ is read on the clock *B* at the instant of emission, then the time read on the clock *O* at the instant of light reception is

(8.2) $$t = \Gamma(u,\theta)\, R/c = r/c.$$

The quantity *t* [*r*] which is actually measured for the duration [length] of the light trip $(b\ at\ B \rightarrow O)$ is called its optical (or *universal or true) duration* [*length*]. The quantity $R/c = T$ appearing in (8.2) represents the duration that light takes from B to O were B a true source. The S system of clocks alone is sufficient of course to specify the characters of the trip $(b\ at\ B \rightarrow O)$ since the readings of the clocks B and O of the events of light's emission and reception respectively determine these characters. Moreover, as evidenced by (8.2), *only the geometric distance R between B and O is sufficient to determine the duration of the trip* $(b\ at\ B \rightarrow O)$, provided the velocity of the source *b* in *S* is already given. Thus the relations (7.10) give rise to transformations within the same frame *S*, between the *geometric length* $R = cT$ (*or geometric distance R*) *and the optical length* $r = ct$ (*or optical distance r*). The directional coordinates $(\emptyset, \theta)$ of the true and virtual trips are obviously the same. When the pulse arrives at *O* the source *b* occupies a point $b' \in S$ with $\overrightarrow{Bb'} = \vec{u}t$. The angle $\delta = \measuredangle\left(\vec{R}, \overrightarrow{Ob'}\right)$ is calculated from (7.7(i)); it is given by $\sin\delta = \beta\sin\theta$; it is the deviation measured within the frame *s*, which is moving in *S*. of the direction of the light trip in terms of its direction in *S*.

Alternatively, the transformations (7.10) hold within a *timed inertial frame s*, with *B* is a true source while *b* can be a true or a virtual source. Here, *R* is the optical (or universal) distance from *o* of a true source *B*, which is moving at velocity $-u\vec{\imath}$ in *s*, and *r* is its geometric distance from *o*. In this case the expression of the optical length in terms of the geometric length is obtained just by interchanging *r* and *R* in (7.10) (or (8.1)) and replacing β by $-\beta$, to obtain

$(8.3a)$ $$\vec{R} = \Gamma(-\beta,\theta)\vec{r} = \vec{r}/\Gamma(\beta,\theta),$$

which is identical to the forms (7.10).

The interpretation of the scaling transformations when the light trip is specified in one reference frame is called the *active view*. In the active view therefore, the specification of the characters of light's trip $(b\ at\ B \rightarrow o\ and\ O)$ is realized through two trips of which one trip is certainly true while the other can be true or virtual. When only one true source is present then the optical quantities belong to the true trip whereas the geometric quantities characterize the virtual one. Only one frame in the active view is necessary for full determination of the optical characters of a light's trip, and the latter coincide with its geometric characters if the source is at rest in that frame. Moreover, no ambiguity arises regarding units, because the same units in one frame, namely in *S*, are used when considering the characters of the trips $(B \rightarrow O)$ and $(b \rightarrow O)$.

The Case of Two Trips: We consider here the case in which b and B are both true sources. We have here in addition to the previous true light's trip $(b\ at\ B \rightarrow O)$ another true trip $(B\ when\ b \rightarrow O)$. It is clear that it makes no difference to the transformations between R [T] and r [t] in S if B was also a true source. The current case digresses from the case of a single trip in that, there are two pulses arriving at O at two different instants, T and t. Both trips follow the *same path* in S, namely the straight segment connecting (B when occupied by b) and O, but with

(8.4) $$t - T = (\Gamma(u,\theta) - 1)T$$

time difference in arrival at O.

9. Lifetime of Meta Stable Particles

The $\mu -$ meson particles are generated at an altitude of $X = 60km$ and move at velocity v close to that of light. Even if these particles have the velocity of light, it can travels during its short lifetime ($\tau \approx 2.10^{-6}s$) only the distance $d \approx c\tau = 0.6km$, which is just 0.01 of the distance from the earth surface. According to active view (iii) the distance of an μ −meson particle generated at an altitude X and approaching the earth surface shrinks to a value

$$x = \Gamma(\beta,\pi)X = \sqrt{\frac{1-\beta}{1+\beta}}X.$$

In order to reach the earth surface the particle should possess a velocity v such that

$$\sqrt{\frac{1-\beta}{1+\beta}}X < c\tau = 0.6.$$

Setting $X=60$ and solving for β yields $\beta > 0.9998$, which is a tangibly probable range in the speed distribution of such particles.

Thus any detector placed at a height 600m of the sea level, or above, will (theoretically) capture every particle heading towards it at a speed exceeding β. Detectors placed lower than 600m have also a chance to register the arrival of such particles, since higher velocities of these particles is statistically possible.

10. The Active View Through Geometric and Optical Units

We may think of the transformations (7.10) as *setting up in S a unit of optical (or universal or true) length* $pLS(u,\theta)$ *associated with a trip* $(b\ at\ B \rightarrow O)$ *in terms of the unit of geometric length LS, whereas the numerical values of the geometric and optical lengths are the same in S.* This means that, if the geometric distance $|\overrightarrow{BO}|$ is $nlen.LS$, then the length of the light's trip $(b\ at\ B \rightarrow O)$ is $nlen.pLS(u,\theta)$. The unit of optical length $pLS(u,\theta)$ of the light's trip $(b\ at\ B \rightarrow O)$ which is determined in terms of LS depends on the velocity of the source and its orientation relative to $\overrightarrow{OB} \equiv \vec{R}$. *The latter assertions remain unchanged regardless of the nature of the source B, true or virtual.* Let's call the length $|\overrightarrow{BO}| = R$ of the trip $(B \rightarrow O)$ in S whether true or virtual, the geometric length of every light trip $(b\ at\ B \rightarrow O)$, and denote the units of geometric length (time) in S by LS (TS), and the associated units of optical length (duration) by $pLS(u,\theta)$ $(pTS(u,\theta))$. We have already asserted that the geometric and optical lengths (durations) of the trip have the same numerical value *nlen* (*ntim*). Consider now a light's trip $(b\ at\ B \rightarrow O)$ of unit geometric length in S, i.e. $|\overrightarrow{BO}| = SL$. Setting $R = SL$ and $r = pSL(u,\theta)$ in (7.10) yields

(10.1) $$\frac{pLS(u,\theta)}{LS} = \frac{pTS(u,\theta)}{TS} = \frac{\Gamma(u,\theta)}{1}.$$

Now, there corresponds to the trip $(b\ at\ B' \to O)$ with geometric length $R' = nlen'.LS$, the optical length $r' = \Gamma(u,\theta)R'$ which is expressible in the form we have asserted:

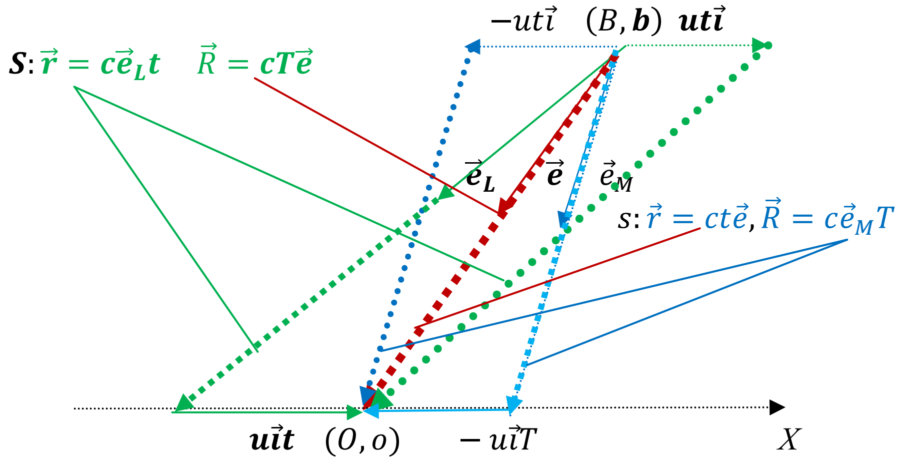
(10.2) $$r' = \Gamma(u,\theta)(nlen'.LS) = nlen'.\big(LS.\Gamma(u,\theta)\big) = nlen'.pLS(u,\theta)$$

The geometric and optical lengths of the trip are equal if and only if $\Gamma(u,\theta) = 1$, which amounts to either $u = 0$, or $\theta = \pi/2$.

If the geometric unit of length in S is taken the wavelength λ_0 of the stationary emission of a given spectral line, and if $|\overrightarrow{BO}| = R$ accommodates *nlen* wavelengths then the optical length $|\overrightarrow{bo}| = r$ accommodates also *nlen* of the optical unit of length $pLS = \Gamma(u,\theta)\lambda_0$.

11. Viewing the Light Trip in the Universal Space -The Passive View

The way in which we derived the scaling transformations allows for the identification of the universal space with either frame *S* or *s* while the other frame is moving. The relations (7.10) express the relations between what is measured in *S* (in *s*) when chosen the stationary frame to what is measured in *s* (in *S*) when chosen stationary; it is valid whether *b* or (exclusive) *B* was the true source. If *b* was the true source of light, then *R* (*r*) is it geometric (universal or optical or true) distance from *O* in *S*. And since the source is stationary in *s*, *r* is also its geometric distance from *o* in *s*. If *B* is the true source then *r* (*R*) is its geometric (universal) distance from *o* in *s*, and *R* is also its geometric distance from *O* in *S*. In any case, the universal or optical length (duration) of the light's trip, which coincides with the geometric length (duration) in the frame in which the source is stationary, is the same in both frames. This guarantees that time flaws equably in both frames and sets up accordingly a universal distance between the source and the observer in the frame in which it is moving. We may thus look on (***b*** *at B*) as one point in the universal space with *one universal, or optical distance,* from (*O and* o), and interpret (7.10) as *defining units of geometric length and time in one frame from the counter units in the other; whereas the universal or optical characters of a true trip are absolute in the universal space.* The absolute characters of the trip, which concern its length and duration, must be the same in both frames.



We denote the unit of geometric length (time) in *S* and *s* by *LS* (*TS*) and *ls* (*ts*) respectively. Setting $R = LS, r = ls$ and $T = TS, t = ts$ in (7.10), we obtain

(11.1) $$\frac{ls}{LS}=\frac{ts}{TS}=\frac{\Gamma(u,\theta)}{1},$$
with $\vec{u}=u\vec{\imath}$ is the velocity of *s* relative to *S*. The latter relation indicates that the unit of length (time) in *s* and *S* must comply with the ratio $1:1/\Gamma(u,\theta)$. Units of length (time) in the frames *s* and *S* that obey the ratio $\Gamma(u,\theta):1$ will be referred to as *universal units.*

Since the optical length of a given light trip is absolute in the universal space, we must have

(11.2a) $$R_c.LS=r_c.ls,$$
(11.2b) $$T_c.LS=t_c.ts,$$
where R_c and T_c (r_c and t_c) are the length and duration of the trip as read in *S* (*s*) respectively. From (11.1) and (11.2) we have

(11.3) $$\frac{R_c}{r_c}=\frac{T_c}{t_c}=\frac{\Gamma(u,\theta)}{1}.$$
According to the last relations the observed characters in *S* are simply the *S* equivalents (means using *S* universal or optical units) of the corresponding observed characters in *s*, and vice versa. It is noted that the relations (11.1) and (11.3) *are valid whether b or B was the source of light, and whether S or s was the universal frame.*

We consider now how the transformations (11.3) can be employed, and what available data are there.

Suppose that $b\in s$ is the source of light.

If ***s* is chosen the stationary** (or timed) frame then the geometric unit of length *ls* is already given and the optical length of the light trip is identical to its geometric length in *s*, and hence $r_c=r_g$ (and $t_c=t_g$) is an available data. The relations (11.3) determine the characters of the light trip as observed by an observer O moving at velocity $-\vec{u}$ using his unit of geometric length $LS(u,\theta)=ls/\Gamma(u,\theta)$, which is direction dependent; it is given by $R_c=r_g\Gamma(u,\theta)$.

Suppose that *b* is a source of monochromatic light of a characteristic wavelength λ_0 as measured in *s*, and that the path $(b\ at\ B)\rightarrow(o\ and\ O)$, which is of length r_g *in s and* R_c in *S* measures *n* wavelengths. If λ is the wavelength as measured in *S*, then by (11.3), $n\lambda=R_c=\Gamma(u,\theta)r_g=\Gamma(u,\theta)n\lambda_0$, or $\lambda=\Gamma(u,\theta)\lambda_0$, which is the Doppler effect.

If ***S* is the timed inertial** frame then *LS* is given, and the situation encountered can be looked on as a body *b* moving in *S*. Now, by the active view on one hand, the optical distance (to be denoted here by R_c) in *S* is $\Gamma(u,\theta)$ times the geometric distance. i.e., $R_c=R_g\Gamma(u,\theta)$. On the other hand, $R_c=r_c\Gamma(u,\theta)$, by (11.3). Comparing the last two relations we have

(11.4) $$r_c=R_g$$
which is an available data. It is to be noted that r_c is measured in *s* through measuring the light trip length by the unit $ls(u,\theta)=LS.\Gamma(u,\theta)$, whereas $R_g$is the distance as measured in *S* had the source been stationary in *S* using of course his unit of length *LS*. In other words *s* reads for *b* what *S* reads for the conjugate source *B* but each using his own units.

12. The Doppler Effect

Let *s* be an inertial frame that is moving relative to *S* at velocity $\vec{u}=u\vec{\imath}$ $(u>0)$, and consider a stationary source of light *b* in *s* that radiates monochromatic light of a characteristic wavelength $\lambda_0.ls$ where *ls* is the unit of length in *s*. Let $o\in s$ be another point in s, and suppose that the path bo accommodates n wavelengths. i.e. $|bo| = r = n.\lambda_0.ls$. Imagine that the source starts radiating when at $B \in S$ and that $O \in S$ is contiguous to $o \in s$ when light arrives at o. The path's length $|BO| = R$ accommodates then n wavelengths $\lambda.LS$, where LS is the unit of length in S. According to the passive view the length of the trip $(b\ at\ B \to O\ and\ o)$ is absolute, and $n\lambda_0.ls = n\lambda.LS$, which yields

(12.1) $$\frac{\lambda}{\lambda_0} = \frac{ls}{LS} = \Gamma(u,\theta),$$

by (11.1). The wavelength as observed in S is therefore

(12.2) $$\lambda = \lambda_0\Gamma(u,\theta).$$

By (7.9c),

(12.3) $$\lambda < \lambda_0\ for\ \pi \geq \theta > \frac{1}{2}\pi, \qquad \lambda > \lambda_0\ for\ \frac{1}{2}\pi > \theta \geq 0,$$
$$\lambda = \lambda_0\ for\ \theta = \frac{1}{2}\pi\,.$$

These correspond to the body approaching the observer in the first case, receding from the observer in the second, and moving at right angle to the position vector of the body in the third case.

We compare here the quantitative Doppler's effect as determined by the scaling theory

(12.4) $$\lambda = \gamma\left(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}\right)\lambda_0,$$

with the relativistic formula [1]

(12.5) $$\lambda_E = \gamma(1+\beta cos\theta)\lambda_0,$$

where $\gamma = 1/\sqrt{1-\beta^2}$, and the relativistic predicted wavelength has been denoted by λ_E to distinguish it from the wavelength λ predicted by the scaling theory. It is clear that the predictions of the two theories coincides for longitudinal motion. Indeed

(12.6a) $$\lambda(\theta=\pi) = \Gamma(\beta,\pi)\lambda_0 = \sqrt{\frac{1-\beta}{1+\beta}}\lambda_0 = \lambda_E(\theta=\pi),$$

(12.6b) $$\lambda(\theta=0) = \Gamma(\beta,0)\lambda_0 = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_0 = \lambda_E(\theta=0).$$

The predictions of the two theories become qualitatively distinct for $\theta = \pi/2$. In this case the relativistic formula (12.5) predicts a red shift given by

(12.7) $$\lambda = \gamma\lambda_0 = \frac{\lambda_0}{\sqrt{1-\beta^2}},$$

whereas the relation (12.4) reduces to

(12.8) $$\lambda = \Gamma(\beta,\pi/2)\lambda_0 = \lambda_0,$$

which contrary to the relativistic prediction, shows that there is *no traverse Doppler's effect.*

In spite of the absence of traverse Doppler's effect in the scaling theory, the prediction of the theory are in excellent agreement with the results of the Ives-Stilwell experiment [5,6]. To specify the goal of the experiment, we denote the wavelengths associated with approaching and receding sources by λ_a and λ_r respectively. The Ives-Stilwell experiment was designed to measure the shift [5,6,7]

(12.9) $$\Delta\lambda = \tfrac{1}{2}(\lambda_a+\lambda_r) - \lambda_0.$$

In the relativistic theory

(12.10) $$\lambda_{Er} = \gamma(1+\beta cos\theta)\lambda_0, \quad \lambda_{Ea} = \gamma(1-\beta cos\theta)\lambda_0,$$

and the shift in wave length is

(12.11) $$\Delta\lambda_E = (\gamma - 1)\lambda_0 \approx \frac{1}{2}\beta^2\lambda_0.$$

In the scaling theory

(12.12) $$\lambda_r = \gamma(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}\ ),$$

(12.13) $$\lambda_a = \gamma(-\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}),$$

and the wavelength shift is

(12.14) $$\Delta\lambda = \left(\gamma\sqrt{1-\beta^2 sin^2\theta} - 1\right)\lambda_0 \approx \frac{1}{2}\beta^2 cos^2\theta\lambda_0 = (\Delta\lambda_E)cos^2\theta.$$

The last relation shows that the scaling theory predicts in general a smaller shift than the relativistic one, and the two prediction coincide for $\theta = 0$ or $\theta = \pi$. In Ives-Stilwell's experiment a small concave mirror is set at an angle $\theta = 7°$with the ions velocity to reflect the emitted radiation backwards. As (12.14) shows, the relativistic prediction should be scaled by a factor $cos^2 7° \approx 0.985$ producing accordingly a smaller shift, and the predicted shifts by the scaling theory can be closer to the experimental observations only when the observed shifts are less than the relativistic predictions.

The following table displays some of the predictions of the special theory of relativity and the scaling theory together with the observed shift in Ives and Stilwell experiment, all measured in angstrom.

The relativistic prediction $\Delta\lambda_E = \frac{1}{2}\lambda_0\beta^2$	*observed shift* $(Ives - Stilwell)$	*The Scaling predictions* $\Delta\lambda = \frac{1}{2}\lambda_0\beta^2 cos^2\theta$
0.0202	0.0185	0.0198
0.0243	0.0225	0.0239
0.0280	0.0270	0.0275
0.0360	0.0345	0.0354
0.0478	0.0470	0.0470
0.0670	0.0670	0.0660
0.0686	0.0675	0.0675
0.0869	0.0900	0.0856

13. Galileotion of Optical Measurements

Let *S* be a timed inertial frame in which an object *b* is moving at a velocity $\vec{u} = u\vec{\imath}\ (u > 0)$. The inertial frame *S* is stationary relative to the fixed stars and it is endowed with a global time. All timers are synchronized with each other and the well-defined concept of "now" (section 4) is corresponded with equal readings of all timers at all points of the space. Every other inertial frame S' will then be moving relative to the fixed stars with velocity that is equal to its translational velocity with respect to *S*. Time in S' is induced from time *S* by contiguity.

Assume that at the initial time $t = 0$ the object *b* was at $B \in S$. At each instant of time *t* the object *b* occupies a position at a distance *ut* from *B*. Literally speaking, time her is read by clocks in *S* that are contiguous to the moving object *b* and light signals have no place. I.e. there are no observations from some point through light's signals, but instead, data concerning the object's motion is collected separately by point-wise observers and transmitted (at any time) to some master observer. This however is artificial if not redundant. In fact, one master timer placed at any point, say at *B* (or *O*) is sufficient to indicate time everywhere in *S* (and consequently in every other inertial frame through contiguity). Any *S*-observer (say *O*) can tell, using his timer, the position of the freely moving body *b* at any instant of time, provided he already knows its velocity and initial position.

As far as the description of motion is S by O is concerned, no additional knowledge is gained by placing S-clocks at various points of S. The latter statements are true even when the object b moves in a known force field.

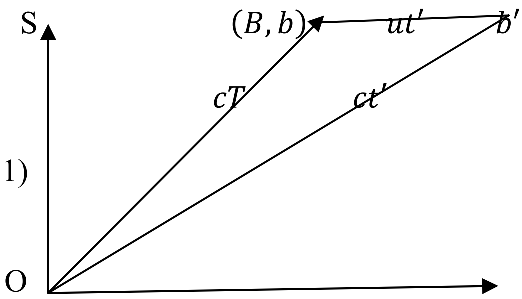


Fig.(13.1)

Assume now that, when b is at $B \in S$ (i.e. at $t = 0$), the source b emits a pulse of light that arrives at the observer $O \in S$ after a period t. The pulse follows the path ($B \in S \rightarrow O \in S$) whose true (or universal) time length is given by

(13.1) $$t = \Gamma(\beta, \theta)T,$$

where T is the geometric time length of the path $\overrightarrow{BO}$ with geometric length $gl(B, O) = R = cT$. When light arrives at O the source occupies a new position $b' \in S$, and the timers at B and b', as well as any other timer, read the same instant of time, namely t. At this instant of time the source b is at a geometric distance ut from B. If the observer $b' \in S$ sends a light pulse to inform O of b being passing by, then when the signal arrives at O the body b is at a further position $b" \in S$.

Suppose now that *the motion is observed from O employing the light signals that are emitted from the source b*. The question is that: when light arrives at O, can we envisage $b' \in S$ at geometric distances ut and ct from B and O respectively? *It is appealing to model the motion, when light arrives at O, such that ct and ut are the present or "now" geometric distances of b from O and B respectively. This model aims to reconcile the Galilean transformations with a constant velocity of light regardless of the velocity of the source.* I.e. light is envisaged as if has emanated from b from its current position, or equivalently, the optical distance ct of b from O is viewed geometrically. If this goal cannot be achieved, which is the case, then is it possible to find a relation between ut and ct and their respective counter geometric lengths $d' = gl(B, b')$ and $r' = gl(O, b')$ which are calculated on the bases of Galilean transformations?

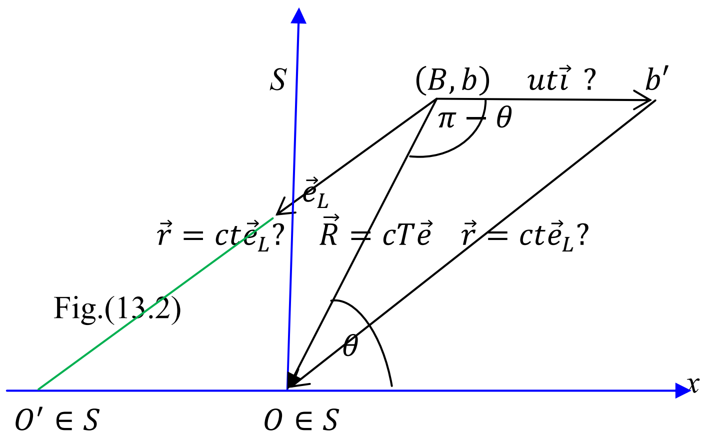


Fig.(13.2)

We shall explore here the cost of implementing this appealing view. Indeed, the proposed natural model, which accepts the Galilean transformation even when motion is observed through light signal, can be true, if and only if the rules of Euclidean trigonometry apply to the triangle $Bb'O$ with sides

(13.2a) $$\overrightarrow{BO} = cT\vec{e},\ \overrightarrow{Bb'} = ut\vec{\imath},\ \overrightarrow{b'O} = ct\vec{e}_L,$$

or equivalently to the vector sum (or Galilean transformation),

(13.2b) $$cT\vec{e} = ut\vec{\imath} + ct\vec{e}_L,\ [\text{or } ct(-\vec{e}_L) = cT(-\vec{e}) + ut\vec{\imath}],$$

and yet the resulting relation between t and T are the same as that prescribed by the scaling transformations. By the law of cosines in Euclidean trigonometry

$$(ct)^2 = (cT)^2 + (ut)^2 + 2ucTtcos\theta,$$

or

$$(1-\beta^2)t^2 - 2\beta Tcos\theta.t - T^2 = 0.$$

This yields

(13.3) $$t = \gamma^2(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta})T.$$

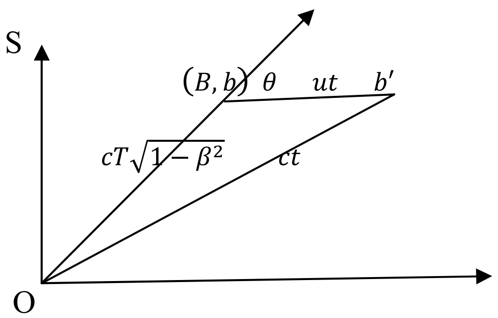
The latter equation can be written in the form

(13.4) $$t = G(\beta, \pi-\theta)T = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}}T,$$

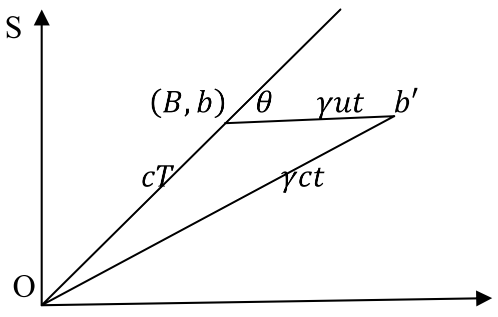
where

(13.5) $$G(\beta, \pi-\theta) = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}},$$

is the Galilean factor.



The relation (13.3) shows that the scaling transformations (13.1) cannot be satisfied by the sides' lengths of the triangle $Bb'O$ as given by (13.2), and our question accordingly has a negative answer. However, the scaling transformations (13.1) can be written in either of two equivalent forms that can be reconciled with (13.4):



(13.6i) $$t = \gamma^{-1}G(\beta,\theta)T = G(\beta,\theta)\left(\sqrt{1-\beta^2}T\right),$$

(13.6ii) $$ct' \equiv \frac{ct}{\sqrt{1-\beta^2}} = G(\beta,\theta)cT.$$

In both forms the vector sum relation (or Galilean transformation)

$$\overrightarrow{BO} = \overrightarrow{Bb'} + \overrightarrow{b'O} \qquad (or\ \ \overrightarrow{Ob'} = \overrightarrow{oB} + \overrightarrow{Bb'})$$

holds, but with the geometric length of its left hand-side, namely *cT*, is contracted by $\gamma^{-1} = \sqrt{1-\beta^2}$ in the first form, or instead, the length of each vector on the right hand-side, namely *ut* and *ct*, is expanded by γ in the second form. Note that either of the latter equations *relate the true time t (or true distance ct) with the geometric distance cT* (or geometric time *T*). It is clear that *T* can be also time; it is the duration of the trip $(B \to O)$ in case *B* was a true source.

The relations (13.6) should be understood from two complementary prospects. The duration *t* registered by the observer *O* of the given light's trip (and by any other observer) is an absolute objective entity. The first prospect concerns the determination of the period t starting from the distances of the moving source from *O* and *B*. The second prospect concerns the evaluation of the distances of the moving source *b* from *O* and *B* when light is received starting from the duration of the light trip to *O*.

The first aspect of the relations (13.6) is expressed by the important direct result: *Employing the Galilean picture to calculate the true (or universal) time length of the trip* $(b\ at\ B \to O)$ *should be accompanied by contraction by a factor* γ^{-1}*; this promotes the following two-step scheme to obtain this period:*

a- Use the Galilean transformations according to which we consider the source as if it is at the instant of time reception at $b' \in S$ *with time distance*

$$(13.7a) \qquad t' = G(\beta,\theta)T$$

from O.

b- Contract the faulty expected time distance t' *by the factor* γ^{-1} *to obtain the true (universal) duration*

$$(13.7b) \qquad t = \sqrt{1-\beta^2}\,t',$$

and the corresponding true distance $ct = \sqrt{1-\beta^2}\,ct'$.

The second prospect involves expanding the observed (or deduced from observations by any method) distances *ct* and *ut* by γ :

$$(13.8) \qquad ct \to ct' \equiv \frac{ct}{\sqrt{1-\beta^2}}, \quad ut \to ut' \equiv \frac{ut}{\sqrt{1-\beta^2}}$$

respectively.

Neglecting to carry out step (b) in the first prospect will result in a longer expected duration, which reflects on the transmitted signal appearing bluer than it should be. On the other hand, and f*ailing to perform the second prospect will convey an apparent anomaly manifested in the source appearing to lack behind its true position; it is behind where it should b; it is closer from it should be if it was receding from O and farer if it was approaching O.*

In practical terms, the latter anomalies between faulty expectations and reality can be detected in a measurable fashion only if the signal is reflected back to *O*.

In conclusion we assert that: *when motion of a source of light b is traced though observation (by light's signals) from an observer O, the Euclidean trigonometric rules apply correctly to the present position* b' *of the source b relative to O and B as deduced by the time readings of the clock O* (*or any S-clock*) *provided we expand the distances ct and ut by the Lorentz factor* γ (*i.e. to become* $\gamma ut \equiv ut'$ *and* $\gamma ct \equiv ct'$). *On the other hand, and when we model the source at the instant of light's reception at O, at distances ut and ct from B and O respectively, then the rules of Euclidean trigonometry apply also to period t of the* *trip provided the calculated duration t' is contracted by γ^{-1}. The latter case amounts to replacing the Galilean factor $G(\beta,\theta)$ by the scaling factor $\Gamma(\beta,\theta)$:*

(13.9) $$G(\beta,\theta) \to \gamma^{-1}G(\beta,\theta) = \Gamma(\beta,\theta).$$

We mention again that if when a pulse emanates from (b *at* B) another pulse emanates from O, then when the first pulse arrives at O the second intercepts the moving object b at a point $b' \in S$, with distances γut *and* γct respectively that conform to the rules of Euclidean trigonometry.

14. There is no Pioneer Anomaly

Suppose that at an instant $t = 0$ the source b is at $B'_0 \in S$ with $\overrightarrow{OB'}_0 = \overrightarrow{R'}_0$, which corresponds to the geometric time distance $T'_0 = R'_0/c$ from O. At the same instant $t = 0$, the observer O sends a light's signal that intercepts the source b at a position $B_1 \in S$ at an instant of time t_1 and bounces back to arrive at O at an instant $\tau_1 = t_1 + t'_1$. The true time length of the forward trip ($O \to b\ at\ B_1$) is

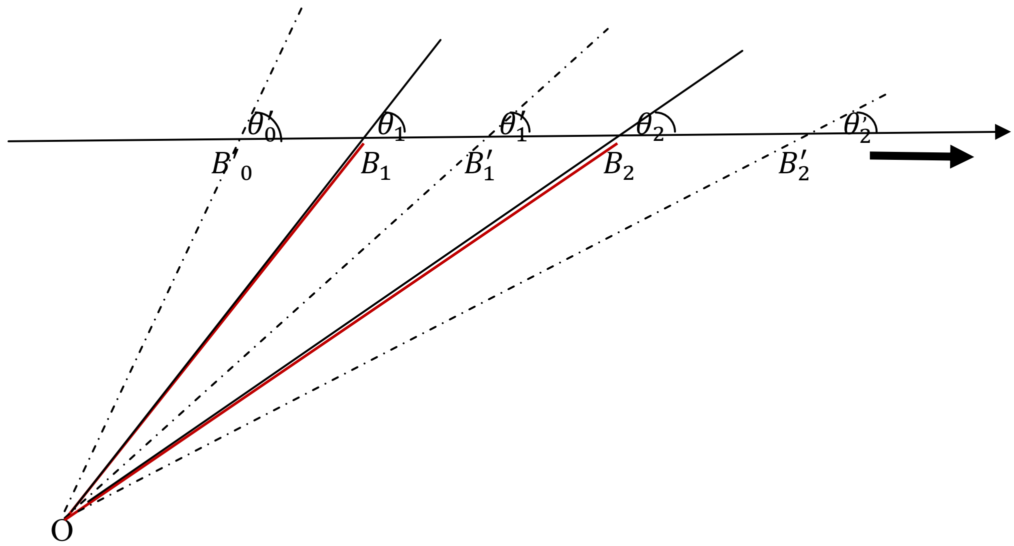
(14.1) $$t_1 = \Gamma(\beta,\theta'_0)T'_0 = \gamma^{-1}G(\beta,\theta'_0)T_0 = \gamma^{-1}G'_0T'_0$$

where $G'_0 = G(\beta,\theta'_0)$. The true time length of the backward trip ($b\ at\ B_1 \to O$) is

(14.2) $$t'_1 = \Gamma(\beta,\theta_1)t_1 = \gamma^{-1}G(\beta,\theta_1)t_1 = \gamma^{-1}G_1t_1$$

(14.2) $$= \gamma^{-2}G_1G'_0T'_0,$$

where $G_1 = G(\beta,\theta_1)$. The duration of the first return trip ($O \to B_1 \to O$) is



$$\tau_1 = t_1 + t'_1 = (1+\gamma^{-1}G_1)t_1$$

(14.3) $$= \gamma^{-1}G'_0T'_0(1+\gamma^{-1}G_1)$$

When the pulse arrives back at O at the instant τ_1, the source is at position B'_1 with distance ct'_1 from O. Suppose that another pulse is sent from O immediately when the first pulse returns. i.e. the second pulse is sent from O at the instant $\tau_1 = t_1 + t'_1$. The pulse intercepts the source b at a position B_2 at an instant $\tau_1 + t_2$ and bounces back to return to O at the instant $\tau_2 + \tau_1 = \tau_1 + t_2 + t'_2$. The time length t_2 of the trip ($O \to b\ at\ B_2$) is

$$t_2 = \Gamma(\beta,\theta'_1)t'_1 = \gamma^{-1}G(\beta,\theta'_1)t'_1 = \gamma^{-1}G'_1t'_1 = \gamma^{-2}G'_1G_1t_1$$

(14.4) $$= \gamma^{-3}G'_1G_1G'_0T'_0$$

where $G'_1 = G(\beta,\theta'_1)$. The time length of the trip back to O, namely, ($b\ at\ B_2 \to O$) is

(14.5) $t'_2 = \Gamma(\beta, \theta_2)t_2 = \gamma^{-1}G(\beta, \theta_2)t_2 = \gamma^{-2}G_2G'_1t'_1$

(14.5) $= \gamma^{-4}G_2G'_1G_1G'_0T'_0$

where $G_2 = G(\beta, \theta_2)$. The duration of the return trip $(O \to B_2 \to O)$ is given by

(14.6) $\tau_2 = t_2 + t'_2 = \gamma^{-2}G'_1G_1t_1 + \gamma^{-2}G_2G'_1t'_1 = \gamma^{-2}G'_1(G_1t_1 + G_2t'_1)$

In general, the measured time length of the n-th return trip is

$$\tau_{n+1} = \gamma^{-2}G'_n(G_nt_n + G_{n+1}t'_n)$$

Now, and if the Galileozation process is not taken into account then the application of the Galilean transformation in the inertial frame *S* to the motion of the source *b* combined with a constant light's velocity that is independent of the source's velocity, results in the following *faulty expected* durations of the return trips $(O \to b \to O)$:

$$\tau_1(expected) = G_1G'_0T'_0 = \gamma^2\tau_1(measured)$$

$$\tau_2(expe) = G'_1(G_1t_1 + G_2t'_1) = \gamma^2\tau_2(meas)$$

and in general

(17.7) $\tau_{n+1}(expe) = G'_n(G_nt_n + G_{n+1}t'_n) = \gamma^2\tau_{n+1}(meas)$

Thus the measured time length of each return trip is less than the expected one by

$$\Delta\tau_{n+1} = \tau_{n+1}(meas) - \tau_{n+1}(expe) = (\gamma^{-2} - 1)G'_n(G_nt_n + G_{n+1}t'_n)$$

(14.8) $= -\beta^2\tau_{n+1}(expe)$

For the full period of the trip $\tau = \sum \tau_n$, we have

(14.9) $\Delta\tau = -\beta^2\tau(expe)$

But since

(14.10) $\tau(expe) = \gamma^2\tau(meas)$

we have

(14.11) $\Delta\tau = -\beta^2\gamma^2\tau(meas)$

This amounts to observing the source lacking behind where it should be by

(14.12) $c\Delta\tau = -\dfrac{c\beta^2}{1-\beta^2}\tau(meas)$

To check this result with the pioneer anomaly [9] we set in (14.12) the following data: For pioneer 10,

$$\tau = 25years = 25 \times 365 \times 24 \times 3600seconds = 7884 \times 10^5 s,$$

$u = 12,$ and $\beta = 4 \times 10^{-5}$.

The apparent spatial retardation is

$$c\Delta t \approx -3 \times 10^5 \times 16 \times 10^{-10} \times 7884 \times 10^5 \approx -378{,}000km$$

For 30 years, the retardation is -454,000km approximately, which corresponds to a hypothetical (and not realistic) acceleration of $-10.005 \times 10^{-10}\, m/s^2$.

The pioneer anomaly is rooted in our expectations which are based on faulty modeling of space and time. In fact if we assign to every light trip corresponding to a duration *t* a distance γct, as it was explained in section 13, then the discrepancy of the spacecraft will disappear.

The explanation of the pioneer anomaly assumes implicitly that the orbital motion of the earth has no effect on the obtained results. In fact, this is true. In a forthcoming part we shall demonstrate an important fact: *If the motion of the earth round the sun is taken into account, then time in the heliocentric frame is induced by contiguity from time in the geocentric frame.*

Gratitude

C P Viazminsky thanks Dr. Kong K. Wan for hosting his sabbatical in the school of physics and astronomy at St-Andrews University in Scotland during which this research approached its final shape; He also acknowledges helpful discussions with Prof. James P Vary from the department of physics and astronomy in Iowa State University at which the pioneer anomaly was partially solved.

References

[1] Rindler W., *Essential Relativity*. Springer-Verlag, New York, 1977. Ch.3.
[2] Viazminsky C.P., General Science Journal, *The Scaling Theory*:
XIII. Feb.1, 2010: The universal space and the illusive Lorentz transformations
XII. Feb.12, 2010: Universal timer – the zeroth law of motion.
XI. Jan.4, 2010: Newton's absolute time and length.
X. Nov.1, 2009: The general Sagnac effect.
IX. Jul. 8, 2009: Explaining the results of Michelson and Morley experiment.
VIII. Jul. 7, 2009: The Sagnac effect and the Michelson and Gale experiment.
VIIb. Jul. 7, 2009: The bound scaling transformations.
VI. Nov. 7, 2009: The Doppler effect and drag effect.
V. Oct. 28, 2008: Aberration
IV. Oct. 28, 2008: Euclidation of the physical space and the Euclidation process.
III. Oct. 9, 2008: Interprtation of the scaling transformations.
II. Oct. 2, 2008: Derivation of the scaling transformations.
I. Sep. 30, 2008: Physical and geometric settings.
[3] Adams Steve, *Relativity, An Introduction to Space-Time Physics*, Taylor and Francis Ltd, London, (1997).
[4] Chappell John E., Natural Philosophy Alliance, www.worldnpa.org (NPA principles)
[5] Faraj A.A., The Ives-Stilwell experiment. The General Science Journal, Jul.1, (2005)
[7] Ives E. Herbert and Stilwell G.R.,An experimental study of the rate of a moving clock, J. opt. soc. Am. (1938) 28, 215-226 (1938) and part II, J. opt. soc. Am. (1941) 28, 369-374.
[8] French A P. *Special relativity*. Butler & Tanner Ltd, Frome and London, 1968.Ch.2&3.
[9] Slava G Turyshev and Viktor T Toth, Pioneer Anomaly, arXiv: 1001.3686v2 [gr-qc] 19 Aug 2010.