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Derivation of the Inverse Fine Structure Constant

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Abstract – The inverse fine structure constant is a quantization number. It can be determined by deriving the energy of the electron orbit at a minimum. This number is not an integer because the electron has a double orbit.

In the hydrogen atom the classical speed of the electron is:

$$v = \frac{c}{137.036} = \frac{c}{N} \quad ; \quad c - \text{Light speed}$$

The perimeter of the orbit is:

$$2\pi R_B = 137.036 x_e \qquad \Leftrightarrow \qquad R_B = \frac{N x_e}{2\pi}$$

$x_e =$ Compton wavelength of the electron; $R_B =$ Bohr's radius

Potential energy of the electron:

$$E_P = m_e g R_B \qquad \text{and} \qquad g = -\frac{q^2}{4\pi\varepsilon_0 R_B{}^2 m_e}$$

$m_e =$ Electron mass; g = Centript acceleration; q = Elementary charge
$\varepsilon_0 =$ Vacuum permittivity

$$E_P = -\frac{q^2}{2\varepsilon_0 x_e N}$$

Kinetic energy:

$$E_K = \frac{1}{2} m_e \frac{c^2}{N^2}$$

Total energy:

$$E = \frac{m_e c^2}{2N^2} - \frac{q^2}{2\varepsilon_0 x_e N}$$

This energy must be a minimum:

$$\frac{dE}{dN} = 0 \quad \Leftrightarrow \quad \frac{dE}{dN} = \frac{q^2}{2\varepsilon_0 x_e N^2} - \frac{m_e c^2}{N^3} = 0$$

$$\Leftrightarrow \quad N = \frac{2m_e c^2 \varepsilon_0 x_e}{q^2} \quad \Leftrightarrow \quad N = 137.036$$

Why is this value not an integer?

Orbital frequency:

$$f_O = \frac{v}{2\pi R_B} = \frac{c}{N^2 x_e}$$

Rydberg frequency:

$$f_R = \frac{f_O}{2} \quad \Leftrightarrow \quad t_R = 2t_O \; ; \qquad f_R = cR_Y$$

R_Y = Rydberg constant

The Rydberg period is two times the orbital period so the electron has a double orbit.

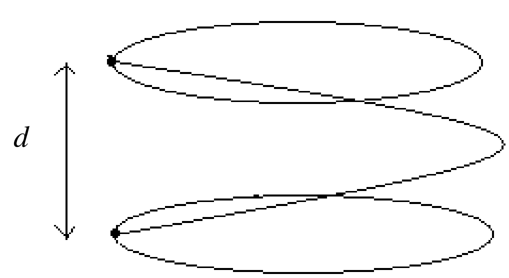
$$N = 137.036 \qquad \text{and} \qquad 0.036 = \frac{\pi^2}{2 \times 137}$$

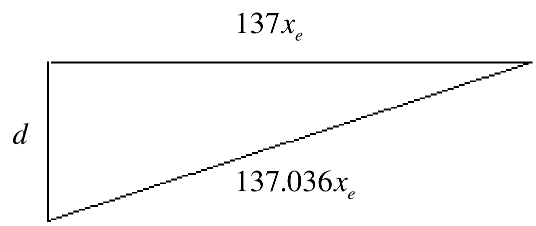
$$N = N_0 + \frac{\pi^2}{2N_0} \; ; \qquad N_0 = 137$$

Total perimeter:

$$P_T = 2x_e 137.036$$

Half orbit:





$$d^2 = (137.036x_e)^2 - (137x_e)^2 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad d = \pi . x_e$$

So we have two orbits with quantized perimeter and an additional length. The orbits are quantized in units of the Compton wavelength of the electron.