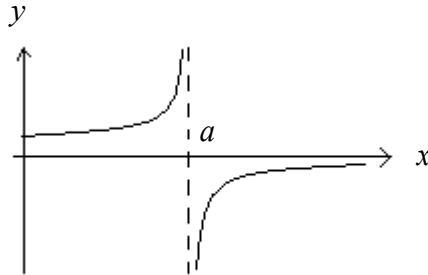




## Mathematical and natural equations

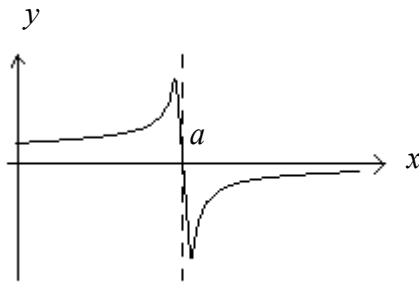
Mathematical equation:

$$y = \frac{1}{a - x}$$



Natural equation:

$$y = \frac{a - x}{(a - x)^2 + a}$$



$$x_{MX} = a \pm \sqrt{a} \quad ; \quad y_{MX} = \pm \frac{1}{2\sqrt{a}}$$

This natural equation happens in several real problems as the double sonic boom and the slow light propagation.

## Group speed in UART

Phase speed:

$$w = \sqrt{c^2 - Sf^2} \quad ; \quad w = \frac{cx}{\sqrt{S + x^2}}$$

Group speed:

$$V = \frac{d\omega}{dk}$$

$$c^2 t^2 - x^2 = S \quad \Leftrightarrow \quad \omega = \frac{2\pi \cdot kc}{\sqrt{4\pi^2 + Sk^2}}$$

$$V = \frac{cx^3}{(S + x^2)^{3/2}} ; \quad V = \frac{(c^2 - Sf^2)^{3/2}}{c^2}$$

$$V = \frac{w^3}{c^2}$$

$w$  -- Phase speed;  $c$  -- Light speed;  $S = 1.9 \times 10^{-34} m^2$ ;  $f$  -- Frequency;  
 $x$  -- Wavelength;  $V$  -- Group speed;  $\omega = 2\pi f$  -- Angular speed;  
 $k = 2\pi / x$ ;  $t$  -- Period.