The Faraday Paradox and Newton's Rotating Bucket

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Abstract. *The Faraday Paradox and the Newton's rotating bucket experiment each concern situations involving relative motion where symmetry might be expected but isn't observed. In the case of the Faraday paradox, a rotating magnet, when rotating about its magnetic axis, will induce no EMF on a stationary charged particle, where on the other hand, the reciprocal motion will. In the case of Newton's rotating bucket, the bucket must be rotating relative to an inertial frame of reference in order for the water in the bucket to be forced outwards and upwards against the insides of the walls. This does not happen if the frame of reference rotates relative to a stationary bucket, and the rotation of the background stars due to the Earth's diurnal motion certainly induces no such effect in a bucket of water. The commonality between these two apparent paradoxes, one electromagnetic and one mechanical, will be investigated.*

Electromagnetic Induction

I. Electromagnetic induction can be induced in two different ways. It can be induced when a stationary conducting wire or charged particle sits in a magnetic field which is changing with time. This is described by the equation $\mathbf{E} = -\partial\mathbf{A}/\partial t$, where $\mathbf{A}$ is the magnetic vector potential and where $\nabla\times\mathbf{A} = \mathbf{B}$. Electromagnetic induction can also occur when a conducting wire or a charged particle moves in a stationary magnetic field. This is described by the equation $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$, or $\mathbf{E} = \mathbf{v}\times\mathbf{B}$. These two kinds of electromagnetic induction are complimentary and can combine into a single total time derivative equation,

$$\mathbf{E}_{total} = -\partial\mathbf{A}/\partial t + \mathbf{v}\times\mathbf{B} \qquad \textbf{(1)}$$

or,

$$\nabla \times \mathbf{E}_{\text{total}} = -\partial \mathbf{B}/\partial t + \nabla \times (\mathbf{v} \times \mathbf{B}) \qquad (2)$$

which can be condensed to the total time derivative form,

$$\nabla \times \mathbf{E}_{\text{total}} = -d\mathbf{B}/dt \qquad \text{(Faraday's Law)} \qquad (3)$$

For the derivation, see the Appendix after the reference section. In equation **(1)**, the first term on the right-hand side, $-\partial \mathbf{A}/\partial t$, is a force which causes a change in the magnitude of an electric charge's speed, whereas the second term on the right-hand side, $\mathbf{v} \times \mathbf{B}$, is a force which changes the direction of an electric charge's velocity.

The Coriolis Force

II. We have a similar situation when dealing with transverse inertial motion as can be observed when expressing the forces in polar coordinates. The expression for the transverse inertial forces, relative to any arbitrarily chosen polar origin is,

$$m\ddot{\mathbf{r}} = m[2v_r\omega + r(d\omega/dt)]\hat{\mathbf{s}} \qquad (4)$$

where v_r is the radial speed, ω is the angular speed, r is the distance from the polar origin, and $\hat{\mathbf{s}}$ is the unit vector in the transverse direction. The term $mr(d\omega/dt)$ is the force that causes a change in the transverse speed, while the term $2mv_r\omega$ is the force that deflects a radial motion into the transverse direction. The two terms are equal and opposite and hence conservation of angular momentum is a feature of the inertial path and we can call the two terms a Coriolis pair. Although the two terms are equal and opposite, they can each be individually observed, especially where one of them is cancelled by a physical restraint.

The Inertial Frame of Reference

III. A Coriolis force causes real physical effects such as are observed in atmospheric cyclones and in the ocean currents. The Coriolis force can also be observed preventing a pivoted precessing gyroscope from toppling under gravity, and in the case of a comet orbiting the Sun, it can be observed deflecting the effects of the Sun's gravity sideways. Since

the inertial forces are simply a consequence of Newton's first law of motion, then there must be some kind of physical reaction within an inertial frame of reference which gives rise to these physically real forces. Putting it another way, there must be a physical cause lurking beneath inertial motion beyond it simply being a consequence of geometry.

James Clerk Maxwell derived the equations of electromagnetism hydrodynamically on the basis that all of space is densely packed with tiny molecular vortices that are pressing against each other with centrifugal force while striving to dilate [1], [2], [3], [4]. These molecular vortices are comprised partly of aether and partly of ordinary matter. If we adopt this approach, we should be able to establish a commonality between the convective force in electromagnetic induction on the one hand and the Coriolis force on the other hand. Consider that the vorticity of such a molecular vortex is $\mathbf{H} = 2\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is the circumferential angular speed of the vortex and where $\mathbf{B} = \mu\mathbf{H}$. We will call $\mathbf{H}$ the magnetic intensity and $\mathbf{B}$ the magnetic flux density where μ is related to density. The convective force in electromagnetic induction, $\mathbf{E} = \mathbf{v}\times\mathbf{B}$, then converts very easily into $\mathbf{E} = 2\mu\mathbf{v}\times\boldsymbol{\omega}$, a form that is uncannily similar to that of the Coriolis force, which in its own familiar form appears as $\mathbf{F} = 2m\mathbf{v}\times\boldsymbol{\omega}$. Remember that Maxwell talked about force per unit volume as opposed to force per unit charge $\mathbf{E}$. In Maxwell's original works, aether density would appear to correspond to the modern concept of electric charge.

Conclusion

IV. The apparent paradoxes arise because we are not dealing with mutual motion as is wrongly assumed. We are dealing with absolute motion relative to a physically real background medium that pervades all of space, and the interaction with this medium is what induces both the electromagnetic forces and the inertial forces. Centrifugal force is the radial inertial force arising out of the same inertial motion that causes the Coriolis force. Hence for the water in a rotating bucket to be induced to press against the inside walls of the bucket, the bucket must be rotating relative to Maxwell's sea of molecular vortices. Likewise, in order for the convective force in electromagnetic induction, $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ to be induced, the $\mathbf{v}$ term needs to be measured relative to Maxwell's sea of molecular vortices. The symmetry in both the Faraday paradox and the Newton's rotating bucket experiment is only apparent because Maxwell's all-pervading sea of molecular vortices is being ignored.

References

[1] Clerk-Maxwell, J., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)
http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf
Equation (77) in Maxwell's paper is his electromotive force equation and it exhibits a strong correspondence to equation **(1)** in this article, differing only by virtue of the inclusion of a third term on the right-hand side which is the electrostatic term. The transverse terms $2mv_r\omega$ (where vorticity $\mathbf{H} = \mathbf{2\omega}$) and $mr(d\omega/dt)$ correspond to the compound centrifugal term $\mu\mathbf{v}\times\mathbf{H}$ and the Faraday term $-\partial\mathbf{A}/\partial t$, with m corresponding to μ, and where $\mathbf{A}$ is the electromagnetic momentum, as in the momentum of the primordial aethereal electric fluid that fills all of space and from which everything is made.

[2] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity"***, Chapter 4, pages 100-102, (1910)
*"All space, according to the younger Bernoulli, is permeated by **a fluid aether, containing an immense number of excessively small whirlpools.** The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; **for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools**."*

[3] O'Neill, John J., ***PRODIGAL GENIUS***, ***Biography of Nikola Tesla***, *Long Island, New York, 15th July 1944*
http://www.rastko.rs/istorija/tesla/oniell-tesla.html
*"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. **The primary substance, thrown into infinitesimal whirls of prodigious velocity,** becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".*

[4] Lodge, Sir Oliver, ***"Ether (in physics)"***, Encyclopaedia Britannica, Fourteenth Edition, Volume 8, Pages 751-755, (1937)
http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105
In relation to the speed of light, *"The most probable surmise or guess at present is that **the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion,** circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed"*

Appendix

The curl of the vector product of two vectors can be expanded by the standard vector identity,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = \mathbf{v}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B} \qquad \textbf{(1A)}$$

Let us consider only the vector $\mathbf{B}$ to be a vector field. If $\mathbf{v}$ represents arbitrary particle motion, the second and the third terms on the right-hand side of equation **(1A)** will vanish. If we consider the vector $\mathbf{B}$ to be solenoidal, the first term on the right-hand side will also vanish due to the fact that the divergence of $\mathbf{B}$ will be zero.

Hence,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}\cdot\nabla)\mathbf{B} \qquad \textbf{(2A)}$$