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E = vXB and Maxwell's Fourth Equation

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Abstract. An abominable by-product of the modern relativity era is the widely circulated notion that a magnetic field is the relativistic component of the electric field. This idea arises out of applying the Lorentz transformation to Heaviside's versions of Maxwell's equations. The result yields both the Biot-Savart law and the Lorentz force along with the relativistic conversion factors.

This article aims to demonstrate that the Lorentz transformation of the Maxwell/Heaviside equations, as regards producing the vXB component of the Lorentz force, is merely doing what a Galilean transformation would also do. It is restoring the convective component that was part of Maxwell's original fourth equation, and which Heaviside and Gibbs took away in 1884.

This article also demonstrates that the Biot-Savart law is a solution to Maxwell's equations independently of the Lorentz transformation.

The Lorentz Force

I. It is a virtually unknown fact in physics circles that the expression that bears the misnomer 'the Lorentz force' is actually equation **(D)** of the original set of eight 'Maxwell's Equations'. The Lorentz force existed when Lorentz was a young boy. It is Maxwell's fourth equation. See part III of 'A Dynamical Theory of the Electromagnetic Field' [1] at,

http://www.zpenergy.com/downloads/Maxwell_1864_3.pdf

The Lorentz force is the solution to Faraday's law of electromagnetic induction and in modern vector format it is written as,

$$\mathbf{E} = -\text{grad}\psi - \partial\mathbf{A}/\partial t + \mathbf{v}\text{X}\mathbf{B} \qquad \textbf{(The Lorentz Force)} \qquad \textbf{(1)}$$

where ψ is the electrostatic potential, **A** is the magnetic vector potential, and where **B** is equal to μ**H**. The Lorentz force also appears as equation (77) in an earlier paper by Maxwell entitled 'On Physical Lines of Force' 1861 [2].

When Oliver Heaviside and JW Gibbs reformatted Maxwell's equations in 1884, they put them into partial time derivative format. Instead of using the Lorentz force, Heaviside and Gibbs chose to use a partial time derivative version of Faraday's law of electromagnetic induction, and in doing so they lost the convective **v**X**B** component of the Lorentz force [3].

This point can be demonstrated by taking the curl of **v**X**B** using the standard theorem for the curl of a vector cross product ie.

$$\text{curl}\,(\mathbf{v}\text{X}\mathbf{B}) = \mathbf{B}(\text{div}\,\mathbf{v}) - \mathbf{v}(\text{div}\mathbf{B}) + (\mathbf{B}.\text{grad})\mathbf{v} - (\mathbf{v}.\text{grad})\mathbf{B} \qquad \textbf{(2)}$$

Since the divergence of **B** is zero and since the velocity terms are of arbitrary value, equation **(2)** reduces to,

$$\text{curl}\,(\mathbf{v}\text{X}\mathbf{B}) = -\,(\mathbf{v}.\text{grad})\mathbf{B} \qquad \textbf{(3)}$$

We know that the total time derivative operator can be split into a local component and a convective component as follows,

$$d/dt = \partial/\partial t + v.\text{grad} \qquad \textbf{(4)}$$

Faraday's law in total time derivative format should appear as,

$$\text{curl}\,\mathbf{E} = -\,d\mathbf{B}/dt = -\partial\mathbf{B}/\partial t - (\mathbf{v}.\text{grad})\mathbf{B} \qquad \textbf{(5)}$$

but in Heaviside's versions of Maxwell's equations, Faraday's law simply appears as,

$$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t \quad (\text{ Partial Time Derivative Faraday's Law}) \qquad \textbf{(6)}$$

We can clearly see that Heaviside has left out the convective **v**X**B** component of the Lorentz force.

The convective component could be reintroduced to Heaviside's versions of Maxwell's equations by a simple Galilean transformation. A Lorentz transformation would also produce the **v**X**B** convective component for the same reasons, but a Lorentz transformation would also introduce additional relativistic effects.

The fact that a Lorentz transformation produces additional relativistic effects doesn't however mean that the **v**X**B** term itself becomes a relativistic effect. It has been suggested in some quarters that the **v**X**B** force takes its name (misnomer) from the fact that it is the consequence of a Lorentz transformation. These quarters are totally overlooking the fact that the Lorentz transformation in respect of producing the **v**X**B** term is only doing what a Galilean transformation would also do. It is restoring the convective component that Heaviside removed from Maxwell's equations.

The electromagnetic field exists independently of any relativistic considerations. It is arrant nonsense to suggest that the magnetic field is the relativistic component of the electric field as if to suggest that magnetism only begins with Einstein's theories of relativity and that nothing that Ampère, Faraday, Weber, or Maxwell did in the nineteenth century is of any significance. The only thing that a Lorentz transformation does to an electric field that a Galilean transformation doesn't do is to add relativistic factors. The Lorentz transformation does not create a magnetic field out of an electric field and it has got no grounds upon which to claim Maxwell's fourth equation as its own creation.

The Biot-Savart Law

II. In 'The Unification of Gravity and Magnetism' at,

http://www.wbabin.net/science/tombe15.pdf

it was shown how the Biot-Savart law is a solution to Maxwell's fifth equation and also to Ampère's Circuital Law provided that it is considered in conjunction with an electron orbiting a positron in a perfectly circular orbit.

In '$E=mc^2$ and Maxwell's Fifth Equation' at,

http://www.wbabin.net/science/tombe14.pdf

it was shown how $E=mc^2$ is a solution to Maxwell's fifth equation, also within the context of a circular rotating electron positron dipole.

Let us once again look at Maxwell's fifth equation.

$R = -4\pi E^2 h$ (Electric Displacement Equation) (105)

For simplicity and familiarity, we will re-write it in modern notation,

$\varepsilon \mathbf{E} = -\mathbf{D}$ (Electric Displacement Equation) **(8)**

where **E** is the electromotive force, **D** is the displacement vector, and ε is the electrical permittivity. The magnetic field vector **H** is essentially the vorticity or angular momentum of a circular rotating electron positron dipole vortex and so we will define it as,

$\mathbf{H} = \mathbf{D}\mathrm{X}\mathbf{v}$ (The Magnetic Vortex Field) **(9)**

Substituting **(8)** into **(9)** we get,

$\mathbf{H} = \varepsilon \mathbf{v}\mathrm{X}\mathbf{E}$ **(10)**

Within the context of a perfectly circular rotating dielectric vortex, the **E** vector must therefore be radial in order to have any effect, since **H** is a vector cross product. If we substitute **E** for the radial Coulomb force, we end up exactly with the Biot-Savart law. The permittivity ε will cancel out, the permeability μ will come in when we convert **H** to **B,** and the charge and velocity will multiply together to yield the electric current.

Let us now take the curl of **v**X**E**,

$$\text{curl}\,(\mathbf{v}\text{X}\mathbf{E}) = \mathbf{E}(\text{div}\,\mathbf{v}) - \mathbf{v}(\text{div}\mathbf{E}) + (\mathbf{E}.\text{grad})\mathbf{v} - (\mathbf{v}.\text{grad})\mathbf{E} \qquad \textbf{(11)}$$

Since the divergence of **E** is ρ/ε and since **v** has got arbitrary value, it follows from equation **(10)** that,

$$\text{curl}\,\mathbf{H} = -\rho\mathbf{v} - (\varepsilon\mathbf{v}.\text{grad})\mathbf{E} \qquad \text{(Ampère's Circuital Law)} \qquad \textbf{(12)}$$

where ρ is charge density. Since the velocity is a tangential velocity and hence at ninety degrees to the Coulomb force, the second term on the right hand side of equation **(12)** will vanish.

If we differentiate equation **(8)** with respect to time we obtain,

$$\varepsilon\partial\mathbf{E}/\partial t = -\rho\mathbf{v} = -\mathbf{J} \qquad \textbf{(13)}$$

and substituting into equation **(12)** we get,

$$\text{curl}\,\mathbf{H} = \varepsilon\partial\mathbf{E}/\partial t \qquad \text{(Heaviside/Maxwell/Ampère law)} \qquad \textbf{(14)}$$

Therefore, when the Biot-Savart law is obtained by applying the Lorentz transformation to Heaviside's versions of Maxwell's equations, it is merely revealing a relationship between the electric and the magnetic field that is already implicit in Maxwell's equations.

Conclusion

III. The Lorentz transformation, if it actually held true at all, would only add relativistic effects to electric and magnetic fields. It would not convert electric fields into magnetic fields nor vica-versa. An electric field can be generated by a charged particle moving in a magnetic field, but this is classical theory described by the convective **v**X**B** component of the Lorentz force and it is most certainly not an effect that is exclusively caused by the Lorentz transformation. The Lorentz transformation is acting on Heaviside's partial time derivative versions of Maxwell's equations and it reintroduces the **v**X**B** convective component to Faraday's law that was taken away by Heaviside and Gibbs in 1884.

The Biot-Savart law is already implicitly contained within the Heaviside versions of Maxwell's equations and also within Maxwell's fifth equation, and is hence totally independent of the Lorentz transformation.

References

[1] Clerk-Maxwell, J., "A Dynamical Theory of the Electromagnetic Field", Philos. Trans. Roy. Soc. 155, pp 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531--536 (1864)

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[2] Clerk-Maxwell, J., "On Physical Lines of Force", Philosophical Magazine, Volume 21, (1861)

http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[3] The full convective component would involve not only a Coriolis (**v**X**B**) term, but also a centrifugal term. However, since the centrifugal term is irrotational, it can reveal nothing as regards this particular analysis. In large scale electromagnetism, the magnetic centrifugal force manifests itself in paramagnetic attraction and diamagnetic repulsion. See "Archimedes' Principle in the Electric Sea" at,

http://www.wbabin.net/science/tombe11.pdf

Ferromagnetic and electromagnetic repulsion are also manifestations of the centrifugal force. The 'so-called' relativistic **v**(**v**.**E**) term that comes with the Lorentz transformation in addition to the **v**X**B** term is almost certainly connected to the centrifugal force.