

The Sagnac Effect - A Classical Explanation

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Although the Sagnac effect has been known for about 100 years and is now successfully used in laser gyroscopes, there are no non-contradictory explanations why a beam traveling against an interferometer's rotation traverses the curve faster than the beam traveling in the opposite direction. Sagnac, as many of today's authors confirm, explained the effect using an ether hypothesis [1, 2, 3]. Relativity theory does not explain the Sagnac effect [4, 5] though some authors try to prove otherwise [2]. There are attempts to explain the effect by non-instant reflection of light from mirrors [5], by Doppler shift of the frequencies [6] and even by Coriolis forces [7].

In this paper, it is shown that the Sagnac effect has a classical explanation based on the fact that light travels from the source to any given point through an optimal (shortest), trajectory.

In a fixed ring interferometer, coherent beams travel identical distances and interference fringes are in some initial position. It is shown below that the Sagnac effect arises because the coherent beams cover **different distances** in a rotating interferometer, although the beams travel at the identical speed C relative to it. Beam 2 traveling against the rotation, covers less distance. Therefore, it traverses the curve faster than beam 1 and the fringe shift arises; that is, the Sagnac effect takes place.

The distances covered by the beams are different because the photons travel in a rotating interferometer with **different trajectories**. Every photon moves between the mirrors on **the trajectory which is rectilinear relative to the inertial frame and which does not depend on the rotation of the interferometer**. Therefore, while the beam travels between the mirrors, the interferometer turns by some angle and the photons reach different points on the mirrors. That is, the Sagnac effect arises because the direction of linear movement of the photons does not depend on the angular movement of the interferometer.

It is well known that light takes the shortest path from source S to some point A ; that is, it goes in a straight line from S to A . If light is reflected from a flat mirror on its way from S to A , the point of reflection M is located on the straight line connecting the source S and the mirror reflection A^* . Because the straight line SA^* is the minimum distance from S to A^* , the distance SMA equal to SA^* is also the minimum distance from S to A .

In a ring interferometer, coherent beams travel from a semitransparent mirror, around the circle in opposite directions and then reach the semitransparent mirror again. The beams are reflected from mirrors and travel minimum-distance trajectories.

When the interferometer is at rest, beam 1 travels from point A through points B_0 and E_0 and then returns to A (Fig.1). Beam 2 reaches the same points in the opposite direction. That is, the beams in a fixed interferometer travel identical trajectories and cover identical distances equal to the sum of the three sides of triangle AB_0E_0 .

We can find the same trajectories and the same distances if we plot a straight line, A_BA_E connecting points A_B and A_E which are the mirror reflections of point A in mirrors B and E. The line A_BA_E intersects mirrors B and E in the same points B_0 and E_0 which determine the same trajectories.

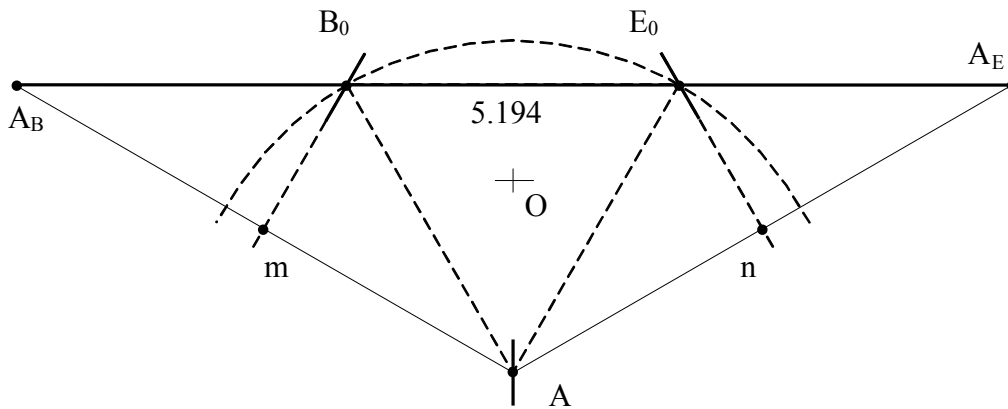


Fig.1.

The length of line A_BA_E is equal to the distance covered by both beam 1 and beam 2 ($A_BB_0 = B_0A$ and $A_EE_0 = E_0A$). The length A_BA_E in Fig.1 is equal in representative scale to 5.194.

When the interferometer is rotating – Fig.2, Fig.3

Fig.2 and Fig.3 show that the optimal trajectories of the beams do not pass through points B_0 and E_0 and therefore the distances traveled by the beams relative to the interferometer are different. The optimal trajectories and distances are determined when the interferometer rotates clockwise. In order to simplify construction, we consider rotation of the interferometer relative to one of its vertexes, point A.

Beam 1 in the rotating interferometer (Fig.2)

While beam 1 travels from fixed point A to mirror B, the mirror shifts along the circle (of radius AB_0) and simultaneously turns by some angle β (in Fig.2, angle β is equal to 5°). Point B_0 arrives at position B_1 . The mirror reflection of point A in mirror B is the point A_B1 ($Am1$ is perpendicular to mirror B and $m1A_B1 = Am1$).

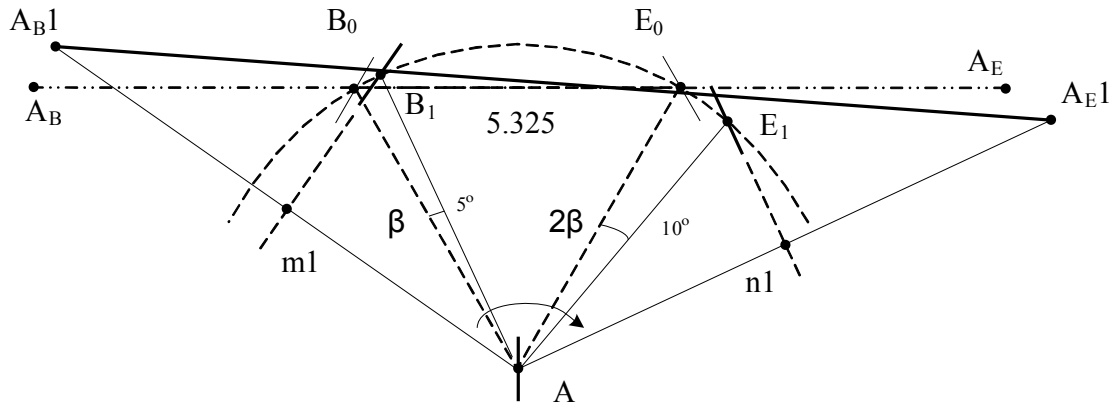


Fig.2.

While beam 1 travels from mirror B to mirror E, the interferometer turns by angle β and mirror E arrives at position E_1 . The mirror reflection of point A in mirror E is on the point A_{E1} (An_1 is perpendicular to mirror E and $n_1A_{E1} = An_1$). The points of intersections of straight line $A_{B1}-A_{E1}$ with the mirrors, determine the trajectory of beam1 and the length of the line $A_{B1}-A_{E1}$ is equal to the length of the trajectory. In Fig.2, the length of $A_{B1}-A_{E1}$ is equal to 5.325.

Beam 2 in the rotating interferometer (Fig.3)

While beam 2 travels from A to mirror E, the mirror shifts along the circle and simultaneously turns by angle β . The point E_0 arrives at position E_2 . The mirror reflection of point A in mirror E is in the point A_{E2} (An_2 is perpendicular to mirror E and $n_2A_{E2} = An_2$).

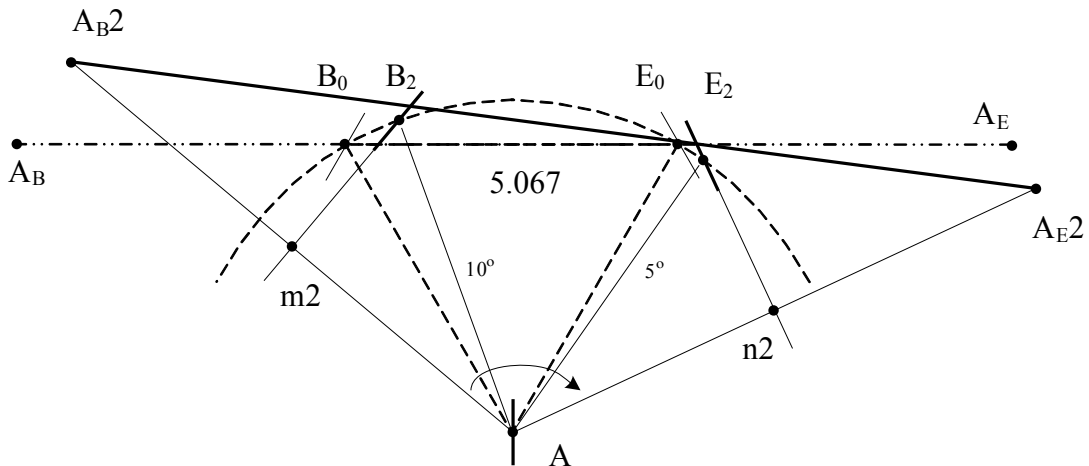


Fig.3.

While beam 2 travels from mirror E to mirror B, the interferometer turns by angle β and mirror B arrives at position B_2 . The mirror reflection of point A in mirror B is in the point A_{B2} (Am_2 is perpendicular to mirror B and $m_2A_{B2} = Am_2$). The points of intersection of straight line $A_{E2}-A_{B2}$ with the mirrors, determine the trajectory of beam2 and the length

of the line $A_E2- A_B2$ is equal to the length of the trajectory. In Fig.2 the length of $A_E2- A_B2$ is equal to 5.067.

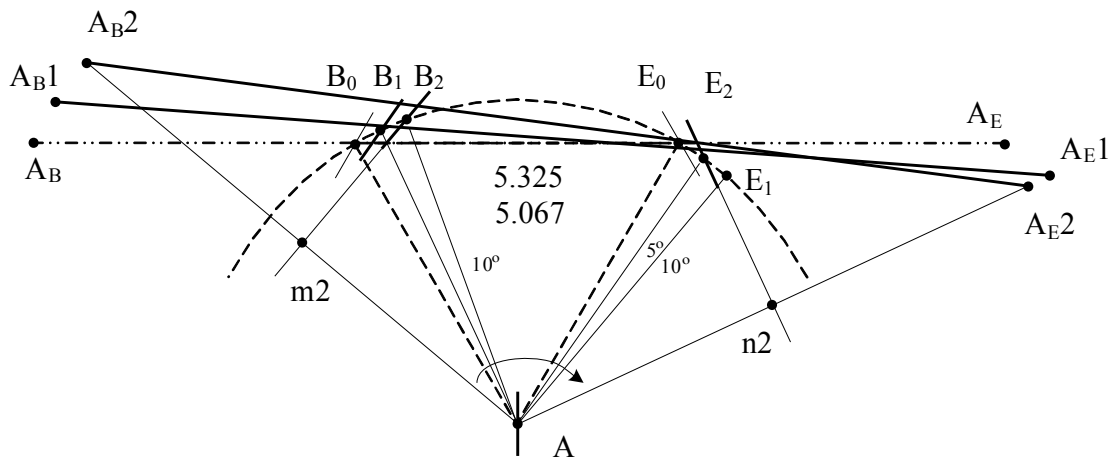


Fig.4

The trajectories of beams 1 and 2 are superimposed in Fig.4. The comparison of the line segments $A_B1- A_E1$ in Fig.2 and $A_E2- A_B2$ on Fig.3 shows that beam 2 covers less distance and therefore arrives at the semitransparent mirror faster than beam 1.

Fig.1.1, Fig.2.1, Fig.3.1, Fig.4.1 on which the trajectories are determined with more accuracy and **the sequence of geometrical constructions** are set out in the **Appendix**.

In **Fig.1.1, Fig.2.1, Fig.3.1** in the **Appendix**: the difference between the distances is $15.15 - 14.85 = 0.3$ and the ratio $L/\Delta L = 15/0.3 = 50$; light speed $C = 5.0$; the mirrors move relative to the interferometer's center O with linear speed $V = 0.10$ and the ratio $C/V = 5.0/0.1 = 50$. That is the ratio of the speeds is the same as the ratio of the distances.

1. <http://www.sciteclibrary.ru/rus/catalog/pages/6730.html>
2. http://www.ufn.ru/ufn88/ufn88_9/Russian/r889e.pdf
3. <http://n-t.ru/tp/iz/os.htm>
4. <http://www.mathpages.com/rr/s2-07/2-07.htm>
5. <http://www.trinitas.ru/rus/doc/0016/001b/00161336.htm>
6. <http://www.vokrugsveta.ru/vs/article/6330/>
7. <http://suhorucov.narod.ru/index2.htm>

- See Appendix for scale drawings
- Download detailed Microsoft Visio drawings: <http://wbabin.net/sokolov/visio.zip>

Appendix

Fig.1.1, Fig.2.1, Fig.3.1, Fig.4.1 and the sequence of geometrical constructions

The trajectories are determined in representative scale for the interferometer with the distances between the mirrors equal to 5.0 and the angle step of 2° .

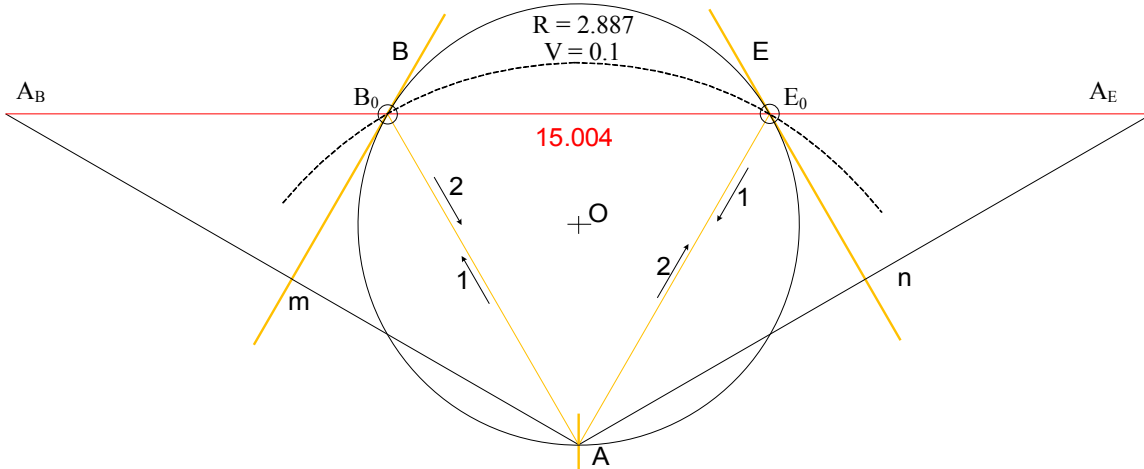


Fig.1.1. Fixed interferometer.

A_B, A_E - the mirror reflections of point A in mirrors B, E.
 A_m is perpendicular to mirror B, $A_B m = A m$,
 A_n is perpendicular to mirror E, $A_E n = A n$
 $A_B A_E = 15.004$ - optimal distance for beams 1 and 2.

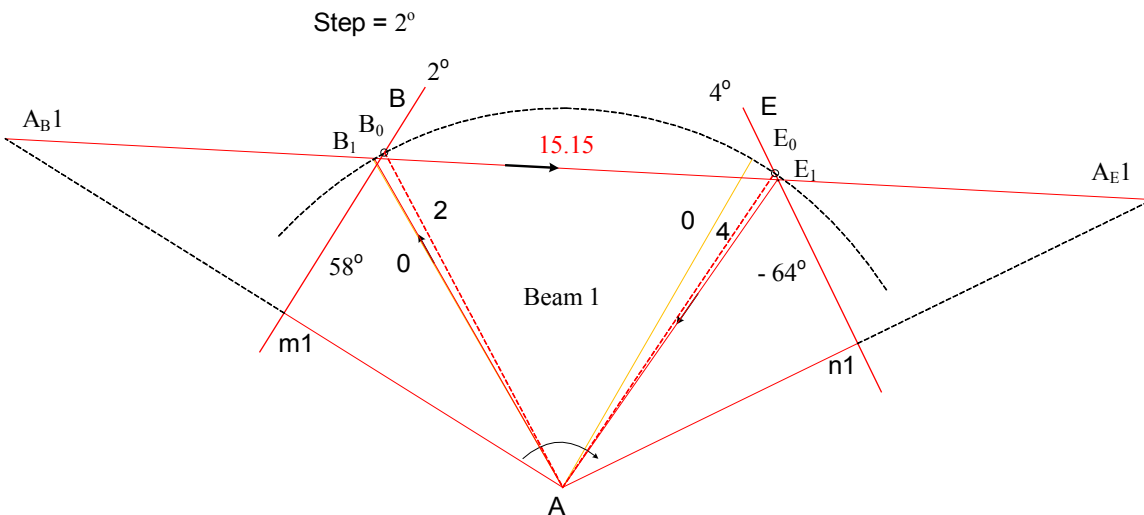


Fig.2.1. Beam 1 in the interferometer rotating around A.

While beam 1 covers distance AB_0 , the interferometer turns relative to the laboratory by 2° and mirror B shifts by $2\pi R/180 = 2\pi \cdot 2.887/180 = 0.1745$ that is, mirror B moves at $V = 0.1745$.

When beam 1 reaches mirror B, it turns 2° and is at position 58° (was at 60° in the fixed interferometer).

A_{B1} - the mirror reflection of A in mirror B

While beam 1 approaches mirror E, the interferometer turns by 2° and mirror E is in position -64°

A_{E1} - the mirror reflection of A in mirror E.

$A_{B1}-A_{E1} = 15.15$ - the distance covered by beam 1.

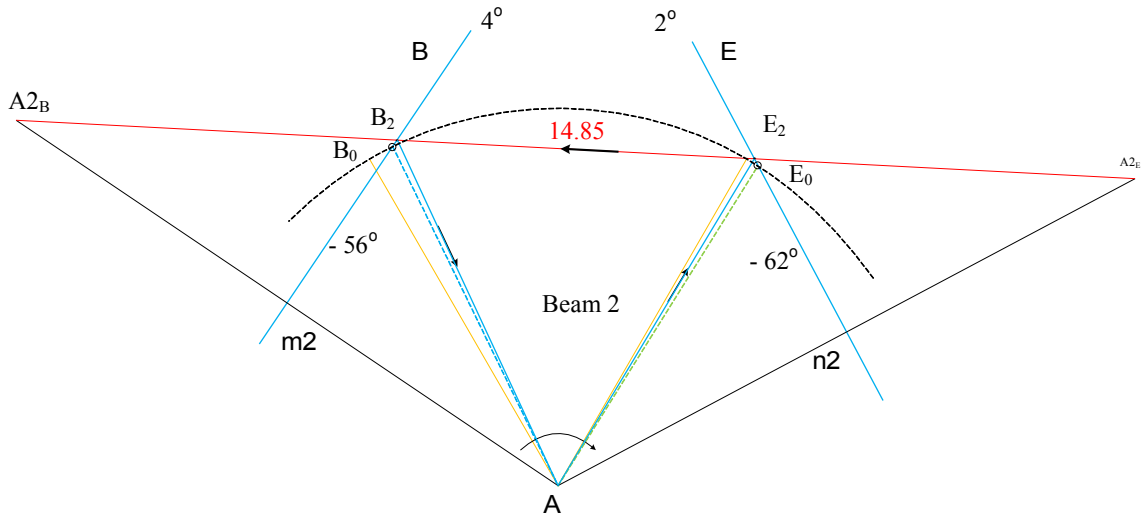


Fig.3.1. Beam 2 in the interferometer rotating around A.

While beam 2 approaches mirror E, the mirror reaches position -62° .

A_{E2} - the mirror reflection of A in mirror E

When beam 2 approaches mirror B, the mirror is at -56° .

A_{B2} - the mirror reflection of A in mirror B

$A_{E2}-A_{B2} = 14.85$ - the distance covered by beam 2.

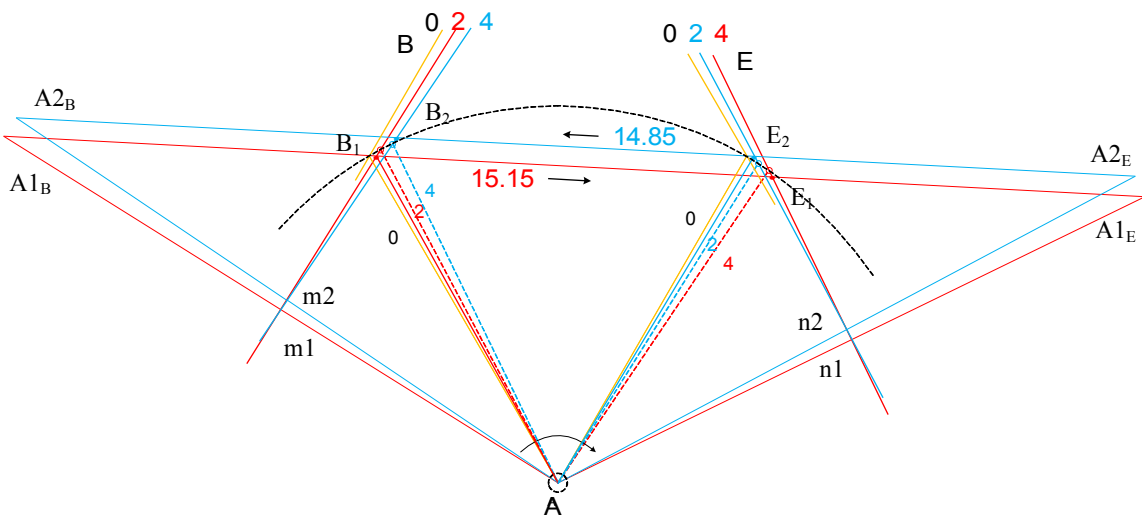


Fig.4.1 - Beam 1 and beam 2 together