The Scaling Transformations, Theory and Applications (II)

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Abstract

In the first part of this work [1] the relations between the lengths and durations of a light's trip in two inertial frames was found in a form of an anisotropic scaling transformations. The latter transformations extend the longitudinal transformations [2-5] to a general case, and yield the speed of light constant within each inertial frame. The concept of relatively absolute units of length and time was introduced. In the current part of this work, the scaling transformations are put into action to give convincing explanations for well-known, mostly optical, phenomena. These include [6-8] the life time of meta-stable particles, the Doppler's effect, the drag's effect, the null results of the Michelson and Morley experiment, and the Sagnac's effect [9]. The stellar aberration [10-12] will be the subject of the third part of this work.

1. Introduction

Before setting out to present explanations of some optical effects, it is useful to outline the common tool employed in all, namely the scaling transformations.

Consider a body *b* moving relative to an inertial frame $S \equiv OXYZ$ with a constant velocity $\vec{u}$. We may choose the rectangular Cartesian frame *S* so that the body's velocity is parallel to *OX*, i.e. $\vec{u} = u\vec{i}$ $(u > 0)$ and $\vec{i}$ is the unit vector of *OX*. If *s* is another inertial frame in standard configuration with *S*, then *b* is stationary in s. We endow *S* and *s* with spherical coordinates (R, θ, ϕ) and (r, θ', ϕ') respectively. Assume that when at $B(R, \theta, \phi)$ in *S*, the source *b* emits a spherical pulse which reaches the conjugate observers [1,3,5] *O* and *o* at $T = t = 0$. It was shown in the first part of this work that the relations between the length and duration of the light's trip $(b\, or\, B \to O\, and\, o)$ in *s* and *S* are

(1.1) $$\frac{r}{R} = \frac{t}{T} = \Gamma(\beta, \theta), \quad \theta = \theta', \quad \phi = \phi'$$

where

(1.2) $$\Gamma(\beta, \theta) = (1 - \beta^2)^{-1/2}(\beta \cos\theta + \sqrt{1 - \beta^2 \sin^2\theta}),$$

with $\beta = u/c$, and

(1.3) $$T = R/c \; and \; t = r/c$$

in (1.1) standing for the duration of the light's trip $(B\, or\, b \to O\, and\, o)$ in *S* and *s* respectively. Noting that *T* and *t* are directly proportional to *R* and *r* respectively, we may restrict our discussions to coordinate transformations.

2. Interpretation of the Scaling Transformations

It was shown in the first part of this work [1] that the light's trip $(B\, or\, b \to O\, and\, o)$, in which one source of light is involved, can be viewed either

1. from a single frame, say *S*, with two possible starting points *B* or *b* and the same ending point *O*. i.e. $(B \to O)$ and $(b \to O)$, with one of these trips is virtual. Assuming *b* is the true source, the relations (1.1) describe the transformations between the set of quantities (R, θ, ϕ, T) pertaining to the location *B* of the true source at emission, and the quantities (r, θ', ϕ', t) pertaining to the true source *b* at the instant of light reception by

O. Equivalently, B can be looked at as a virtual source, and hence the transformations (1.1) determine the relation between the coordinates of the virtual and the true source. The quantities pertaining to the body were referred to as "mobile or proper", and to those pertaining to its location as "geometric or stationary". The transformations (1.1), thus relate the geometric and mobile coordinates of a body in one frame to each other.

Since the transformations (1.1) preserve the direction of the light's ray and affect an anisotropic scaling of the geometric distance in S by a factor $\Gamma(\beta,\theta)$, it was accordingly called the scaling transformations. The view of the light's trip from one frame formally as two trips one of which is virtual, is called the *active view*.

2. The single light trip $(b\,at\,B \to O\,and\,o)$ that involves one source b is envisaged to take place in an absolute 3-pysical space [1], but viewed from two frames of reference S and s. The length of the trip is thus absolute and identical to its geometric length in the frame in which the source is stationary, which is the frame s when b is the source, whereas the relations (1.1) determine the transformations between the units of length and time in S and s. For a given choice of units of length and time in S, say *ULS* and *UTS*, there correspond units of length and time *uls* and *uts* in s such that

(2.1) $$uls = ULS.\Gamma(u,\theta), \quad uts = UTS.\Gamma(u,\theta).$$

Conversely, and for a given choice *uls* and *uts* of the units of length and time in s there correspond the units of length and time

(2.2) $$ULS = uls/\Gamma(u,\theta), \quad UTS = uts/\Gamma(u,\theta)$$

in S.

Now, if $r = r_g.uls$ is the geometric length of the trip $(b \to o)$ in s, then the S observers assign to the trip $(b \to O)$ the same length $R = r$ using however their own unit of length, namely *ULS*. Setting $R = R_r.ULS$, where r refers to the length "read" by the scale used in S, we may write

(2.3) $$R = r \Leftrightarrow r_g uls = R_r ULS \Leftrightarrow r_g \Gamma.ULS = R_r ULS \Leftrightarrow R_r = \Gamma.r_g$$

It follows that the scaling transformations are expressed in the form

(2.4) $$\frac{R_r}{r_g} = \frac{T_r}{t_g} = \Gamma(u,\theta), \quad \theta = \theta', \quad \phi = \phi'$$

The following explanatory remarks regarding a single light's trip from one source (b or B) to (O and o) are necessary:

(i)- The dimensionless quantities r_g and R_r refer to the readings of the lengths of the light's trip in s and S respectively, with $r = r_g uls$ is its absolute length in the 3-physical space, or its geometric length in s.

(ii)- If the body and its location are interchanged, so that B is the source and b is its location in s, which is equivalent to replace g by r and vice-versa, we obtain

(2.5) $$\frac{R_g}{r_r} = \frac{T_g}{t_r} = \Gamma(u,\theta), \quad \theta = \theta', \quad \phi = \phi',$$

which are the same transformations (2.4). In this case the absolute length of the trip is its geometric length $R = R_g ULS$ in S.

(iii)- It follows from (i) and (ii) that the readings of the length and duration of the trip $(B\,and\,b \to O\,and\,o)$ in S and s relate to each other by

(2.6) $$\frac{R_r}{r_r} = \frac{T_r}{t_r} = \Gamma(u,\theta),$$

whether b or B is the source, and where θ is the angle between the radius vector and the vector velocity $\vec{u}$ of s relative to S.

(iv)- For a single light's trip, or as to say one true source, the choice of the set of geometric quantities in the numerators or denominators in (2.6) is a mere convention. We may choose, as we have already did, the frame in which the body is at rest to accommodate the geometric length and duration of the light's trip. Or alternatively, we may adopt the frame S in which the body b is moving as the site of geometric measurements; the relations (2.6) is valid for either choice. The following equivalent forms of the scaling relation

(2.7) $$R = r\Gamma(u,\theta) \Leftrightarrow r = R\Gamma(-u,\theta),$$

in which the index r has been dropped, show that the radial coordinate of a source in a frame is obtained through scaling its radial coordinate in the other frame by reciprocal scaling factors.

3. The Case of Two Conjugate Sources.

If B and b are both sources of light then we have two light's pulses emanating simultaneously from B and b. Now each observer O and o receives two pulses but not simultaneously, and hence O and o are conjugate observers only for one pulse. To see that this is indeed the case, we use the fact that the length of a light's trip is absolute and equal to its geometric length. The length of the light's trip $(b \to o)$ is $r = r_g uls$, and the length of the light's trip $(B \to O)$ is $R = R_g ULS$. It follows that

(3.1) $$\Gamma(\beta,\theta) = \frac{r}{R} = \frac{r_g.uls}{R_g.ULS} = \frac{r_g}{R_g}\Gamma(\beta,\theta),$$

and hence

(3.2) $$r_g = R_g = a_l$$

Similarly

(3.3) $$t_g = T_g = a_t$$

This means that the readings of the geometric lengths and durations of the true trips $(B \to O)$ and $(b \to o)$ in S and s respectively are equal. The lengths of these trips however and their durations are not equal. Indeed

(3.4) $$r = a_l uls = a_l(\Gamma(u,\theta).ULS) = \Gamma(u,\theta).a_l ULS = \Gamma(u,\theta)R$$

(3.5) $$t = a_t uts = a_t(\Gamma(u,\theta).UTS) = \Gamma(u,\theta).a_t UTS = \Gamma(u,\theta)T$$

The last relation means that the trip $(b \to o)$ takes longer time than the trip $(B \to O)$ provided that $\Gamma(u,\theta) > 1$, and shorter time provided that $\Gamma(u,\theta) < 1$. Confining ourselves to the case $\Gamma(u,\theta) > 1$ with $u > 0$, and $0 < \theta < \pi/2$, we see that the conjugate observers O and o receive first the pulse emanating from B, and after they individuate, each receive the pulse emanating from b. Each of the observers O and o can claim itself stationary while receiving the two pulses, whereas the other has moved a certain distance after receiving the first pulse. Assuming that the emission of light from (b and B) takes place at $T = t = 0$, the observer O register $T = a_t ULS$ at receiving the first pulse and $\Gamma(u,\theta)a_t ULS = \Gamma(u,\theta)T$ at receiving the second. The difference between these is

$$\Delta T(O) = T(\Gamma(u,\theta) - 1)ULS = a_t(\Gamma(u,\theta) - 1)ULS \approx a_t\beta\cos\theta.ULS$$

The observer o register $(a_t/\Gamma(u,\theta))uls = t/\Gamma(u,\theta)$ at receiving the first pulse and $t = a_t uls$ at receiving the second. The difference is

$$\Delta t(o) = t(1 - 1/\Gamma(u,\theta))uls = a_t(1 - 1/\Gamma(u,\theta))uls \approx a_t \beta \cos\theta .uls$$

For motion along the X-axis,

$$\Delta X(O) = (\Gamma(u,0) - 1)X.ULS \approx \beta X.ULS = uT.ULS$$

3. The Scaling Transformations in One Dimension

In many practical physical problems we encounter the case of a source and an observer moving on the same line towards, or away from, each other. If we stick to the case in which the velocity of *s* with respect to *S* is positive, then the angle $\theta \equiv \angle(OX, \vec{R})$ coincides with the angle between the radius vector and the velocity vector $\vec{u}$ of the frame *s*. Under the latter choice of the direction of velocity of *s*, we recognize two particular important cases:

(i) When the observer and the source recede from each other on the same line then $\theta = 0$, (2.6) reduces to the elongation formula [2-5]

(4.1) $$\frac{X}{x} = \frac{T}{t} = \Gamma(u,0), \quad Y = Z = 0,$$

where

(4.2) $$\Gamma(u,0) = \sqrt{\frac{1+\beta}{1-\beta}}.$$

(ii) When the observer and the source approach each other on the same line then $\theta = \pi$, (2.6) reduces to

(4.3) $$\frac{X}{x} = \frac{T}{t} = \Gamma(u,\pi) = \Gamma(-u,0), \quad Y = Z = 0.$$

In part (III) of this work we set out to explain some challenging well-known physical effects using the scaling transformations. These include the life time of meta-stable particles, the Sagnac's effect [9], the Doppler's effect, the drag effect, and the null results of the Michelson's and Morley's experiment. The stellar aberration which will also receive a novel treatment without the need to introduce a sparse gas filling the space [10-12] will be postponed to part (IV) of this work.

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