Treatment of Some Optical Phenomena Using Scaling Transformations (I)

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Abstract

In earlier works [1,4,5] the transformations that yield the speed of light constant within each inertial frame was found in a form of an anisotropic scaling transformations. The latter transformations extend the longitudinal transformations [2-5] to a general case. The concept of absolute physical space which had been introduced in previous works [1,4], was promoted to replace the traditional concept of "absolute space", and the associated concepts of geometric distance and absolute units were discussed. In the current part of this work, the scaling transformations is put in action to give convincing explanations for well-known, mostly optical, phenomena.

1. Introduction

Consider a body *b* moving relative to an inertial frame $S \equiv OXYZ$ with a constant velocity $\vec{u}$. We may choose a rectangular Cartesian frame in *S* so that the body's velocity is parallel to *OX*, i.e. $\vec{u} = u\vec{i}$ $(u > 0)$ and $\vec{i}$ is the unit vector of *OX*. If *s* is another inertial frame in standard configuration with *S*, then *b* is stationary in *s*. We endow *S* and *s* with spherical coordinates (R, θ, ϕ) and (r, θ', ϕ') respectively, with *OX* and *ox* are the polar axes. Assume that when at $B(R, \theta, \phi)$ in *S*, the source *b* emits a spherical pulse which reaches the conjugate observers[1] *O* and *o* at $T = t = 0$. It was shown in the first part of this work that the light trip $(B\,and\,b \to O\,and\,o)$ can be viewed either

1. From a single frame, say *S*, with two possible starting points *B* or *b* and the same ending point *O*. i.e. $(B \to O)$ and $(b \to O)$. The transformations between the set of quantities (R, θ, ϕ, T) pertaining to location of the source at emission, and the quantities (r, θ', ϕ', t) pertaining to the body at the instant of light reception by *O*, are

(1.1)
$$\frac{r}{R} = \frac{t}{T} = \Gamma, \quad \theta = \theta', \quad \phi = \phi',$$

where

(1.2)
$$\Gamma(\beta, \theta) = (1 - \beta^2)^{-1/2}(\beta \cos\theta + \sqrt{1 - \beta^2 \sin^2\theta}),$$

and

(1.3)
$$T = R / c, t = r / c.$$

The quantities pertaining to the body were referred to as "mobile or proper", and to those pertaining to its location as "geometric or stationary". The transformations (1.1), thus, relate the geometric and mobile coordinates of a body in one frame to each other. Since the transformations (1.1) preserve the direction of the light's ray and affects an anisotropic scaling of the geometric distance by a factor $\Gamma(\beta, \theta)$, it was accordingly called the scaling transformations. The view of the light trip from one frame formally as two trips, is called the active view.

2. The light trip $(B\,and\,b \to O\,and\,o)$ is envisaged to take place in an absolute physical space[1], but viewed from two frames of reference *S* and *s*. The quantities pertaining to this trip are thus absolute and do not depend on the inertial frame within

which they are measured, whereas (1.1) are looked at as the transformations between the units of length and time in each frame. For a given choice of units of length and time in *S*, say *ULS* and *UTS*, there correspond units of length and time *uls* and *uts* in *s*, such that

(1.4) $$uls = ULS.\Gamma(u,\theta), \quad uts = UTS.\Gamma(u,\theta).$$

Conversely, and for a given choice *uls* and *uts* of the units of length and time in *s* there correspond the units of length and time

(1.5) $$ULS = uls / \Gamma(u,\theta), \quad UTS = uts / \Gamma(u,\theta)$$

in *S*. If b is a stationary source of light in s, then the length $r_g = r_a.uls$ of the trip $(b \to o)$ is geometric and is already known. If $R = R_r.ULS$ is the observed length of the trip $(b \to O)$ in S, with R_r is the read length using *S* scale, then the absoluteness of length yields

(1.6) $$R_r = r_a \, \Gamma \,,$$

and thus we have

(1.7) $$\frac{R_r}{r_a} = \frac{T_r}{t_a} = \Gamma(u,\theta), \quad \theta = \theta', \quad \phi = \phi'$$

2. Life Time of the Cosmic Ray Mesons

The scaling transformations corresponding to one dimensional motion will be employed to explain the seemingly abnormal bountiful existence of the μ-mesons particles at the sea level.

The following data characterize a typical cosmic ray meson

- It is produced at an altitude ($X \approx 60km)$,
- Its velocity is close to the velocity of light *c*.
- Its stationary lifetime is about $\tau \approx 2 \times 10^{-6} s$ during which it can approximately cover the distance $x \approx \tau.c = 0.6km$,
- The μ-mesons particles are detected abundantly at the sea level.

Let $S = OX$ be a reference frame with *O* at the sea level and *OX* directed upwards, and s be an inertial frame that is moving with the particle.

In the following the passive view is implemented.

- The geometric life time of the particle, which is its lifetime in the frame at which it is at rest is $\tau = t_g$. And the observed life time of the particle is given by $T_r = X / c$.

- by the passive view

(2.2) $$T_r = t_a \Gamma(v,0) .$$

- a particle can reach the earth surface if $T_r \geq (60 / c)\sec$, or

(2.3) $$\Gamma(v,0) \geq \frac{T_r}{t_g} = \frac{60 / c}{0.6 / c} = 100 \,,$$

which yields $\beta \geq 0.9998$.

3. The Sagnac Effect

Consider two electromagnetic waves emitted from a point at the earth's equator parallel to the equator and in opposite directions. It is experimentally verified that the spinning of the earth about its axis amounts for a complete round to approximately 207 *ns* advance (delay) for a wave propagating westward (eastward) parallel to the earth's equator[9].

Let $S \equiv OXYZ$ be the inertial frame of fixed stars with origin *O* at a point *o* of the earth's equator, and take *OX* tangent to the equator at *o* and directed eastward, so that the linear velocity *u* of *o* in *S*, when *o* is contiguous to *O*, is positive. Suppose that two pulses of light are emitted simultaneously from *o* in opposite directions parallel to the equator. Let X_e (X_w) be an *S* observer that is contiguous to *o* when light emitted eastward (westward) is received back by *o* and hence by $X_e(X_w)$. In other words, *o* and X_e (*o* and X_w) are conjugate observers when light emitted eastward (westward) is received by *o*.

The path of light circling the equator can be decomposed into straight segments with two conjugate observers, an *S* observer and an equatorial observer, at the end of each segment. The problem can thus be visualized as a linear one. Let $s \equiv oxyz$ be an inertial frame in standard configuration with *S* and moving relative to *S* with velocity *u* $(u > 0)$. We may envisage the pulse emitted eastwards from (*o* and *O*) and received by (*o* and X_e), as if received by the conjugate *S* and *s* observers (*o'* and X_e), with *o'* on the *x*-axis of *s* and at a distance *x* from *o* that is equal to the circumference of the earth, as measured in *s*. For the westward trip, the emitter *o* is approaching the observer X_w and hence the absolute units of time in *S* and *s* are in the proportion $1:1/\Gamma(u)$. For the eastward trip the emitter is receding from the observer X_e, and the absolute units of time in *S* and *s* are in the proportion $1:\Gamma(u)$.

It is important to note that the *S* time is the fixed stars time, which is also the time we use on earth, i.e. the time read by our clocks on earth. To the unit "second" of the stars time, there correspond two absolute units of time in *s*, which we name the east and west equatorial seconds. Let's denote the latter absolute units by *Esec* and *Wsec* respectively. According to the passive view of interpretation, the latter absolute units relate to the unit of time in *S* by

(3.1) $$1\sec = \Gamma(u).W\sec, \quad 1\sec = \Gamma(-u).E\sec = E\sec/\Gamma(u).$$

Similar relations hold for the units of length, say "meter":

(3.2) $$1m = \Gamma(u).Wm, \quad 1m = \Gamma(-u).Em = Em/\Gamma(u).$$

Let t_e be the geometric durations of the eastward trip $(o\ and\ O \to o'\ and\ X_e)$ in *s*. Since the length of the trip in *s* is the earth circumference in absolute *s* units, we have

(3.3) $$t_e = \frac{circum}{c} = \frac{40{,}000\,(Ekm)}{300{,}000(Ekm/E\sec)} = \frac{2}{15}E\sec.$$

Similarly $t_w = (2/15)W\sec$. The difference between these

(3.4) $$\begin{aligned} t_e - t_w &= \frac{2}{15}(E\sec - W\sec) = \frac{2}{15}(\Gamma(u) - \Gamma(-u))\sec \\ &= \frac{2}{15}\frac{2u/c}{\sqrt{1-(u/c)^2}}\sec \approx 2\times 206.33n\sec, \end{aligned}$$

represents the delay period between receiving the two waves. When calculating (3.4), the linear velocity *u* of a point of the equator was taken

$$u = 40{,}000km/23h\,56\min\times 60(\min/h)\times 60(s/\min) \approx 0.46425\,km/s.$$

4. Doppler's Effect

Let $S \equiv OXYZ$ and $s \equiv oxyz$ be inertial frames in standard configuration, and assume that *s* translates parallel to *OX* with a constant velocity *u* (*u*>0). Let *b* be a source of light that is stationary in *s*, and hence moving with a constant velocity $\vec{u} = u\vec{i}$ relative to *S*. Suppose that the source *b* is radiating a monochromatic light of a characteristic wave-length λ_0. The light emitted from *b* is received by any *s* observer and in particular by the observer *o*, as a monochromatic light of the same wave-length λ_0. If (R,θ,φ) and (r,θ,φ) are the spherical coordinates of *b* in *S* and *s* respectively, then at any instant of actual observation,

(4.1) $$R = \Gamma(\theta,u)r, \quad \theta = \theta, \quad \phi = \phi,$$

where the passive view is adopted. If the distance *r* corresponds to one wave length λ_0 in *s*, then the distance *R* corresponds to one wave length λ in *S* (which is the distance between two nodes, for example). Setting $R = \lambda$ and $r = \lambda_0$ in (4.1) yields the generalized Doppler's formula

(4.2) $$\lambda = \Gamma(\beta,\theta)\lambda_0 ,$$

which determines the wave length as measured by the stationary observer *O*. The last relation shows that $\lambda > \lambda_0$ for $0 < \theta < \pi/2$, and $\lambda < \lambda_0$ for $\pi/2 < \theta < \pi$. The source *b* is receding from the observer in the first case, and approaching it in the second.

The generalized formula (4.2) reduces, for $\theta = 0$, to the red shift relativistic Doppler's formula

(4.3) $$\lambda = \Gamma(\beta,0)\lambda_0,$$

corresponding to the source and the observer receding from each other. For $\theta = \pi$, the relation (4.2) reduces to the blue shift relativistic Doppler's formula

(4.4) $$\lambda = \Gamma(\beta,\pi)\lambda_0,$$

corresponding to the source and the observer approaching each other.

For $\theta = \pi/2$, the relation (4.2) reduces to

(4.5) $$\lambda = \Gamma(\beta,\tfrac{\pi}{2})\lambda_0 = \lambda_0,$$

which, contrary to the relativistic prediction, shows that there is no traverse Doppler's effect.

5. The Drag Effect

This effect is explained in the special theory of relativity using the law of velocity addition [8], which is also valid in the current theory as it is shown below.

Let $b(x,0,0) \in s$, with $x > 0$, be a source of light moving with a velocity $u\,(u > 0)$ in *S*. Assume that *S* is moving with velocity *v* (*v*>0) relative to a third reference frame S' whose origin *O'* is contiguous to *O* at the instant of light's reception. Adopting the passive view, we find that the transformations from *s* to S' is

(5.1) $$X' = \Gamma^{-1}(v)X = \Gamma^{-1}(v)\Gamma^{-1}(u)x = \Gamma^{-1}(V)x,$$

where

(5.2) $$\Gamma(V) = \sqrt{\frac{1+V/c}{1-V/c}},$$

and

(5.3) $$V = \frac{v+u}{1+uv/c^2}.$$

Equation (5.2) is interpreted as asserting that the frame *s* moves relative to S' with velocity *V* given by (5.3), which is the same law of velocity addition in special relativity.

References

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