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Implication of Contiguous Equivalence On Inter-Bodies' Time and Distance

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Abstract
We introduce first the concept of inertial frames that are contiguous to an inertial frame. It is argued that an inertial frame is physically equivalent to the set of inertial frames that are contiguous to it. Utilizing this equivalence in conjunction with Galilean transformation of velocity requires on one hand adopting besides the familiar time and distance new m-time and m-distance, and on the other hand leads to a contradiction that is resolved in accord with the symmetric status of the inertial frame and that which is contiguous to it. A major conclusion from this formulation is that the Euclidean geometry of the physical space must be reserved only to the static space, i.e. to the space composed of all points that are stationary with respect to each other. The distance at some instant t between any two points that have a non-zero relative velocity is no more equal to the distance separating their stationary locations at that instant. The new m-distance equal to the product of a factor $\gamma > 1$ by the stationary distance. It shown that the m-distance shrinks by a factor $1/\gamma$ if the points are approaching each other, and expands by the factor γ if the points are receding from each other. Similar statement holds for the time associated with this distance. The factor γ tends at both infinities to the corresponding relativistic Doppler's factors. As the law of relativistic velocity addition is a direct consequence of the transformation between the m-time and distance and the s-time and distance, the drag effect is readily explained. The Doppler's and aberration effects are explained in a straightforward manner. The independence of the velocity of light of the relative motion between the source and observer is a consequence of the theory, and accordingly all experiments which require for its explanation this assumption are also explicable by the current theory.

1. Introduction
We begin with a simple example in which we consider the Galilean transformations from a slightly different point of view. This will lead us eventually to the concept of contiguous inertial frames. Let us imagine a long train passing by a station deck with a uniform velocity $\vec{u}$. Let $S \equiv (S, O_0) \equiv O_0xyz$ be a rectangular Cartesian frame of coordinates attached to the deck, with O_0 is a fixed point of the deck and O_0x is along the train's motion. Let O_0' be a point of the train that coincides with O_0 at $t = 0$, and $(S', O_0') \equiv O_0'X'Y'Z'$ be a rectangular Cartesian system of coordinates fixed to the train and in standard configuration with (S, O_0). The Galilean transformations between (S, O_0) and (S', O_0') are

$$x = X' + ut, \quad y = Y', \quad z = Z', \quad t = t'.$$

Let b' be a point with coordinates (x, y, z) in the deck frame (S, O_0). Instead of specifying the coordinates of b' relative to the train frame (S', O_0'), i.e. relative to the train frame with origin at O_0', we specify it relative to the frame (S', O_t'), which is a frame moving with the train, but it has its origin at a point O_t' that is contiguous to

O_0 at the instant t. According to this way of coordinatization, an infinite set of adjacent inertial frame all moving with the train is employed, and of which one at a time is used to specify the coordinates of the point b'. If the coordinates of b' are (x, y, z) in S and (x', y', z') in (S', O'_t), then

$$x = x', \quad y = y', \quad z = z', \quad t = t'$$

The Galilean transformations are recovered on noting that the coordinates of O'_t, or O_0, with respect to (S', O'_0) are (-ut,0,0), so that

$$x = x' = X' + ut, \quad y = Y', \quad z = Z', \quad t = t'.$$

It is clear that the set of measurements $\{(x', t')\}$ gathered by the observers (S', O'_t) is identical to the set $\{(x, t)\}$ gathered by (S, O_0), and consequently, the velocity, acceleration, and force related to the material point b' are the same. Assuming b' is a point fixed to the train, then its velocity relative to (S, O_0) is u. Although b' is at rest relative to each frame (S', O'_t), its velocity with respect to the systems of frames (S', O'_t) is

$$dx'/dt = dx/dt = u.$$

We may imagine b' as a point of a train that is composed of extremely tiny compartments, and the position of b' is determined at each instant with respect to the compartment that is contiguous to the point O_0 of the deck. The point b' is at rest relative to each compartment contiguous to O_0 but has velocity u when more than one compartment is involved. On the other hand, a particle which is at rest in the deck frame, has velocity (-u) relative to each compartment contiguous to O_0, but is stationary when many compartments are involved. We should keep in mind that the compartments' size is as small as we please. Also, it is important to note that the velocity v of a particle that is measured in (S, O_0) is the same as that measured by the set of observers $\{(S', O'_t)\}$. However, and relative to one contiguous frame the velocity of the particle will be $v - u$.

Definition: Let (S, O_0) be an inertial frame. The set of inertial frames (S', O'_t) having the properties:

(i) O'_t is coincident with O_0 at the instant t, and $O'_t O'_0 = ut\vec{i}$.

(ii) At each instant t, the measurements of the position of the particle and the corresponding time are made with respect to (S', O'_t),

shall be called a u-contiguous frame to (S, O_0).

Note that if (S', O'_t) is u-contiguous to (S, O_0), Then (S, O_t) is –u-contiguous to (S', O'_0).

We shall assume that the velocity of light within each inertial frame equals to a constant c. By this we shall mean that if light is emitted at an instant t_0 from the point $\vec{r}_0$ and received at an instant t at the point $\vec{r}$, where the points $\vec{r}_0$ *and* $\vec{r}$ are stationary in the inertial frame S, then $|\vec{r} - \vec{r}_0|/(t - t_0) = c$. We have assumed implicitly that clocks in each inertial frame can be synchronized by the usual procedure of light signals (Rindler,1977 ; Mould, 1994) so that time is absolute in each inertial frame. This latter assumption regarding the constancy of the light's velocity within each inertial frame may hardly be counted as a postulate. It may be instead, considered as a natural consequence that follows from the equivalence of inertial frames. In comparison, the special theory of relativity assumes that the velocity of light emitted from a source is the same in all inertial frames, and thus, is independent of the state of relative motion between the source of light and the observer. In our assumption the source is stationary relative to the observer. For start we shall apply the Galilean law of velocity addition to light signals emitted from a moving source and show that this results in a contradiction, which requires for its elimination, adopting a new transformation on one hand and introducing two types of time intervals on the other hand.

2. Longitudinal Motion

Consider a source of light b' moving relative to the inertial frame $S \equiv (S, O_0)$ along the x-axis with constant velocity $\vec{u} = u\vec{i}$, where $\vec{i}$ is the unit vector of the x-axis, and suppose that b' is approaching O_0 from left. The light emitted by the source b' and received latter by O_0 at some instant $t = 0$, may be envisaged by the observer O_0, as well as by any S-observer, in either of the following ways:

(i) Light has been emitted from the source b', and in accord with the Galilean law of velocity addition, it should acquire a velocity $c + u$ relative to O_0, and to any *S*-observer.

(ii) Light has been emitted from the stationary point b_0 in *S*, which was occupied by the source at an earlier instant $t = x_0 / c$, where x_0 is the coordinate of b_0. The pulse takes by the first point of view a duration $t_0 = -x_0 / c$ to reach O_0, and $t'_0 = -x_0 /(c+u)$ by the second view. Thus we have $-x_0 = ct_0 = (c+u)t'_0$, and hence

(2.1) $$t_0 = (1 + u/c)t'_0$$

This current view is unaltered had we introduced a true source of light fixed at b_0 and sends a pulse of a very short duration as b' passes through the point b_0. We shall describe the latter source as virtual although it may be a true source.

Consider the inertial observer O'_0 who is moving relative to S with velocity $\vec{u} = u\vec{i}$ and just passing by O_0 when light reaches O_0 (and of course O'_0). Suppose that the observer O'_0 is endowed with an inertial frame $(S', O'_0) \equiv O'_0 x'y'z'$ in standard configuration with S. Relative to (S', O'_0) the source b' is stationary, whereas b_0 is moving away from O'_0 with velocity (-u). Using the Galilean law of velocity addition, the observer O'_0 deduces that the time intervals t_0 *and* t'_0, considered above, are related by the equation $(c-u)t_0 = ct'_0$, or

(2.2) $$t'_0 = (1 - u/c)t_0.$$

Being physical quantities for both observers O_0 *and* O'_0, the intervals t_0 *and* t'_0 refer to the same quantities in equations (2.1) and (2.2). Indeed t_0 is the time interval taken by light if emitted from b_0 to reach O_0 *and* O'_0, whereas t'_0 is such time interval if light is emitted from b'. However, and if we substitute for t_0 *and* t'_0 from one equation into the other, we obtain

(2.3) $$t_0 = (1-\beta^2)t_0, \quad t'_0 = (1-\beta^2)t'_0,$$

where $\beta = u/c$. To resolve this contradiction and maintain at the same time the symmetric status of the observer O_0 *and* O'_0, we have to scale the right hand-sides of equations (2.1) and (2.2) through multiplying by $1/\sqrt{1-\beta^2}$. This scaling process yields the relations

(2.4) $$t_0 = \sqrt{\frac{1+\beta}{1-\beta}}\,t'_0, \qquad t'_0 = \sqrt{\frac{1-\beta}{1+\beta}}\,t_0.$$

Setting

(2.5) $$\gamma = \sqrt{\frac{1+\beta}{1-\beta}},$$

we write the latter equations as

(2.6) $$t_0 = \gamma t'_0, \quad t'_0 = \gamma^{-1} t_0.$$

Since the true source is stationary relative to (S', O'_0), its distance form O'_0 (and $O_0)$ when light reaches these two points is $-x'_0 = ct'_0$. Similarly, and since the virtual source is stationary relative to (S, O_0), its distance from $O_0\,(and\, O'_0)$ when light reaches these points is $-x_0 = ct_0$. Thus we have

(2.7) $$x_0 = \gamma x'_0, \quad x'_0 = \gamma^{-1} x_0 \ .$$

When light reaches $O_0\, and\, O'_0$ the observer O_0 describes the process of light emission and reception as follows: At an earlier instant $t = x_0 / c$ the true and virtual sources were adjacent at b_0 and each emitted a pulse of light. At the moment O'_0 was contiguous to me(which I have already taken as $t = 0$), light has reached both of us. At this moment (t=0) the virtual source is at a distance $(-x_0)$ from me, whereas the true source is at a distance $-x'_0 = -\gamma^{-1} x_0$. The observer O'_0 describes the same process as: at an earlier instant $t' = x'_0 / c$ the true and virtual sources were adjacent at b' and each emitted a pulse of light. At the moment O_0 was contiguous to me (which I take t=0) light has reached both of us. At this moment $(t = 0)$ the true source is at a distance $(-x'_0)$ from me, whereas the virtual source is at a distance $-x_0 = -\gamma^{-1} x'_0$. The relations (2.6) and (2.7) hold as long as b' approaches O_0. However, as b' bypasses O_0 and travels away, these relations become

(2.8) $$t_0 = \gamma^{-1} t'_0, \quad t'_0 = \gamma t_0$$

(2.9) $$x_0 = \gamma^{-1} x'_0, \quad x'_0 = \gamma x_0.$$

We have seen that the nature of b_0 as a true or a virtual source has no effect on the results we have obtained. Thus our treatment may be considered to involve a system of two sources, one of each is stationary in one frame and moving in the other. The moving (true) source in S approaches O_0, while the moving (virtual) source moves away from O'_0. The observers O_0 and O'_0 associate with each body (source), or more precisely with the distance separating him from the source, a certain time interval $t\ or\ t'$, so that the same time interval is associated with the same body. Both observers $O_0\, and\, O'_0$ agree that: the time intervals $(b_0 \to O_0), (b_0 \to O'_0)$ are equal to t_0; the time intervals $(b' \to O_0)$, $(b' \to O'_0)$ are equal to t'_0, and that $t_0 = \gamma t'_0$, provided that b' is approaching O_0. An equivalent alternative view is the following: denote by $s_0 = -x_0$

the distance between $b_0\, and\, O_0\,(or\, O'_0)$ and by $s'_0 = -x'_0$ the distance from $b'\, to\, O_0\,(or\, O'_0)$, then $s_0 = \gamma s'_0$. The observer O_0 finds that the moving body b' (towards $O_0)$ is closer than its location b_0 by a factor $1/\gamma$. Similarly, O'_0 finds the distance of the hypothetical source b_0, which is moving away from O'_0, is larger than the distance of its location b' by a factor γ.

Instead of considering the body b' is moving relative to (S, O_0) we may assume that O_0 is moving relative to the frame (S', O'_0) in which b' is momentarily stationary, and we get the same result: the body b' seems closer to $O_0\,(and\, O'_0)$ than its location b_0.

We shall refer to $x_0\, and\, x'_0$ as the body's stationary and moving coordinates respectively, or in brief, the s- and m-coordinate respectively. Note that if we let $x_0\, and\, x'_0$ refer to the s- and m-coordinate of b' at an arbitrary instant t, then

$$\frac{dx_0}{dt_0} = \frac{dx_0'}{dt_0'} = u.$$

Note also that the velocity of light calculated as the quotient of the distance it travels to the relevant time is c, whether this quotient involves the s-coordinate and s-time, or the m-coordinate and m-time:

$$\frac{-x_0}{t_0} = \frac{-x_0'}{t_0'} = c$$

3. The Inter-Bodies Time

Let b_1 be a point of the x-axis with coordinate x_1, so that $x_1 - x_0 = \Lambda > 0$. Suppose that the source emits its first and second pulse at $x_0 \, and \, x_1$ respectively. Following reasoning similar to that we have just carried out in section 2, we obtain

(3.1) $$x_1 = \gamma x_1' \qquad x_1' = \gamma^{-1} x_1,$$

where $(-x_1)$ is the distance traveled by light had it been emitted from b_1 and till it reaches $O_0 \, (or \, O_1'$ which is a point of (S', O_0') just passing by O_0 when light from b_1 reaches $O_0)$. Similarly, $(-x_1')$ is the distance traveled by light considered as emitted from b' at the position b_1 and till it reaches $O_0 \, (or \, O_0')$. Subtracting (2.9) from (3.1) yields

(3.2) $$\Lambda = x_1 - x_0 = \gamma (x_1' - x_0')$$

This shows that the s-distance $x_1 - x_0$ is γ times greater than the m-distance $x_1' - x_0'$. In the S-frame or $O_0 -$ time, a duration $t = \Lambda / u$ elapses between the passage of the body from $b_0 \, to \, b_1$, and hence

(3.3) $$ut = \gamma (x_1' - x_0') = \gamma u t',$$

where t' is the time corresponding to the m-distance $x_1' - x_0'$. From (3.3) we have

(3.4) $$t = \gamma t', \quad t' = \gamma^{-1} t.$$

The relations (3.4) and (3.2) shows that the m-time and the m-distance of $b' \, from \, O_0$ are $1/\gamma$ of the corresponding s-time and s-distance. It is tempting to identify the m-time as the body time as endowed by the the observer O_0. This however, will lead to the following contradiction: Consider both $b' \, and \, O_0$ as sources and observers, and assume that b' sets out from O_0 with velocity u relative to O_0, moves to apoint at distance s, and returns with the same velocity to O_0. The observer O_0 register fro this trip aperiod $2t = 2s/u.$ The m-time of the first stage of the trip (b' is moving away from $O_0) \, is \, t_1' = \gamma t,$ and the m-time of the second stage (b' is coing back) is $t_2' = t/\gamma.$ Thus

$$t_1' + t_2' = \gamma t + t/\gamma = \frac{2t}{\sqrt{1-\beta^2}}.$$

Thus O_0 finds that b' ages more. Similarly, b' finds the duration of the trip of O_0 is $1/\sqrt{1-\beta^2}$ longer than the duration of his own trip, and accordingly finds that O_0 ages more. This is the twin paradox in special relativity, however with opposite results.

We shall thus interpret t' as an attribute of the state of the spatial separtion between $b' \, and \, O_0$, i.e. t' depends on the rate of change in the distance $b'O_0$. The of the body b' is the same as the time of the observer

O_0 since both can be regarded as inertial labs. It is only the durations of signals transmitted from b' in its route to or vice versa, that undergo contraction or expansion by a factor γ.

If x_0 is the body's stationary coordinate at $\tau_0 = -x_0 / c$ and X is its stationary coordinate at t $(t > \tau_0)$, then

(3.5) $$X = x_0 + u(t - \tau_0)$$

(3.6) $$= \gamma[x_0' + u(t' - \tau_0')],$$

where x_0' is the body's m-coordinate at $\tau_0' = -x_0' / c.$ The m-coordinate X' at an instant t' is

(3.7) $$X' = x_0' + u(t' - \tau_0')$$

The relation (3.6) is valid as long as the body is approaching O_0, i.e. as long as $X < 0.$ For $X > 0, \gamma$ has to be replaced in (3.6) by $1/\gamma$.

4. Velocity Addition

Suppose that the velocity of a body as measured in an inertial frame (S', O_0') is v, and that the inertial frame (S, O_0) measures the velocity of (S', O_0') as u. If x' is the body's s-location in S', then its m-location in S' is $x'' = \gamma^{-1}(v)x'$. Since the s-location of the body in S' is moving relative to S with a velocity u, we have $x' = \gamma^{-1}(u)x,$ and $x'' = \gamma^{-1}(v)\gamma^{-1}(u)x,$ where x is its s-location in S. If the initial position of the body is to the left of O_0, u and v are positive, then

$$\gamma^{-1}(v)\gamma^{-1}(u) = \sqrt{\frac{1-\beta_1}{1+\beta_1}}\sqrt{\frac{1-\beta_2}{1+\beta_2}} = \sqrt{\frac{1-\beta}{1+\beta}} \ ,$$

where $\beta_1 = v/c, \beta_2 = u/c,$ and

(4.1) $$\beta = \frac{V}{c} = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$$

The last relation, which is easily verifiable, is the law of velocity addition in our theory as well as in special relativity. Thus the observer O_0 assigns to the body a moving coordinate $x'' = \gamma^{-1}(V)x$ corresponding to the motion of the body with velocity V given by (4.1) relative to S.

The Drag Effect

This effect is explained in the special theory of relativity by making use of the law of velocity addition (4.1), which is in common between the current theory and the special relativity theory (see Rindler, 1977; Mould, 1994).

5. The Doppler Effect

Suppose that a source of light is stationary in S at the point x_0 and radiating a monochromatic light of frequency ν and period $\tau = 1/\nu$. This is received also at O_0 as a monochromatic light of frequency $\nu.$ Suppose now that the source is in motion with velocity u relative to S. The period of light will be $\tau' = \gamma^{-1}\tau,$ and hence

$$\frac{\nu'}{\nu} = \frac{\tau}{\tau'} = \gamma = \sqrt{\frac{1+\beta}{1-\beta}} \ ,$$

which is Doppler's effect.

The latter result can also be reached through the following analysis. Had the body been stationary in S, an n periods would have covered an interval $\Delta t = n\tau$. As the body is moving there corresponds to Δt the inter-time interval $\Delta t' = \gamma^{-1}\Delta t = n(\tau/\gamma)$, and hence $\tau' = \tau/\gamma$. Should the body be moving away from O_0, the radiation will have the frequency $\nu'' = \gamma^{-1}\nu$.

6. The General Case

If the source b' has an arbitrary vector velocity $\vec{u}$ relative to the inertial frame (S, O_0) we may assume without loss of generality that $\vec{u}$ is parallel to the x-axis, for we may always rotate the S-axes so that the x-axis in the direction of $\vec{u}$. Let $\{S'_t\}$ be a set of u-contiguous frames to S. The light reaching the observers O_0 *and* O'_0 at an instant taken by both observer as $t = 0$, may be viewed by the observer O_0 (as well as by any other S-observer) in either of the following two ways:

(i) Light had emanated from a point b_0 in S which was occupied by the source b' at an earlier instant $t = -r_0/c$, where $r_0 = |O_0 b_0|$ is the length of the vector $O_0 b_0$. By this view $r_0 = ct_0$, where t_0 is the time interval between the emission of light at b_0 and receiving it at O_0. In accordance with this view, the ray reaching O_0 should had been ejected in a direction $\vec{e} = b_0 O_0 / |b_0 O_0|$. Had b_0 been a true source, nothing would have changed regarding the previous view. We may therefore and whenever it is convenient, suppose that there exists, in addition to the moving source b', a stationary source b_0 that emits, only when b' is passing by it, a pulse of a very short duration.

(ii) Alternatively, an S-observer may consider the ray received at O_0 as had been ejected from b' in a direction $\vec{e}_L$ to the left of $\vec{e}$ (Fig.1), so that $c\vec{e}_L + u\vec{i}$ is along $\vec{e}$. We have used in the latter deduction the law of velocity addition in Newtonian mechanics. An equivalent argument is the following: The observer O_0 can consider himself to be moving with velocity $-\vec{u}$ relative to a frame $\bar{S}$ whose origin is b'. In its own frame, b' emits spherical waves. The velocity of a photon in a beam reaching O_0 can be decomposed in (S, O_0) into two components: The first, $u\vec{i}$, is along the x-axis; the second is in a direction $\vec{e}_L$, so that $c\vec{e}_L + u\vec{i}$ is along $b_0 O_0$. The triangle of velocity addition is similar to the triangle $O_0 b_0 B'$. If t'_0 is such that

$$t'_0 \left| c\vec{e}_L + u\vec{i} \right| = |b_0 O_0| = r_0,$$

then t'_0 is the time taken by light since its emission from the moving source b' and till it reaches O_0. From the triangle $O_0 b_0 B'$, whose sides are

$$|B' O_0| = ct'_0, \quad |b_0 B'| = ut'_0, \qquad |b_0 O_0| = ct_0$$

we calculate $|B' O_0| = ct'_0$

(6.1) $$c^2 t_0'^2 = c^2 t_0^2 + u^2 t_0'^2 + 2uct_0 t'_0 \cos\theta$$

where θ is the angle between the vector $O_0 b_0$ and the x-axis. Solving for t'_0 we get

(6.2) $$t'_0 = (1-\beta^2)^{-1}(\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta})t_0$$

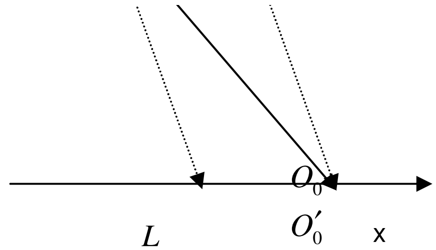

Fig.1

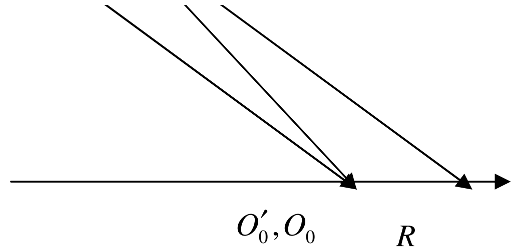

Fig.2

Fig 1. Emission of light from b_0 and b' as viewed by O_0.

The point B' is the position of b' when the light reaches O_0.

$\angle(O_0x, O_0b_0) \equiv \theta,\ |b_0O_0| = ct_0, |b_0B'| = |LO_0| = ut'_0, |b_0L| = |B'O_0| = ct'_0.$

Fig.2. Emission of light from b_0 and b' as viewed by O'_0.

The point B_0 is the position of b_0 when light reaches O'_0.

$\angle(O'_0R, O'_0b') = \theta,\ |b'O'_0| = ct'_0, |b'B_0| = |O'_0R| = ut_0,\ |b'R| = |BO'_0| = ct_0.$

Before proceeding to consider the view of O'_0, it is constructive to state afresh that the location of b' at the instant of light emission is the same in S and S'_0 (which inherits the values already measured by S'_t). The same fact holds for the remaining points occupied by the traveling pulse. However, the velocities of each point when looked at from S and S'_0 (including b_0) are related by $\vec{v}' = \vec{v} - \vec{u}$ $(or\ \vec{v} = \vec{v}' - \vec{u})$. In the frame (S', O'_0) which is u-contiguous to (S, O_0) the source b' is stationary and b_0 is moving with velocity $-\vec{u}$. The $S'-$ observers' version of description of light emission from b' and b_0 and its reception at O'_0 is shown in Fig.2. With respect to these observers:

(i) When light is emitted from the stationary source b', we have

$$ct'_0 = |b'O'_0| = r'_0.$$

(ii) Had light been emitted from the point b_0, then the ray reaching O'_0 must had been ejected in a direction $\vec{e}_R = b'R/|b'R|$, so that $c\vec{e}_R - u\vec{i}$ is along $b'O'_0 \equiv b_0O_0$. If t_0 is such that

$$r'_0 = |b'O'_0| = t_0|c\vec{e}_R - u\vec{i}|,$$

then t_0 is the time taken by light to reach O'_0 (or O_0), had it been emitted from b_0. From the triangle of velocity addition, which is similar to $b'BO'_0$ we calculate $|BO'_0| = ct_0$:

(6.3) $$c^2t_0^2 = c^2t_0'^2 + u^2t_0^2 - 2uct_0t'_0\cos\theta,$$

and hence

(6.4) $$t_0 = (1-\beta^2)^{-1}(-\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta})t'_0$$

This result could have been obtained by replacing $\vec{u}$ in (6.2) by $-\vec{u}$. The quantities t_0 and t'_0 in (6.2) and (6.4) have the same meaning. Indeed, t_0 refers to the time interval taken by light to travel the distance from b_0 to O'_0 (or O_0), had light been emitted from b, and t'_0 is the time interval taken by light to travel the distance from b' to O' (or O) had light been emitted from b'. Substituting for t'_0 from (6.2) into (6.4) yields the contradiction

$$t_0 = t_0 /(1-\beta^2),$$

which is resolved evenly through multiplying the right hand-sides of (6.2) and (6.4) by $\sqrt{1-\beta^2}$, to obtain

(6.5) $$t_0 = \frac{1}{\sqrt{1-\beta^2}}(-\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta})t'_0$$

Setting

(6.6) $$\gamma(\beta,\theta) = (1-\beta^2)^{-1/2}(-\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta})$$

We write the relations between t_0 and t'_0 in the form

(6.7) $$t_0 = \gamma(\beta,\theta)t'_0, \quad t'_0 = \gamma(\beta,\theta)^{-1}t_0,$$

where

(6.8) $$\gamma(\beta,\theta)^{-1} = (1-\beta^2)^{-1/2}(\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}) = \gamma(-\beta,\theta)$$

Since the speed of light within each frame is c, we have $r_0 = |O_0 b_0| = ct_0$ and $r'_0 = |O'_0 b'| = ct'_0$, and hence

(6.9) $$r_0 = \gamma(\beta,\theta)r'_0, \qquad r'_0 = \gamma(\beta,\theta)^{-1}r_0$$

Aberration

Reverting to equation (6.2) which on multiplying its right hand-side by $\sqrt{1-\beta^2}$ conform with the correct relation between t_0 and t'_0 we deduce that Fig.1 conform with this relation only when the length $|b_0 O_0|$, or equivalently t_0, is multiplied by $\sqrt{1-\beta^2}$. Similarly, equation (6.4) conforms with the relation (6.5) only if the length $|O'_0 b'|\ (or\,|O_0 b'|)$ is multiplied with the latter factor. In Fig.1 the angle $\phi \equiv \angle b'O_0 B' = \angle O_0 b'L$ is the aberration angle. Using the relevant triangle we get

$$\frac{\sin\phi}{ut'_0} = \frac{\sin\theta}{ct'_0}$$

This yields

$$\phi \approx \sin\phi = \frac{u}{c}\sin\theta,$$

which is the familiar relation (French, 1968) that determines the aberration angle.

7. The factor γ

The expression (6.6) shows that γ exists and real if $|\beta| < 1;$ which is equivalent to the limitation $|u| < c$ on the velocity of any material body. Now

$$\sqrt{1-\beta^2\sin^2\theta} > \sqrt{1-\sin^2\theta} = |\cos\theta| > |\beta\cos\theta|,$$

and hence γ is positive as it should be. The latter inequality also justifies why we did discard the root corresponding to the minus sign when solving the quadratic equations (6.1) and (6.3).

To see how does γ varies with θ we consider its partial derivative $\partial\gamma/\partial\theta$. It is easy to derive the following expression

$$\frac{\partial\gamma}{\partial\theta} = \gamma\frac{\beta\sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}$$

which is positive for all values of θ in the range $(0,\pi)$. Thus γ increases monotonously with θ from the value

$$\gamma(\beta,0) = \sqrt{\frac{1-\beta}{1+\beta}}$$

to the value

$$\gamma(\beta,\pi) = \sqrt{\frac{1+\beta}{1-\beta}} \ .$$

Since $\gamma = 1$ at $\theta = \pi/2,$ the value of γ is less than 1 for $\theta < \pi/2,$ and greater than 1 $\theta > \pi/2.$ There follows that if a body is moving relative to S with velocity $u > 0,$ then for $\theta \in (\pi/2, \pi],$ its m-distance r' from O_0 is less than its s-distance r from O_0 by a factor $\gamma(\beta,\theta).$ An identical statement holds for the s-time and the m-time. The converse situation is encountered if $u < 0,$ because $\gamma(-\beta,\theta) = \gamma(\beta,\theta)^{-1}.$

For $\beta << 1$ we have

$$\gamma = (1+\tfrac{1}{2}\beta^2 ....)(-\beta\cos\theta + 1 - \tfrac{1}{2}\beta^2\sin^2\theta....) \cong 1-\beta\cos\theta + \frac{1}{2}\cos^2\theta.$$

Hence

$$\gamma(\beta,0) \cong 1-\beta+\tfrac{1}{2}\beta^2$$
$$\gamma(\beta,\pi/2) = 1$$
$$\gamma(\beta,\pi) \cong 1+\beta+\tfrac{1}{2}\beta^2.$$

Since

$$\frac{\partial\gamma}{\partial\theta}(\frac{\pi}{2}) = \beta(1-\beta^2)^{-1/2} \cong \beta + \frac{1}{2}\beta^3,$$

the approximate value of γ at $\Delta\theta + \tfrac{1}{2}\pi$ is

$$\gamma(\Delta\theta + \pi/2) \cong 1 + \beta\Delta\theta.$$

The variation of γ is shown in the following table

θ	0	$\pi/2$	π
γ	$\sqrt{\frac{1-\beta}{1+\beta}}$	1	$\sqrt{\frac{1+\beta}{1-\beta}}$

Note that all bodies with the same velocity $\vec{u}$ have the same factor $\gamma(u,\theta)$ when they lie on a circular half-cone with half-vertex angle $\theta.$ The same value of $\gamma(u,\theta)$ is possessed by all bodies lying on the other half of the cone but having a velocity $(-\vec{u}),$ because $\gamma(u,\theta) = \gamma(-u,\pi-\theta).$ The latter statements show that

(i) The transformation $(r,\theta) \to (r',\theta)$ is axially symmetric about ox.

(ii) The transformation is effected by the same factor γ for all points of the half-cone with half-vertex angle $\theta,$ and by $1/\gamma$ on the other half.

(iii) The transformation reduces to the identity on the plane $x = 0.$

We consider next the variation of γ with u. Since

$$\frac{\partial\gamma}{\partial u}=-\frac{\gamma}{c}(1-\beta^2)^{-2}\cos\theta$$

and $\gamma>0,$ the sign of $\partial\gamma/\partial u$ is opposite to that of $\cos\theta.$ Thus we have

$$\partial\gamma/\partial u<0 \qquad for\ \ 0\le\theta<\pi/2$$

$$\partial\gamma/\partial u>0 \qquad for\ \frac{\pi}{2}<\theta\le\pi$$

It is easy to check that

$$\lim_{\beta\to 0}\gamma(\beta,\theta)=1 \qquad \theta\in[0,\pi],$$

and

$$\lim_{\beta\to 1}\gamma(\beta,\theta)=0 \qquad \theta\in[0,\pi/2]$$

$$=+\infty \qquad \theta\in(\pi/2,\pi]$$

For $\theta=\pi/2,\ \gamma(u,\pi/2)=1$ for any value of u.

8. Conclusion

The work we have presented hinges on two starting points. The first is the concept of an inertial frames $S'_t\equiv(S',O'_t)$ that are u-contiguous to an inertial frame $S=(S,O_0)$, together with the realization of its classical equivalence to S as far as positions and time intervals are concerned. The Galilean law of velocity addition which proves valid for low velocities was our second starting point. This law was used to combine the velocity of light and the relative velocity of the source and the observer. The self-contradictory result obtained was evenly resolved, giving rise to specific relations between the stationary time (and distance) and the moving time (and distance). After we have found the relations between the s-time (s-distance) and the m-time (m-distance) the contiguous frame (S',O'_t) becomes completely redundant, and is dismantled.

As far as the results obtained in favour of the work we have developed, we mention that
(i)The velocity of light, which is in relativity theory postulated constant and independent of the relative velocity between the source and observer, turns out to be a consequence of formulation of the current work. There follows from this fact that any phenomena which requires merely the constancy of the velocity of light for its explanation is readily explicable by the current work.
(ii) The familiar relativistic law of velocity addition is also valid in the present work.
(iii) The theory was put to test for explaining the well-known optical phenomena that challenged pre-relativity physics, such as drag effect and aberration. The ease at which these phenomena were explained promotes confidence in the new approach.

The current work, and up to this stage, is clearly distinct of relativity. The concepts of time and distance are radically different from that in relativity. Subsequent works by the author however, will show that most of the relativistic dynamical facts, including the mass energy equivalence, are preserved in the current work.

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