Opposing hypotheses of the reflection of light applied to the Michelson interferometer with a particular geometry

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Abstract: The derivation of light paths in the Michelson interferometer is based on the hypothesis that the incident speed and reflected speed of the wavefront of a ray of light are equal in the frame at absolute rest. In this case, the Michelson–Morley experiment predicts a fringe shift of 0.40. With the hypothesis that the incident speed and reflected speed of the wavefront of a ray of light are equal in the inertial frame of a mirror at the instance of collision, the Michelson interferometer with a particular geometry predicts zero fringe shift, which is in agreement with the result of the Michelson–Morley experiment. © *2021 Physics Essays Publication.*
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Résumé: La dérivation des chemins lumineux dans l'interféromètre de Michelson est basée sur l'hypothèse que la vitesse incidente et la vitesse réfléchie du front d'onde d'un rayon de lumière sont égales dans le cadre au repos absolu. Dans ce cas, l'expérience de Michelson-Morley prédit un décalage marginal de 0.40. Avec l'hypothèse que la vitesse incidente et la vitesse réfléchie du front d'onde d'un rayon de lumière sont égales dans le cadre inertiel d'un miroir au moment de la collision, l'interféromètre de Michelson avec une géométrie particulière prédit un décalage de frange nul, qui est en accord avec le résultat de l'expérience de Michelson-Morley.

Key words: Geometrical Optics; Speed of Light; Reflection of Light; Interference of Light; Michelson Interferometer; Michelson–Morley Experiment; Elastic Collision Ball-Wall.

I. INTRODUCTION

The speed of light is considered independent of the motion of the light source. Therefore, the speed of light from a source is the constant speed c in the frame at absolute rest if the source is within this frame or an inertial frame.

An inertial frame travels at a speed v in the frame at absolute rest. A wavefront of a ray of light from a source travels at speed c and collides with a mirror at rest in the inertial frame. It is hypothesized that the incident speed and reflected speed of the wavefront of a ray of light are equal in the inertial frame of the mirror at the instance of collision. With this hypothesis, the reflected speed of the wavefront in the frame at absolute rest can be calculated.

This paper presents a detailed theoretical analysis of the reflection of light in the empty space of the frame at absolute rest. This paper aims to derive the formula for the speed of a wavefront of a ray of light reflected by a mirror in motion, as seen by a hypothetical observer in the frame at absolute rest; then, the formula is applied to the Michelson interferometer.

The formula for the speed of the reflected wavefront applied to the Michelson interferometer with a particular geometry predicts zero fringe shift, which is supported by the result of the Michelson–Morley experiment.[1,2] The velocity of light is considered to be independent of the velocity of its source, which is in accordance with astronomers' observations of binary stars[3,4] and the experiment performed at CERN, Geneva, in 1964.[5]

Section II derives in the empty space of the frame at absolute rest the formula for the speed of a wavefront of a ray of light reflected by a mirror in motion. The derivation applies the hypothesis that the incident speed and reflected speed of the wavefront of a ray of light are equal in the inertial frame of a mirror at the instance of collision.

Section III applies the result obtained in Section II to the Michelson interferometer with a particular geometry. Section IV applies the hypothesis that the incident speed and reflected speed of the wavefront of a ray of light are equal in the frame at absolute rest to the same interferometer. The Michelson interferometer with a particular geometry has the beam splitter placed at an angle of 45° and the two opaque mirrors placed at angles of 90° and 0° to the direction of light from the source. The length of the interferometer arms is 11 m, as in the Michelson–Morley experiment.

Per the notations used in this study, the points indicated by letters without an index correspond to points seen by an observer in the inertial frame. The points indicated by letters with an index are instances of inertial frame points in the frame at absolute rest. The points with the same index are associated with the same instance but not necessarily in the time-sequential order.

II. FORMULA FOR THE SPEED OF THE REFLECTED WAVEFRONT DERIVED WITH THE HYPOTHESIS THAT THE INCIDENT SPEED AND REFLECTED SPEED OF A WAVEFRONT ARE EQUAL IN AN INERTIAL FRAME

The figures in this section show a mirror and a source of coherent light both at rest in an inertial frame that travels at

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the speed v in the frame at absolute rest. For each figure, the mirror and source have a different setup.

The velocity vectors are illustrated for $v = 1$ m/s and $c = 2$ m/s at the instance when a wavefront of a ray of light collides with the mirror.

A. Reflection of light when the velocity of light from a source is in the opposite direction to the velocity of an inertial frame

Figure 1 illustrates the initial position of the mirror and source. The source emits rays of light that travel at velocity c perpendicular to the mirror and in the opposite direction to the velocity v of the inertial frame. A wavefront of a ray of light from the source travels at speed c and collides with the mirror at point A_1, and it is reflected back in the opposite direction.

The velocity of the mirror in the opposite direction of the incident velocity of the wavefront v_i and the velocity of the mirror in the direction of the reflected velocity of the wavefront v_r are concepts that are used extensively in this study. In Fig. 1, v_i and v_r are the same velocity v illustrated by vector A_1B_1.

In the inertial frame, the speed of the incident wavefront with respect to the mirror is $c_{ii} = c + v_i = c + v$.

It is hypothesized that, in the inertial frame of the mirror, at the instance of collision, the speed of the reflected wavefront c_{ri} is equal to the speed of the incident wavefront c_{ii}; therefore, $c_{ri} = c_{ii} = c + v_i = c + v$.

The velocities c, c_{ii}, c_{ri}, v, v_i, and v_r are shown at the instance of collision at point A_1, and the speed of the reflected wavefront in the frame at absolute rest c_{ra} is obtained below.

After time t from the instance of collision, in the frame at absolute rest, the mirror travels the distance A_1A_2 at speed $v_r = v$, and the reflected wavefront travels distance A_1E_2 at speed c_{ra}; in the inertial frame, the reflected wavefront travels distance A_2E_2 at speed $c_{ri} = c + v_i$.

The distance $A_1E_2 = A_1A_2 + A_2E_2 \Rightarrow c_{ra}t = v_rt + c_{ri}t \Rightarrow c_{ra} = c_{ri} + v_r = (c + v_i) + v_r$. This expression yields the equation in the variable c_{ra}

$$c_{ra} = c + v_i + v_r. \tag{1}$$

In Fig. 1, the solution of Eq. (1) is $c_{ra} = c + v_i + v_r = c + v + v = c + 2v$ that is identical to the elastic collision of a massless ball with a wall derived in classical mechanics,[6] as detailed in the Appendix.

For the following examples, the solution of Eq. (1) consists of determining the velocities v_i and v_r.

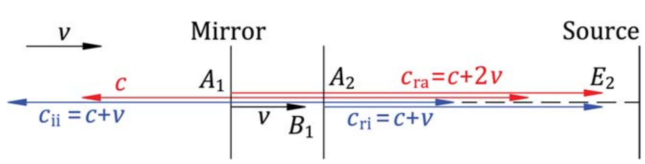

FIG. 1. (Color online) Reflection of light when the velocity of light from a source is in the opposite direction to the velocity of an inertial frame.

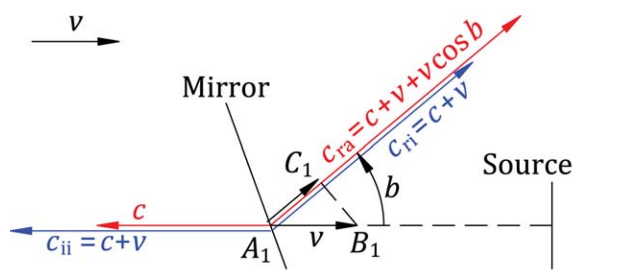

FIG. 2. (Color online) Reflection of light when the direction of the reflected wavefront creates an angle b with the initial position.

When the direction of the reflected wavefront creates an angle b with the initial position, the schematic shown in Fig. 1 can be replaced with that in Fig. 2.

The magnitude of vector A_1B_1 is v_i; therefore, $v_i = v$ and $c_{ri} = c_{ii} = c + v_i = c + v$. The magnitude of vector A_1C_1 is v_r; therefore, $v_r = v \cos b$.

The solution of Eq. (1) $c_{ra} = c + v_i + v_r = c + v + v \cos b$ verifies $c_{ra} = c + 2v$ derived for the setup illustrated in Fig. 1, when $b = 0°$; $c_{ra} = c$ for $b = 180°$ when there is no collision.

The solution $c_{ra} = c + v + v \cos b$ yields $c_{ra} = c + v$ for $b = 90°$. Figure 3 is a modified version of Fig. 1 when the reflected wavefront creates an angle $b = 90°$ with the initial position.

B. Reflection of light when the mirror-source direction creates an angle *a* with the initial position

Figure 4 depicts the mirror-source direction, as illustrated in Fig. 1 at the initial position, at an angle a from the initial position. The incident wavefronts from the source travel perpendicular to the mirror at speed c. A wavefront of a ray of light collides with the mirror at point A_1.

The magnitude of vector A_1C_1 is v_i and v_r; therefore, $v_i = v_r = v \cos a$ and then $c_{ri} = c_{ii} = c + v_i = c + v \cos a$.

The solution of Eq. (1) $c_{ra} = c + v_i + v_r = c + v \cos a + v \cos a = c + 2v \cos a$ verifies $c_{ra} = c + 2v$ derived for the setup in Fig. 1, when $a = 0°$; $c_{ra} = c - 2v$ for $a = 180°$ that is identical to the elastic collision of a massless ball with a wall derived in classical mechanics,[6] as detailed in the Appendix.

The schematic shown in Fig. 4 is modified when the reflected wavefront creates an angle b with the initial position, as illustrated in Fig. 5.

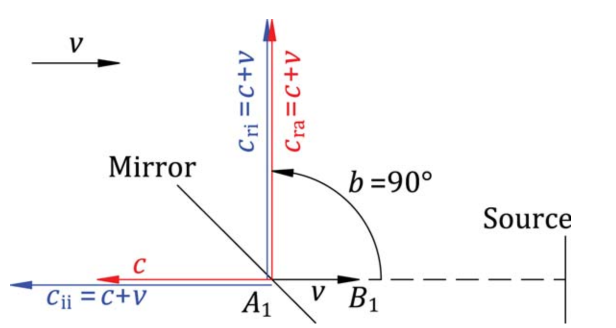

FIG. 3. (Color online) Reflection of light when the direction of the reflected wavefront creates an angle $b = 90°$ with the initial position.

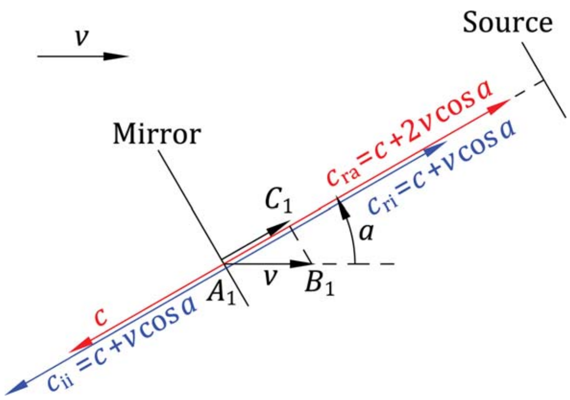

FIG. 4. (Color online) Reflection of light when the mirror-source direction creates an angle a with the initial position.

The magnitude of vector A_1C_1 is v_i; therefore, $v_i = v \cos a$ and $c_{ri} = c_{ii} = c + v_i = c + v \cos a$. Magnitude of vector A_1D_1 is v_r; therefore, $v_r = v \cos b$.

The solution of Eq. (1) $c_{ra} = c + v_i + v_r = c + v \cos a + v \cos b$ is applicable for any angle a and b. This solution verifies $c_{ra} = c + 2v \cos a$ derived for the geometry in Fig. 4 when $b = a$; $c_{ra} = c + v$ as seen in Fig. 3 for $a = 0°$ and b=90° and $c_{ra} = c + v \cos a + v \cos (a + 180°) = c + v\cos a - v \cos a = c$ for $b = a + 180°$ when there is no collision.

The solution $c_{ra} = c + v \cos a + v \cos b$ yields $c_{ra} = c + v \cos a + v \cos (a + 90°) = c + v \cos a - v \sin a$ for $b = a + 90°$. Figure 6 depicts the modified version of Fig. 4 when the reflected wavefront creates an angle $b = a + 90°$ with the initial position.

C. Discussion

Angle a corresponds to the opposite direction of the incident wavefront, and angle b corresponds to the direction of the reflected wavefront. The directions of these angles are outward in space of the point of collision and measured counterclockwise from the direction of the velocity vector v with the origin at the point of collision, as illustrated by vector A_1B_1 in the above figures.

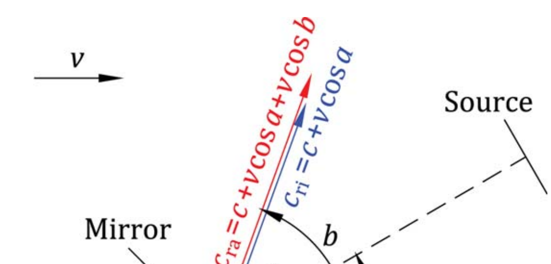

FIG. 5. (Color online) Reflection of light when the direction of a reflected wavefront creates an angle b with the initial position.

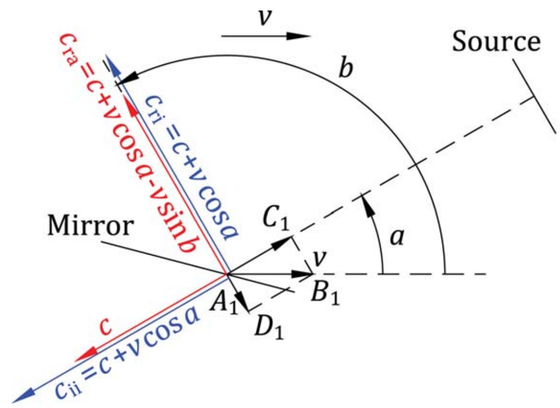

FIG. 6. (Color online) Reflection of light when the direction of a reflected wavefront creates an angle $b = a + 90°$ with the initial position.

A mirror in motion reflects a wavefront from a source of light with a speed other than the constant speed c. The reflected wavefront may become the incident wavefront for another mirror. The final solution of Eq. (1) is $c_{ra} = c_s + v_i + v_r = c_s + v \cos a + v \cos b$, where speed c_s is the speed of light from a source or a mirror. This expression yields a practical alternative to Eq. (1)

$$c_{ra} = c_s + v \cos a + v \cos b. \tag{2}$$

The hypothesis that the incident speed and reflected speed of the wavefront of a ray of light are equal in the inertial frame at the instance of collision is similar to the hypothesis that the reflection of light is a mechanical phenomenon[6] if the hypothetical massless balls are included in mechanics, as detailed in the Appendix.

III. HYPOTHESIS THAT THE INCIDENT SPEED AND REFLECTED SPEED OF A WAVEFRONT ARE EQUAL IN AN INERTIAL FRAME APPLIED TO THE MICHELSON INTERFEROMETER WITH A PARTICULAR GEOMETRY

A. Derivation of light paths

Figure 7 presents the Michelson interferometer with a particular geometry rotated counterclockwise by an angle a from the initial position. At the initial position, angle $a = 0°$. The beam splitter M creates an angle of 45° with the direction of wavefronts from the source; mirror M_1 is perpendicular to this direction, and mirror M_2 is parallel to this direction.

The source emits coherent rays of light that are split by the beam splitter M. The transmitted rays from the source travel through M to M_1, and the reflected rays from the source are directed by M to M_2; both rays travel back to M and create interference. From the multitude of transmitted and reflected rays, there is one pair of rays that continuously intercepts at point A of M; this is true for all points on M. Figure 7 depicts the pair of transmitted and reflected rays that interfere at point A and angle a.

The derivation of the light paths starts when the wavefront of a beam of light from the source arrives at line E_1D_1. At this instance, the wavefront of the reflected ray is at point

E_1 associated with the beam splitter. The derivation of each wavefront path employs Eq. (2).

The lengths of the interferometer arms $AB = A_1B_1$ and $AC = A_1C_1$ are equal to L, and the distances A_1F_1 and E_1F_1 are equal to l. Thus, the distance $D_1I_1 = E_1K_1 = L + l$.

The reflected wavefront of light at E_1 travels to J_3 at speed $c_{ra} = c_{21} = c_{s21} + v\cos\, a_{21} + v\, \cos\, b_{21}$ in time t_{21}. The speed from the source $c_{s21} = c$. Consider a velocity vector v with the origin at E_1. The angle measured counterclockwise from the direction of the velocity vector v to the opposite direction of the incident wavefront is $a_{21} = a$. The angle measured counterclockwise from the direction of the velocity vector v to the direction of the reflected wavefront is $b_{21} = a + 90°$. Thus, the speed $c_{21} = c + v\cos\, a + v\cos\,(a + 90°) = c + v\cos a - v\sin a$.

In time t_{21}, mirror M_2 travels distance $J_1J_3 = vt_{21}$; thus, distance $J_3K_1 = vt_{21}\sin a$.

$E_1K_1 = E_1J_3 + J_3K_1 \Rightarrow L + l = c_{21}t_{21} + vt_{21}\,\sin a \Rightarrow L + l = (c + v\cos a - v\sin a)t_{21} + vt_{21}\sin a$, and this expression yields the equation

$$t_{21} = (L + l)/(c + v\cos a). \tag{3}$$

The wavefront reflected at J_3 travels toward A_5 and the screen at speed $c_{ra} = c_{22} = c_{s22} + v\cos\, a_{22} + v\, \cos\, b_{22}$ in time t_{22}. The speed of the incident wavefront at J_3 is $c_{s22} = c_{21}$. Consider a velocity vector v with the origin at J_3. The measured angle $a_{22} = b_{22} = a + 270°$. Thus, the speed $c_{22} = c_{21} + 2v\cos\,(a + 270°) = (c + v\cos a - v\sin a) + 2v\sin a = c + v\cos a + v\sin a$.

In time t_{22}, mirror M_2 travels distance $C_3C_5 = vt_{22}$; thus, distance $C_5J_3 = vt_{22}\sin a$. The distance $A_5C_5 = L$.

$A_5J_3 = A_5C_5 + C_5J_3 \Rightarrow c_{22}t_{22} = L + vt_{22}\,\sin a \Rightarrow (c + v\cos\, a + v\sin a)t_{22} = L + vt_{22}\sin a$. This expression yields the equation

$$t_{22} = L/(c + v\cos a). \tag{4}$$

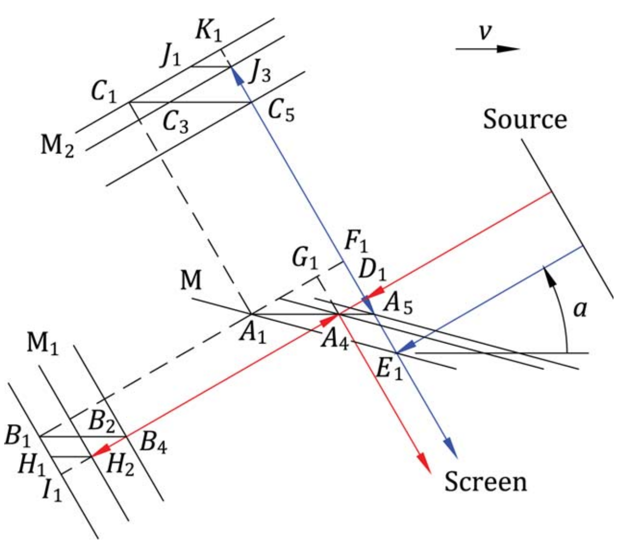

FIG. 7. (Color online) The hypothesis that the incident speed and reflected speed of a wavefront of a ray of light are equal in the inertial frame of a mirror at the instance of collision is applied to the Michelson interferometer with a particular geometry.

The solutions of Eqs. (3) and (4) give time $t_2 = t_{21} + t_{22} = (L + l)/(c + v\cos a) + L/(c + v\cos a)$ that yields the equation

$$t_2 = (2L + l)/(c + v\cos a). \tag{5}$$

In time t_2, M travels the distance A_1A_5. The distance $l = A_1F_1 = A_1A_5\cos a = vt_2\cos a$ that gives the equation

$$t_2 = l/(v\cos a). \tag{6}$$

The equality of Eqs. (5) and (6) yields Eq. (7).

$$\begin{aligned} &\frac{2L + l}{c + v\cos a} = \frac{l}{v\cos a} \Rightarrow l\left(\frac{1}{v\cos a} - \frac{1}{c + v\cos a}\right) \\ &\quad = \frac{2L}{c + v\cos a} \Rightarrow \\ &l = 2Lv\cos a/c. \end{aligned} \tag{7}$$

Substituting the solution of Eq. (7) in Eq. (6), time t_2 becomes a constant for any angle a, $t_2 = 2L/c$.

The transmitted wavefront of light travels from D_1 to H_2 at speed $c_{s11} = c_{11} = c$ in time t_{11}. Simultaneously, mirror M_1 travels the distance $H_1H_2 = vt_{11}$; thus, the distance $H_2I_1 = vt_{11}\cos a$.

$D_1I_1 = D_1H_2 + H_2I_1 \Rightarrow L + l = c_{11}t_{11} + vt_{11}\cos a$. This expression yields the equation

$$t_{11} = (L + l)/(c + v\cos a). \tag{8}$$

The wavefront reflected at H_2 travels to A_4 at speed $c_{ra} = c_{12} = c_{s12} + v\cos\, a_{12} + v\, \cos\, b_{12}$ in time t_{12}. The speed $c_{s12} = c_{11} = c$. Consider a velocity vector v with the origin at H_2. The measured angle $a_{12} = b_{12} = a$. Thus, $c_{12} = c_{11} + 2v\cos\, a = c + 2v\cos a$.

In time t_{12}, mirror M_1 travels distance $B_2B_4 = vt_{12}$; thus, distance $B_4H_2 = vt_{12}\cos a$. The distance $A_4B_4 = L$.

$A_4H_2 = A_4B_4 + B_4H_2 \Rightarrow c_{12}t_{12} = L + vt_{12}\cos a \Rightarrow (c + 2v\cos a)t_{12} = L + vt_{12}\cos a$, and this expression gives the equation

$$t_{12} = L/(c + v\cos a). \tag{9}$$

The solutions of Eqs. (8) and (9) give time $t_1 = t_{11} + t_{12} = (L + l)/(c + v\cos a) + L/(c + v\cos a)$ that yields the equation

$$t_1 = (2L + l)/(c + v\cos a). \tag{10}$$

Substituting the solution of Eq. (7) in Eq. (10), time t_1 becomes a constant for any angle a, $t_1 = 2L/c$.

The wavefront reflected at A_4 travels to the screen at speed $c_{ra} = c_{13} = c_{s13} + v\cos\, a_{13} + v\, \cos\, b_{13}$. The speed $c_{s13} = c_{12}$. Consider a velocity vector v with the origin at A_4; then, the measured angles are $a_{13} = a + 180°$ and $b_{13} = a + 270°$. Thus, the speed $c_{13} = c_{12} + v\cos\,(a + 180°) + v\,\cos\,(a + 270°) = (c + 2v\cos a) - v\cos a + v\sin a = c + v\cos a + v\sin a$.

Because time $t_1 = t_2 = 2L/c$, points A_4 and D_1 coincide with point A_5, and point G_1 coincides with point F_1.

The difference in time $\Delta t = t_2 - t_1 = 0$. Thus, the predicted fringe shift is zero.

The classical derivation, based on the hypothesis that the incident speed and reflected speed of a wavefront of a ray of light are equal in the frame at absolute rest, predicts an observable fringe shift of 0.40 when the Michelson interferometer with a particular geometry is rotated in increments of 90° starting from the initial position, as detailed in Section IV.

B. Discussion

The pair of rays that interfere at point A and at any point on M changes continuously with interferometer rotation.

Along the path to the screen, the transmitted and reflected rays are parallel, and their speeds are equal for any angle a because $c_{13} = c_{22} = c + v\cos a + v\sin a$. Thus, the transmitted and reflected rays interfere.

The difference in time Δt for any angle a and at any point on M is given by $\Delta t = t_2 - t_1 = 0$. Thus, the interference image is an illuminated area at maximum brightness that does not change with the rotation of the interferometer.

If the length of the interferometer arms is not equal, i.e., $AB = A_1B_1 = L_1$ and $AC = A_1C_1 = L_2$, then $\Delta t = t_2 - t_1 = 2(L_2 - L_1)/c$. The difference in the arm's length $L_2 - L_1$ is the same constant for any angle a and at any point on M. Thus, the interference is not in phase, and the illuminated area has less brightness than that of the maximum that does not change with the rotation of the interferometer.

The same image without any fringe shift observed in the frame at absolute rest is observed in the inertial frame. Apparently, in any inertial frame, the pairs of transmitted and reflected wavefronts travel the same path length $2L$ at the same speed c and same time $2L/c$, independent of angle a and the speed of the inertial frame v, as if the interferometer is part of the frame at absolute rest.

IV. HYPOTHESIS THAT THE INCIDENT SPEED AND REFLECTED SPEED OF A WAVEFRONT ARE EQUAL IN THE FRAME AT ABSOLUTE REST APPLIED TO THE MICHELSON INTERFEROMETER WITH A PARTICULAR GEOMETRY

The derivation of the fringe shift follows the same steps as introduced in Section III A, but the speed of light before and after reflection is the constant speed c.

For the reflected wavefront,

$E_1K_1 = E_1J_3 + J_3K_1 \Rightarrow L + l = ct_{21} + vt_{21}\sin a$. This expression gives the equation

$$t_{21} = (L + l)/(c + v\sin a). \tag{11}$$

$A_5J_3 = A_5C_5 + C_5J_3 \Rightarrow ct_{22} = L + vt_{22}\sin a$. This expression yields the equation

$$t_{22} = L/(c - v\sin a). \tag{12}$$

The solutions of Eqs. (11) and (12) give time $t_2 = t_{21} + t_{22}$ that offer the equation

$$t_2 = (L+l)/(c+v\sin a)+L/(c - v\sin a). \tag{13}$$

The distance $l = A_1F_1 = A_1A_5\cos a = vt_2\cos a$ that gives the equation

$$t_2 = l/(v\cos a). \tag{14}$$

The equality of Eqs. (13) and (14) yields Eq. (15).

$$(L + l)/(c + v\sin a) + L/(c - v\sin a) = l/(v\cos a) \Rightarrow$$
$$l = 2Lcv\cos a/((c - v\sin a)(c + v\sin a - v\cos a)). \tag{15}$$

Substituting the solution of Eq. (15) in Eq. (14), the equation in variable of time t_2 becomes

$$t_2 = 2Lc/((c - v\sin a)(c + v\sin a - v\cos a)). \tag{16}$$

For the transmitted wavefront,

$D_1I_1 = D_1H_2 + H_2I_1 \Rightarrow L + l = ct_{11} + vt_{11}\cos a$, and this expression offers the equation

$$t_{11} = (L + l)/(c + v\cos a). \tag{17}$$

$A_4H_2 = A_4B_4 + B_4H_2 \Rightarrow ct_{12} = L + vt_{12}\cos a$. This expression gives the equation

$$t_{12} = L/(c - v\cos a). \tag{18}$$

The solutions of Eqs. (17) and (18) give time $t_1 = t_{11} + t_{12} = (L + l)/(c + v\cos a) + L/(c - v\cos a)$ that yields the equation

$$t_1 = (L + l)/(c + v\cos a) + L/(c - v\cos a). \tag{19}$$

The distance l can be calculated for any angle a with Eq. (15) followed by that of time t_1 and t_2 with Eqs. (19) and (16), respectively. The difference in time for a value of $a = a_1$ is $\Delta t_{a_1} = t_2 - t_1$, the light period is $T = \lambda/c$, and the number of wavelengths in Δt_{a_1} is $N_{a_1} = \Delta t_{a_1}/T = c\Delta t_{a_1}/\lambda$ for any angle a.

If the number of wavelengths for a value of angle $a = a_2$ is $N_{a_2} = c\Delta t_{a_2}/\lambda$, then the fringe shift obtained by rotating the interferometer from a_1 to a_2 is $\Delta N_{a_1,a_2} = N_{a_2} - N_{a_1}$ that can be calculated for any angle a_1 and a_2.

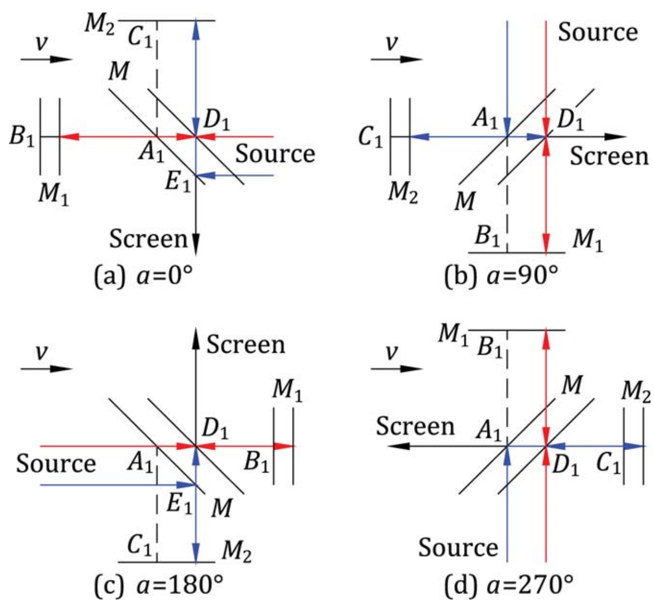

FIG. 8. (Color online) Schematic of the light paths for angle a at 0°, 90°, 180°, and 270°.

TABLE I. Numerical calculation of the fringe shift at four positions.

a (rad)	0	$\pi/2$
l (m)	$2.2002200220022000 \times 10^{-3}$	$1.3476633138761200 \times 10^{-19}$
t_2 (s)	$7.3340667400073300 \times 10^{-8}$	$7.3333334066666700 \times 10^{-8}$
t_1 (s)	$7.3340667400073300 \times 10^{-8}$	$7.3333333333333300 \times 10^{-8}$
$\Delta t_0,\ \Delta t_{\pi/2}$ (s)	$0.0000000000000000 \times 10$	$7.3333334569908100 \times 10^{-16}$
$N_0,\ N_{\pi/2}$	$0.0000000000000000 \times 10$	$4.0000000674495300 \times 10^{-1}$
$\Delta N_{3\pi/2,0},\ \Delta N_{0,\pi/2}$	$-4.0000000674495300 \times 10^{-1}$	$4.0000000674495300 \times 10^{-1}$
a (rad)	π	$3\pi/2$
l (m)	$-2.1997800219978000 \times 10^{-3}$	$-4.0429899416283500 \times 10^{-19}$
t_2 (s)	$7.3326000733260000 \times 10^{-8}$	$7.3333334066666700 \times 10^{-8}$
t_1 (s)	$7.3326000733260000 \times 10^{-8}$	$7.3333333333333300 \times 10^{-8}$
$\Delta t_\pi,\ \Delta t_{3\pi/2}$ (s)	$0.0000000000000000 \times 10$	$7.3333334569908100 \times 10^{-16}$
$N_\pi,\ N_{3\pi/2}$	$0.0000000000000000 \times 10$	$4.0000000674495300 \times 10^{-1}$
$\Delta N_{\pi/2,\pi},\ \Delta N_{\pi,3\pi/2}$	$-4.0000000674495300 \times 10^{-1}$	$4.0000000674495300 \times 10^{-1}$

Figure 8 illustrates a schematic of the light paths for angle a at 0°, 90°, 180°, and 270°.

For calculations, the length of the interferometer arms $L = 11$ m, the speed $v = 3.00 \times 10^4$ m/s, the speed $c = 3.00 \times 10^8$ m/s, and the wavelength of the light $\lambda = 550 \times 10^{-9}$ m; these values are the same as those used by Michelson and Morley in their experiment.

Table I gives the numerical calculation results, obtained in Excel, for a fringe shift in steps of 90° starting from the initial position for the cases presented in Fig. 8.

For their experiment, Michelson and Morley predict a fringe shift of 0.40, as well.[1]

APPENDIX: ELASTIC COLLISION OF A BALL WITH A WALL

In the frame at absolute at rest, a wall of mass m_1 travels at speed v_1, and a ball of mass m_2 travels at speed v_2 in the opposite direction of speed v_1; $m_1 > m_2$ as illustrated in Fig. 9. At the instance of the elastic collision, the wall applies momentum to the ball, and the ball is reflected back at speed v_2'. The wall continues traveling in the same direction with a speed v_1'.

The speed v_1' of the wall and the speed v_2' of the ball after an elastic collision are derived here.

The equation for the law of conservation of momentum and kinetic energy yields the solution for speed v_1' and speed v_2'.[6]

The equation for the law of conservation of momentum is

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'. \tag{A1}$$

The equation for the law of conservation of kinetic energy is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2. \tag{A2}$$

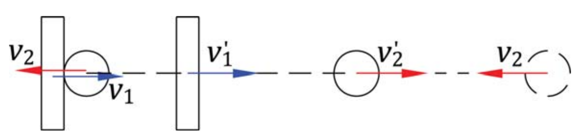

FIG. 9. (Color online) Elastic collision of a ball with a wall.

The two equations yield the following solutions:

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2,$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2.$$

The solutions are derived in mechanics without knowing the direction of speeds v_1' and v_2' after the collision. If the mass of the ball m_2 decreases toward zero or $m_1 \gg m_2$ and the direction of v_1, as a reference, is considered to be positive, then the direction of v_2 is negative and the direction of v_1' and v_2' is positive. The simplified solutions are approximated with $v_1' \cong v_1$ and $v_2' \cong v_2 + 2v_1$. If the wall travels in the same direction with the ball, $v'_1 \cong v_1$ and $v'_2 \cong v_2 - 2v_1$. These solutions are given as an approximation, because the Newtonian mechanics, as we understand it, excludes the massless entities. At the limit when the mass of the ball is zero or massless, the simplified solutions are $v_1' = v_1$, $v_2' = v_2 + 2v_1$, and $v_1' = v_1$, $v_2' = v_2 - 2v_1$, respectively.

Nevertheless, the massless ball respects the equation for the law of kinetic energy, $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1$, and the equation for the law of conservation of momentum, $m_1 v_1 = m_1 v_1'$. These solutions emphasize that there is no need for energy to change the state of a massless ball, and a massless ball does not have momentum.

From a theoretical perspective, the hypothetical massless entities, and in particular, the massless balls, may be included in mechanics. Therefore, the reflection of light as a massless entity or the reflection of photons as massless particles could be considered mechanical phenomena.

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