Article

Reflection of Light as a Mechanical Phenomenon Applied to a Particular Michelson Interferometer

Filip Dambi Filipescu *

Independent Researcher, Surprise AZ, USA
* Correspondence: filipdambi1@gmail.com

Abstract: Derivation of light paths in the Michelson interferometer is based on the hypothesis that the speed of light does not change after reflection by a mirror in motion. The Michelson-Morley experiment predicts a fringe shift of 0.40. The same fringe shift is predicted for a particular Michelson interferometer in which the beam splitter of the interferometer makes an angle of 45° with the direction of light from the source. Light behaves like a wave and also as a particle. Thus, it is reasonable to consider the reflection of light as a mechanical phenomenon. With this hypothesis, the speed of light changes after reflection, and the predicted fringe shift for the particular Michelson interferometer is zero which is in accordance with the result of the Michelson-Morley experiment. Apparently, light travels in any inertial frame as if this particular interferometer belongs to a fixed frame. The velocity of light is considered independent of the velocity of its source, which is in accordance with astronomers' observations of the binary stars, and the experiment performed at CERN, Geneva, in 1964.

Keywords: geometrical optics; reflection of light; speed of light; interference of light; Michelson interferometer; Michelson–Morley experiment; elastic collision ball wall

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1. Introduction

The hypothesis that the reflection of light is a mechanical phenomenon explains the negative result of the Michelson–Morley experiment [1,2]. The velocity of light is considered independent of the velocity of its source, which is in accordance with the astronomers' observations of the binary stars [3,4], and the experiment performed at CERN, Geneva, in 1964 [5].

Section 2 includes a detailed theoretical analysis of the reflection of light as a mechanical phenomenon. The purpose of this section is to obtain the formula for the speed of a reflected ray of light in the fixed frame by a mirror in motion.

Section 3 applies the result of Section 2 to a particular Michelson interferometer. The derivation of the light paths and the fringe shift is achieved in the fixed frame.

2. Reflection of light as a mechanical phenomenon

The drawings in this section present a stationary frame consisting of a mirror and a source of coherent light belonging to an inertial frame that travels at speed v. For each drawing, the mirror and source have a different setup.

The speed of light from its source in a fixed frame is the constant c. The velocity vectors are illustrated for $v = 1\ \mathrm{m/s}$ and $c = 2\ \mathrm{m/s}$ at the instance when light collides with the mirror. The velocity vectors of the reflected rays are red in the fixed frame and blue in the inertial frame.

As per the notations used in this study, points marked by a letter without an index correspond to the points as seen by an observer in the inertial frame. Points marked by a letter with an index are instances of inertial frame points in the fixed frame. Points with the same index belong to the same instance, not necessarily in time–sequential order.

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2.1. Reflection of light when the velocity of light from the source has an opposite direction to the velocity of the inertial frame

Figure 1 illustrates the initial position of the mirror–source frame. The source emits parallel rays of light with velocity c perpendicular to the mirror, in the opposite direction of velocity v of the inertial frame. A ray of light from the source travels with speed c and collides with the mirror at point A_1.

The velocity of the mirror along the opposite direction of the incident ray v_{mi} and the velocity of the mirror along the reflected ray direction v_{mr} are concepts used extensively in this study. For the case of Figure 1, v_{mi} and v_{mr} are the same velocity v that is illustrated by vector A_1B_1.

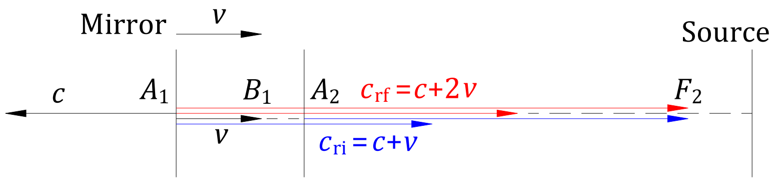

Figure 1. Reflection of light when the velocity of light from the source has an opposite direction to the velocity of the inertial frame.

In the fixed frame, the relative speed of light with respect to the mirror c_{r} is the speed of light c plus the speed v_{mi}, $c_{\mathrm{r}} = c + v_{\mathrm{mi}} = c + v$. It is also the speed of the incident ray with respect to the mirror in the inertial frame $c_{\mathrm{ii}} = c_{\mathrm{r}} = c + v_{\mathrm{mi}} = c + v$.

Considering the reflection of light as a mechanical phenomenon, the speed of the reflected ray in the inertial frame c_{ri} is equal to the speed of the incident ray c_{ii}, $c_{\mathrm{ri}} = c_{\mathrm{ii}} = c + v_{\mathrm{mi}} = c + v$.

The velocities $c_{\mathrm{ri}}, v_{\mathrm{mi}}$ and v_{mr} that are the velocity v illustrated by vector A_1B_1, and the velocity of the reflected ray in the fixed frame c_{rf} are shown at the instance of collision at point A_1.

In time t from the instance of collision, in the fixed frame, the mirror travels the distance A_1A_2 with speed $v_{\mathrm{mr}} = v$, and the reflected ray travels the distance A_1F_2 with speed c_{rf}; in the inertial frame, the reflected ray travels the distance A_2F_2 with speed $c_{ri} = c + v_{\mathrm{mi}}$. The distance $A_1F_2 = A_1A_2 + A_2F_2 \;\Rightarrow\; c_{rf}t = v_{\mathrm{mr}}t + c_{ri}t \;\Rightarrow\; c_{rf} = c_{ri} + v_{\mathrm{mr}} = c + v_{\mathrm{mi}} + v_{\mathrm{mr}} = c + v + v = c + 2v$. The speed $c_{rf} = c + 2v$ is identical to the elastic collision of a ball with a rigid wall derived in classical mechanics if the ball is massless [6], Appendix A.

For the following examples, the derivation of the formula $c_{\mathrm{rf}} = c_{\mathrm{ri}} + v_{\mathrm{mr}} = c + v_{\mathrm{mi}} + v_{\mathrm{mr}}$ consists of identifying the velocities v_{mi} and v_{mr}.

When the reflected ray makes an angle b with the initial position, the schematic shown in Figure 1 can be replaced with that in Figure 2. Point E_1 rotates on a circle of radius v shown partially with a dashed line, when angle b varies from $0°$ to $360°$.

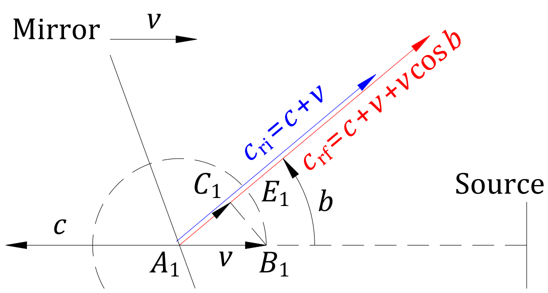

Figure 2. Reflection of light when the reflected ray makes an angle b with the initial position.

Vector A_1B_1 has a magnitude of v; thus, the speed of the mirror along the opposite direction of the incident ray $v_{\mathrm{mi}} = v$. In the inertial frame, at the instance of collision, the speed of the reflected ray $c_{\mathrm{ri}} = c_{\mathrm{ii}} = c + v_{\mathrm{mi}} = c + v$. At the instance of collision, the velocity vector c_{ri} belongs to the inertial and fixed frame.

Vector A_1C_1 has a magnitude of $v\cos b$; thus, the speed of the mirror along the direction of the reflected ray $v_{\mathrm{mr}} = v\cos b$ for this non–frontal collision. In the fixed frame, the speed of the reflected ray $c_{\mathrm{rf}} = c_{\mathrm{ri}} + v_{\mathrm{mr}} = c + v_{\mathrm{mi}} + v_{mr} = c + v + v\cos b$.

The formula $c_{\mathrm{rf}} = c + v + v\cos b$ verifies $c_{\mathrm{rf}} = c + 2v$ derived for the setup illustrated in Figure 1, for the case when $b = 0°$; $c_{\mathrm{rf}} = c$ for $b = 180°$, for the case when there is no collision.

The formula $c_{\mathrm{rf}} = c + v + v\cos b$ yields $c_{\mathrm{rf}} = c + v$ for $b = 90°$. Figure 3 is a modified version of Figure 1 when the reflected ray makes an angle $b = 90°$ with the initial position.

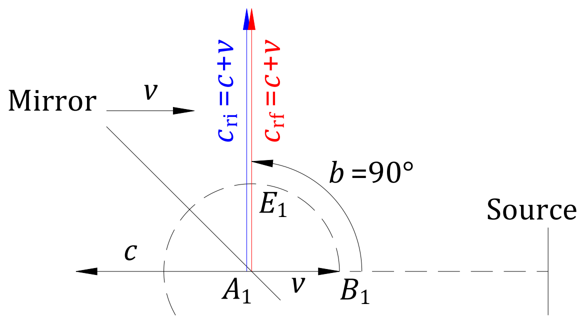

Figure 3. Reflection of light when the reflected ray makes an angle $b = 90°$ with the initial position.

2.2. Reflection of light when the mirror–source frame makes an angle a with the initial position

Figure 4 depicts the mirror–source frame, as illustrated in Figure 1 at the initial position, making an angle a with this initial position. The rays of light from the source travels perpendicular to the mirror with speed c. A ray of light from the source collides with the mirror at point A_1.

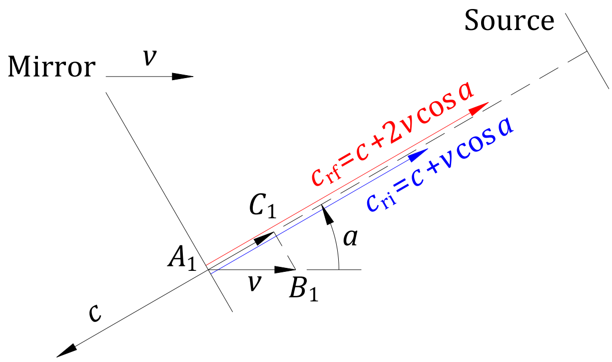

Figure 4. Reflection of light when the mirror–source frame makes an angle a with the initial position.

Vector A_1C_1 has a magnitude of $v\cos a$; thus, the speed $v_{\mathrm{mi}} = v_{\mathrm{mr}} = v\cos a$ for this frontal collision. In the inertial frame, at the instance of collision, the speed $c_{\mathrm{ri}} = c_{\mathrm{ii}} = c + v_{\mathrm{mi}} = c + v\cos a$. At the instance of collision, the velocity vector c_{ri} belong to the inertial and fixed frame. In the fixed frame, the speed $c_{\mathrm{rf}} = c_{\mathrm{ri}} + v_{\mathrm{mr}} = c + v_{\mathrm{mi}} + v_{\mathrm{mr}} = c + v\cos a + v\cos a = c + 2v\cos a$.

The formula $c_{\mathrm{rf}} = c + 2v\cos a$ verifies $c_{\mathrm{rf}} = c + 2v$ derived for the setup in Figure 1, wherein $a = 0°$; $c_{\mathrm{rf}} = c - 2v$ for $a = 180°$, which is identical to the elastic collision of a ball with a rigid wall derived in classical mechanics if the ball is massless [6], Appendix A.

The schematic shown in Figure 4 gets modified when the reflected ray makes an angle b with the initial position, and can be represented as Figure 5.

Vector A_1C_1 has a magnitude of $v\cos a$; thus, the speed $v_{\mathrm{mi}} = v\cos a$. In the inertial frame, at the instance of collision, the speed $c_{\mathrm{ri}} = c_{\mathrm{ii}} = c + v_{\mathrm{mi}} = c + v\cos a$.

Vector A_1D_1 has a magnitude of $v\cos b$; thus, the speed $v_{\mathrm{mr}} = v\cos b$. In the fixed frame, the speed $c_{\mathrm{rf}} = c_{\mathrm{ri}} + v_{\mathrm{mr}} = c + v_{\mathrm{mi}} + v_{\mathrm{mr}} = c + v\cos a + v\cos b$ that applies for any angle, both a and b.

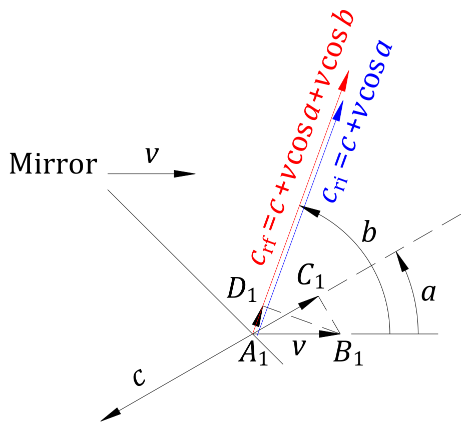

Figure 5. Reflection of light when the reflected ray makes an angle b with the initial position.

The formula $c_{rf} = c + v\cos a + v\cos b$ verifies $c_{rf} = c + 2v\cos a$ derived for the geometry in Figure 4, for the case when $b = a$; $c_{rf} = c + v$ as seen in Figure 3, for $a = 0°$ and $b = 90°$; $c_{rf} = c + v\cos a + v\cos(a + 180°) = c + v\cos a - v\cos a = c$ for $b = a + 180°$ when there is no collision.

The formula $c_{rf} = c + v\cos a + v\cos b$ yields $c_{rf} = c + v\cos a + v\cos(a + 90°) = c + v\cos a - v\sin a$ for $b = a + 90°$. Figure 6 depicts the modified version of Figure 4 when the reflected ray makes an angle $b = a + 90°$ with the initial position.

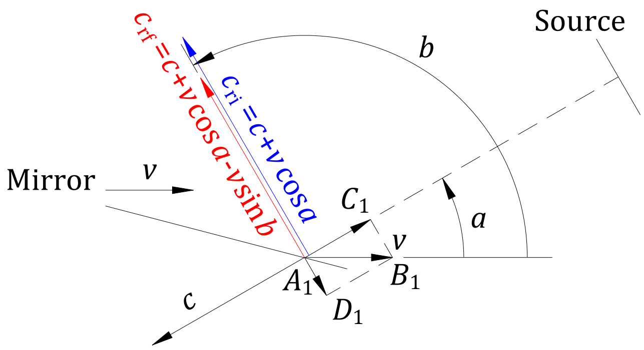

Figure 6. Reflection of light when the reflected ray makes an angle $b = a + 90°$ with the initial position.

2.3. Discussions

Angle a corresponds to the opposite direction of the incident ray, and angle b to the direction of the reflected ray. The direction of angles a and b are outward in space from the point of collision. Angles a and b are measured counterclockwise from the direction of the velocity vector v with its origin at the point of collision, illustrated by vector A_1B_1 in the above figures.

In the fixed frame, a mirror at rest reflects the rays of light from a source with the constant speed c. A mirror in motion reflects the rays of light from a source with a speed different from the constant speed c, and the reflected rays may become the incident rays for another mirror. The final formula of speed c_{rf} is $c_{rf} = c_s + v_{mi} + v_{mr} = c_s + v\cos a + v\cos b$, where speed c_s is the speed of light from a source or from a mirror.

If this study starts with the initial position of Figure 1 in which velocity v has the same direction as velocity c, then $c_{rf} = c_s - v_{mi} - v_{mr} = c_s - v\cos a - v\cos b$.

3. Reflection of light as a mechanical phenomenon applied to a particular Michelson interferometer

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3.1. Derivation of the light paths

Figure 7 presents a particular Michelson interferometer rotated counterclockwise by an angle a from the initial position. At the initial position, angle $a = 0°$. The beam splitter M makes an angle of 45° with the direction of the rays from the source, mirror M_1 is perpendicular and mirror M_2 is parallel to this direction, respectively.

The transmitted rays from the coherent source of light travel through M to M_1, and the reflected rays from the source are directed by M to M_2; both rays travel back to M where they interfere. From the multitude of transmitted and reflected rays, there is one pair of rays that continuously intercepts at a point A of M; this is true for all points on M. Figure 7 depicts the pair of the transmitted ray from X_1 in red and the reflected ray from Y_1 in blue that interferes at point A, for angle a.

The initial instance of the light paths derivation is considered when the ray from Y_1 is reflected at point E_1 of M. At the initial instance, the transmitted ray from X_1 is at point D_1 on line E_1K_1.

The length of the interferometer arms $AB = A_1B_1$ and $AC = A_1C_1$ are equal to L, and the distances A_1F_1 and E_1F_1 are equal to l. Thus, the distance $D_1I_1 = E_1K_1 = L + l$.

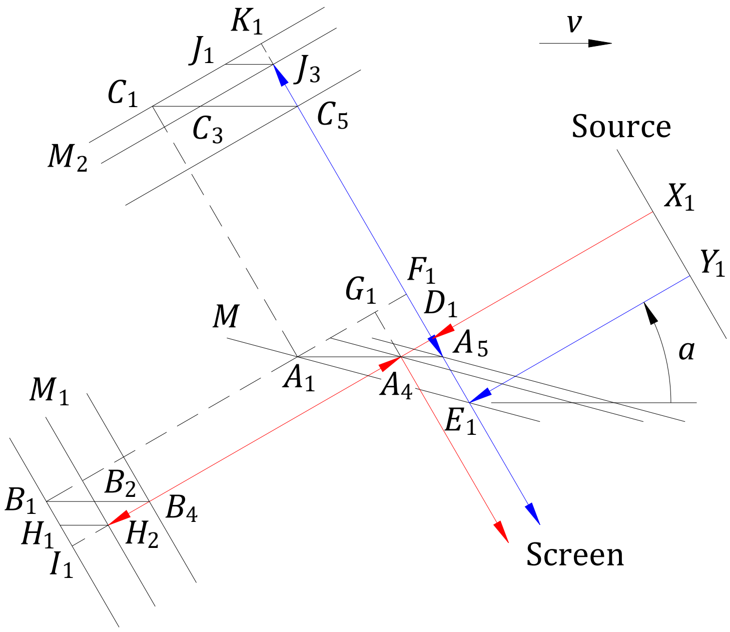

Figure 7. Reflection of light as a mechanical phenomenon applied to a particular Michelson interferometer.

The reflected ray travels from E_1 to J_3 with speed $c_{\mathrm{rf}} = c_{21} = c_{s21} + v\cos a_{21} + v\cos b_{21}$, in time t_{21}. The speed from the source $c_{s21} = c$. Imagine a velocity vector v with its origin at E_1. The angle measured counterclockwise from the direction of the velocity vector v to the opposite direction of the incident ray is $a_{21} = a$. The angle measured counterclockwise from the direction of the velocity vector v to the reflected ray direction is $b_{21} = a + 90°$. Thus, the speed $c_{21} = c + v\cos a + v\cos(a + 90°) = c + v\cos a - v\sin a$.

In time t_{21}, mirror M_2 travels from J_1 to J_3 with speed v, and K_1 travels to J_3 with speed $v\sin a$.

$$E_1K_1 = E_1J_3 + J_3K_1 \quad \Rightarrow \quad L + l = c_{21}t_{21} + vt_{21}\sin a \quad \Rightarrow \quad L + l$$

$$= (c + v\cos a - v\sin a)t_{21} + vt_{21}\sin a \quad \Rightarrow \quad t_{21} = \frac{L + l}{c + v\cos a}$$

The ray reflected by M_2 travels from J_3 toward A_5 and the screen with speed $c_{\mathrm{rf}} = c_{22} = c_{s22} + v\cos a_{22} + v\cos b_{22}$, in time t_{22}. The speed of the incident ray at J_3 is $c_{s22} = c_{21}$. Consider a velocity vector v with its origin at J_3, then the measured angle $a_{22} = b_{22} = a + 270°$. Thus, the speed $c_{22} = c_{21} + 2v\cos(a + 270°) = (c + v\cos a - v\sin a) + 2v\sin a = c + v\cos a + v\sin a$.

In time t_{22}, mirror M_2 travels from C_3 to C_5 with speed v, and J_3 travels to C_5 with speed $v\sin a$. The distance $A_5C_5 = L$.

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$$A_5J_3 = A_5C_5 + C_5J_3 \quad \Rightarrow \quad c_{22}t_{22} = L + vt_{22}\sin a \quad \Rightarrow \quad (c + v\cos a + v\sin a)t_{22} = L + vt_{22}\sin a$$

$$\Rightarrow \quad t_{22} = \frac{L}{c + v\cos a}.$$

$$t_2 = t_{21} + t_{22} = \frac{L + l}{c + v\cos a} + \frac{L}{c + v\cos a} = \frac{2L + l}{c + v\cos a}.$$

The distance $l = A_1F_1 = A_1A_5\cos a = vt_2\cos a \quad \Rightarrow \quad t_2 = l/(v\cos a)$.

The equality of the two formulas of time t_2 yields the distance l.

$$\frac{2L + l}{c + v\cos a} = \frac{l}{v\cos a} \quad \Rightarrow \quad l\left(\frac{1}{v\cos a} - \frac{1}{c + v\cos a}\right) = \frac{2L}{c + v\cos a} \quad \Rightarrow \quad l = \frac{2Lv\cos a}{c}.$$

With the formula of distance l, the formula of time t_2 becomes

$$t_2 = \frac{2L + l}{c + v\cos a} = \frac{2L + \frac{2Lv\cos a}{c}}{c + v\cos a} = \frac{2L}{c}.$$

The transmitted ray travels from D_1 to H_2 with speed $c_{s11} = c_{11} = c$, in time t_{11}. Simultaneously, mirror M_1 travels from H_1 to H_2 with speed v, and I_1 travels to H_2 with speed $v\cos a$.

$$D_1I_1 = D_1H_2 + H_2I_1 \quad \Rightarrow \quad L + l = c_{11}t_{11} + vt_{11}\cos a \quad \Rightarrow \quad t_{11} = \frac{L + l}{c + v\cos a}.$$

The ray reflected by M_1 travels from H_2 to A_4 with speed $c_{rf} = c_{12} = c_{s12} + v\cos a_{12} + v\cos b_{12}$, in time t_{12}. The speed $c_{s12} = c_{11} = c$. Imagine a velocity vector v with its origin at H_2, then the measured angle $a_{12} = b_{12} = a$. Thus, $c_{12} = c_{11} + v\cos a + v\cos a = c + 2v\cos a$.

In time t_{12}, mirror M_1 travels from B_2 to B_4 with speed v, and H_2 travels to B_4 with speed $v\cos a$. The distance $A_4B_4 = L$.

$$A_4H_2 = A_4B_4 + B_4H_2 \quad \Rightarrow \quad c_{12}t_{12} = L + vt_{12}\cos a \quad \Rightarrow \quad (c + 2v\cos a)t_{12} = L + vt_{12}\cos a$$

$$\Rightarrow \quad t_{12} = \frac{L}{c + v\cos a}.$$

$$t_1 = t_{11} + t_{12} = \frac{L + l}{c + v\cos a} + \frac{L}{c + v\cos a} = \frac{2L + l}{c + v\cos a} = \frac{2L}{c}.$$

The ray reflected by M travels from A_4 to the screen with speed $c_{rf} = c_{13} = c_{s13} + v\cos a_{13} + v\cos b_{13}$. The speed $c_{s13} = c_{12}$. Consider a velocity vector v with its origin at A_4, then the measured angle $a_{13} = a + 180°$ and $b_{13} = a + 270°$. Thus, the speed c_{13} is $c_{13} = c_{12} + v\cos(a + 180°) + v\cos(a + 270°) = (c + 2v\cos a) - v\cos a + v\sin a = c + v\cos a + v\sin a$.

As time $t_1 = t_2$, points A_4 and D_1 coincide with point A_5 and point G_1 coincides with point F_1.

The difference of time $\Delta t = t_2 - t_1 = 0$. Thus, the predicted fringe shift is zero.

The classical derivation, based on the hypothesis that the speed of light does not change after reflection, predicts an observable fringe shift of 0.40, when this particular interferometer is rotated in increments of $90°$ starting from the initial position, Appendix B.

3.2. Discussions

The pair of rays that interfere at point A, and any point on M, change continuously with interferometer rotation.

Along the path to the screen, the transmitted and reflected rays are parallel, and their speeds are equal for any angle a, $c_{13} = c_{22} = c + v\cos a + v\sin a$. Thus, the transmitted and reflected rays interfere.

The difference in time for any angle a and at any point on M is given by $\Delta t = t_2 - t_1 = 0$. Thus, the interference image is an illuminated area at maximum brightness that does not change by the rotation of the interferometer. For this particular interferometer, the fringe shift is displayed by the changes in the brightness of the illuminated area.

If the length of the interferometer arms is not equal, i.e., $AB = A_1B_1 = L_1$ and $AC = A_1C_1 = L_2$, then $\Delta t = t_2 - t_1 = 2(L_2 - L_1)/c$. The difference in the arm's length $L_2 - L_1$ is the same constant for any angle a and at any point on M. In conclusion, the interference is not in phase, and the illuminated area has less brightness than that of the maximum. The rotation of the interferometer does not display a change in the brightness of the illuminated area.

In the fixed frame, the pairs of the transmitted and reflected rays travel the same path length $2L$ with the same speed c in the same time $2L/c$, in any inertial frame, independent of angle a and the speed of the inertial frame v.

The same image without any fringe shift is observed in the inertial frame as in the fixed frame. Apparently in any inertial frame, the pairs of the transmitted and reflected rays travel the same path length $2L$ with the same speed c in the same time $2L/c$, for any angle a, as if the interferometer belongs to the fixed frame.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. The speed of a ball after an elastic collision with a rigid wall

In Figure 1, consider that the mirror is replaced with a rigid wall and the ray of light or photon with a ball. The speed of the wall and ball after an elastic collision are derived here.

The wall of mass m_1 travels at speed $v_1 = v$ and the ball of mass m_2 travels at speed $v_2 = c$ in the opposite direction to v_1. The speed of the wall and the speed of the ball after the elastic frontal collision are v_1' and v_2', respectively. The equation for the law of conservation of momentum and kinetic energy yield the solution for speed v_1' and speed v_2'.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'. \tag{1}$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2. \tag{2}$$

The two equations yield the following solutions:

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \quad \text{and}$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2.$$

For ray of light or photon, $m_2 = 0$ and the simplified solutions are $v_1' = v_1$ and $v_2' = 2v_1 - v_2$.

The solutions are derived in mechanics without knowing the direction of the speeds v_1' and v_2' after the collision. For mirror and light, the direction of the speeds v_1' and v_2' are known.

If the direction of v_1 as reference is considered to be positive, then the direction of v_2 is negative and the direction of v_1' and v_2' are positive. The simplified solutions turn out to be $v_1' = v_1$ and $v_2' = 2v_1 - (-v_2) = v_2 + 2v_1$. For the case of mirror and light, v_1 is v, v_2 is c, and v_2' is c_{rf}. Thus, $v_1' = v$ and $c_{\text{rf}} = c + 2v$.

If v_1 has an opposite direction to that illustrated in Figure 1, and this direction as reference is considered to be negative, then the direction of v_2 and v_1' are negative, and the direction of v_2' is positive. The simplified solutions turn out to be $-v_1' = -v_1$ or $v_1' = v_1$ and $v_2' = 2(-v_1) - (-v_2)$ or $v_2' = v_2 - 2v_1$. For the case of mirror and light, v_1 is v, v_2 is c, and v_2' is c_{rf}. Thus, $v_1' = v$ and $c_{\text{rf}} = c - 2v$.

Appendix B. Classical derivation of the fringe shift for the particular Michelson interferometer

The derivation here follows Figure 7; it adopt the same steps as in Section 3, but the speed of light before and after reflection is taken to be the constant c.

For the reflected ray:

$$E_1K_1 = E_1J_3 + J_3K_1 \quad \Rightarrow \quad L + l = ct_{21} + vt_{21} \sin a \quad \Rightarrow \quad t_{21} = \frac{L + l}{c + v \sin a}.$$

$$A_5J_3 = A_5C_5 + C_5J_3 \quad \Rightarrow \quad ct_{22} = L + vt_{22} \sin a \quad \Rightarrow \quad t_{22} = \frac{L}{c - v \sin a}.$$

$$t_2 = t_{21} + t_{22} = \frac{L + l}{c + v \sin a} + \frac{L}{c - v \sin a}.$$

$$l = A_1F_1 = A_1A_5 \cos a = vt_2 \cos a \quad \Rightarrow \quad t_2 = l / v \cos a.$$

The equality of the two formulas of time t_2 yields the distance l.

$$\frac{L + l}{c + v \sin a} + \frac{L}{c - v \sin a} = \frac{l}{v \cos a} \quad \Rightarrow \quad l = \frac{2Lcv \cos a}{(c - v \sin a)(c + v \sin a - v \cos a)}.$$

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For the transmitted ray:

$$D_1I_1 = D_1H_2 + H_2I_1 \quad \Rightarrow \quad L + l = c_{11}t_{11} + vt_{11}\cos a \quad \Rightarrow \quad t_{11} = \frac{L + l}{c + v\cos a}.$$

$$A_4H_2 = A_4B_4 + B_4H_2 \quad \Rightarrow \quad ct_{12} = L + vt_{12}\cos a \quad \Rightarrow \quad t_{12} = \frac{L}{c - v\cos a}.$$

$$t_1 = t_{11} + t_{12} = \frac{L + l}{c + v\cos a} + \frac{L}{c - v\cos a}.$$

The distance l can be calculated for any angle a, followed by the calculations of times t_1 and t_2. The difference of time $\Delta t_{a_1} = t_2 - t_1$. The period of the light wave is given by $T = \lambda/c$; thus, the number of periods or wavelengths in Δt_{a_1} is $N_{a_1} = \Delta t_{a_1}/T = c\Delta t_{a_1}/\lambda$ for any angle a.

If the number of wavelengths for another angle a is N_{a_2}, then the fringe shift by rotating the interferometer from one angle to another is $\Delta N_{a_1,a_2} = N_{a_2} - N_{a_1}$.

Figure 8 illustrates the schematic of the light paths for angle a at 0°, 90°, 180°, and 270°.

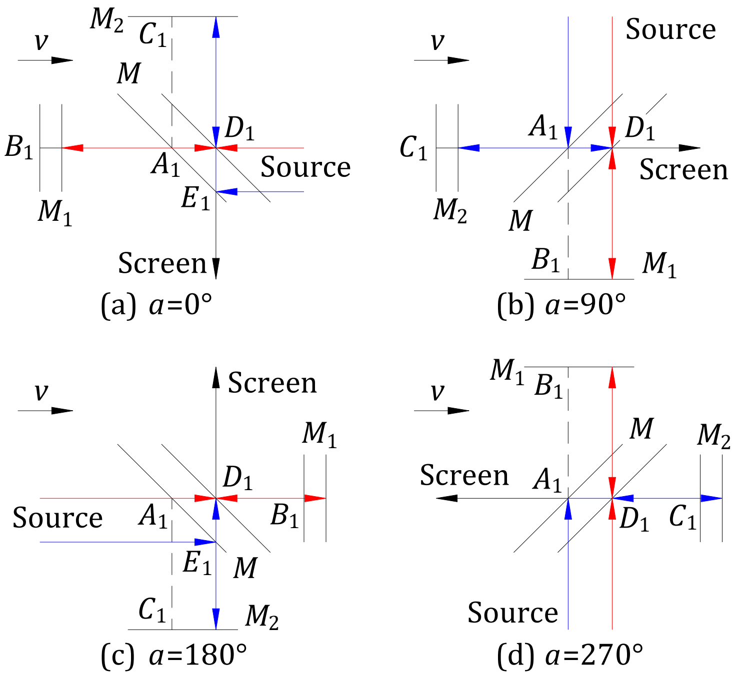

Figure 8. Schematic of the light paths for angle a at 0°, 90°, 180°, and 270°.

For calculations, the length of the interferometer's arms $L = 11$ m, the speed $v = 3.0E + 04$ m/s, the speed $c = 3.0E + 08$ m/s, and the wavelength of the light $\lambda = 550E - 09$ m, which is the same data used by Michelson and Morley for their experiment.

Table 1 gives the numerical calculation, performed in Excel, of the fringe shift in steps of 90° starting from the initial position, for the cases presented in Figure 8.

Table 1. Numerical calculation of the fringe shift for four positions

a [rad]	0	π/2
l [m]	2.2002200220022000E–03	1.3476633138761200E–19
t_2 [s]	7.3340667400073300E–08	7.3333334066666700E–08
t_1 [s]	7.3340667400073300E–08	7.3333333333333300E–08
$\Delta t_0, \Delta t_{\pi/2}$ [s]	0.0000000000000000E+00	7.3333334569908100E–16
$N_0, N_{\pi/2}$	0.0000000000000000E+00	4.0000000674495300E–01
$\Delta N_{3\pi/2,0}$, $\Delta N_{0,\pi/2}$	–4.0000000674495300E–01	4.0000000674495300E–01

a [rad]	π	3π/2
l [m]	–2.1997800219978000E–03	–4.0429899416283500E–19

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t_2 [s]	7.3326000733260000E–08	7.3333334066666700E–08
t_1 [s]	7.3326000733260000E–08	7.3333333333333300E–08
$\Delta t_\pi, \Delta t_{3\pi/2}$[s]	0.0000000000000000E+00	7.3333334569908100E–16
$N_\pi, N_{3\pi/2}$	0.0000000000000000E+00	4.0000000674495300E–01
$\Delta N_{\pi/2,\pi}, \Delta N_{\pi,3\pi/2}$	–4.0000000674495300E–01	4.0000000674495300E–01

The geometry of the light paths as described by Michelson and Morley in their experiment also predicts a fringe shift of 0.40 [1].

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