the general science journal

A unit of magnetic flux density produces a constant electromotive force

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Abstract

Faraday's law of electromagnetic induction states that variations in the magnetic flux permeating a coil induce an electromotive force (EMF). While it is accurate to say that a changing instantaneous magnetic flux density induces an EMF, it is important to recognize that the strength of this force, is directly proportional to the rate of change in magnetic flux. Therefore, a unit magnetic flux density generates a constant EMF only if the magnetic flux is changing at a constant rate.

To illustrate this concept, the electrons in the coil must align uniformly, and the Lorentz force must propel them in the same direction. Examining the center of the bar magnet, reveals that the magnetic field lines are approximately straight and parallel. The outermost electrons of the copper atoms in a copper wire coil are free electrons with a spin of ↑. Thus, it can be deduced that when the coil is positioned in the magnetic field at that location, the electrons will uniformly align from the north pole to south pole. Because it is easy to calculate magnetic flux density, a triangular coil with straight segments was used. To determine the magnetic flux density passing through each side of the triangular coil, measurements were taken of the distance from the bar magnet's center to various points on the coil and the corresponding magnetic flux density. Using these measurements, an approximate function relating distance to magnetic flux density was derived, and the integral value of the magnetic flux density along each side was calculated. It was observed that the EMF induced on each side of the triangular coil varied depending on the angle of the triangle. In addition, I calculated the EMF per unit magnetic flux density by varying the orientation of the triangular coil and its distance from the center of the bar magnet, finding that the values remained approximately consistent.

Keywords

electromagnetic induction; magnetic flux density; obtuse triangle coil; integral value

1 Introduction

Emanating from the magnetic pole (N) of a bar magnet, the magnetic field lines disperse in multiple directions, as depicted in Figure 1a. Consequently, the electrons in the coil within this magnetic field also align in various orientations along the magnetic field lines. Therefore, even though an electromotive force (EMF) is induced in the electrons, it is difficult to understand the overall electron movement. However, upon examining the center of the bar magnet, as shown in Figure 1b, it was observed that the magnetic field lines are approximately parallel. Hence, it was hypothesized that if the coil were positioned in this region, the electrons in the coil would align uniformly, and any induced EMF would cause the electrons to move in a consistent direction. A voltmeter measures the EMF of the entire coil but cannot determine the EMF's magnitude in each coil segment. Consequently, I aimed to determine a method for calculating the EMF's magnitude in each coil segment. Considering that the magnetic flux density spreads concentrically from the bar magnet's center, as illustrated in Figure 1c, to simplify the calculations, it would be advantageous to have a coil with at least one straight side. Therefore, a triangular coil was constructed. Positioning side A of the triangular coil directly beneath the center of the bar magnet, I contemplated the nature of the EMF that would be generated by the electrons in side A at varying magnetic flux density strengths. The magnetic flux density decreases with increasing distance from the bar magnet's center, resulting in a corresponding decrease in the EMF. However, owing to the Lorentz force, all electrons experience an upward EMF, and the cumulative effect of these individual EMFs constitutes the EMF of the entire side A. Consequently, by calculating the integral value of the magnetic flux density permeating side A, the total magnetic flux density for side A can be obtained. Applying this approach to sides B and C allows for the determination of the magnetic flux density for the entire coil. Subsequently, dividing the EMF of the entire coil by the magnetic flux density of the entire coil yields the EMF per unit magnetic flux density (V/T). Furthermore, multiplying the EMF per unit magnetic flux density by the integral of the magnetic flux density of each side also provides the EMF generated on each side. With this in mind, it was decided to construct an obtuse triangular coil with three vertices having angles of 20°, 51°, and 109°, respectively. By calculating the integral of the magnetic flux density for each side and performing an experiment, the consistency of this approach with the EMF of the entire coil could be verified. The coil was constructed with 100 turns. To determine the integral of the magnetic flux density passing through each coil side, measurements were taken of the magnetic flux density spreading concentrically from the bar magnet's center at varying distances. Using these measurements, a function relating distance to magnetic flux density was derived. The bar magnet under examination was positioned in the magnetic flux density measuring apparatus depicted in Figure 1d, and a gaussmeter was placed at the bar magnet's center. The bar magnet was then displaced in 5 mm increments, and the corresponding magnetic flux density y (G) was recorded for each distance x (mm). Using the acquired data an approximate formula for the function was derived using Microsoft Excel. Subsequently, the formula was refined using a software application called GeoGebra, resulting in the following approximation: $f(x) = 253.8e^{-0.02798x} + 2$. When side A of the obtuse triangular coil was positioned directly below the center of the bar magnet, approximate equations for the lengths of each side and the magnetic flux density function were derived using $f(x) = 253.8e^{-0.02798x} + 2$. The integral values of the magnetic flux density along each side were then calculated. Because side A aligns directly below the magnet's center, this approximation formula was applied without modification. To measure the coil's EMF, a custom measuring device was constructed, as shown in Figure 1e. The EMF was recorded at the instant side A of the coil reached the position directly beneath the bar magnet's center. For comparison, measurements were also taken by varying the coil's orientation and its distance from the magnet. The rationale behind changing the coil's orientation in three different ways was the hypothesis that the EMF might vary depending on the coil's orientation. Experimental results demonstrated that the EMF generated on each coil side increased or decreased in accordance with the side's length and inclination (angle). The total EMF V of the coil, as measured by a voltmeter, was determined to be V = A-side integral value × V/T + C-side integral value × V/T - B-side integral value × V/T. Based on these findings, the EMF per unit magnetic flux density, denoted as V/T, can be calculated as V/T = V/ (A-side integral value + C-side integral value - B-side integral value). Experimental observations revealed that the V/T values were within approximately the same range. It is important to note that measurements were taken using a gaussmeter, resulting in values expressed in mV/G. However, for consistency in the paper, these values were later converted to V/T.

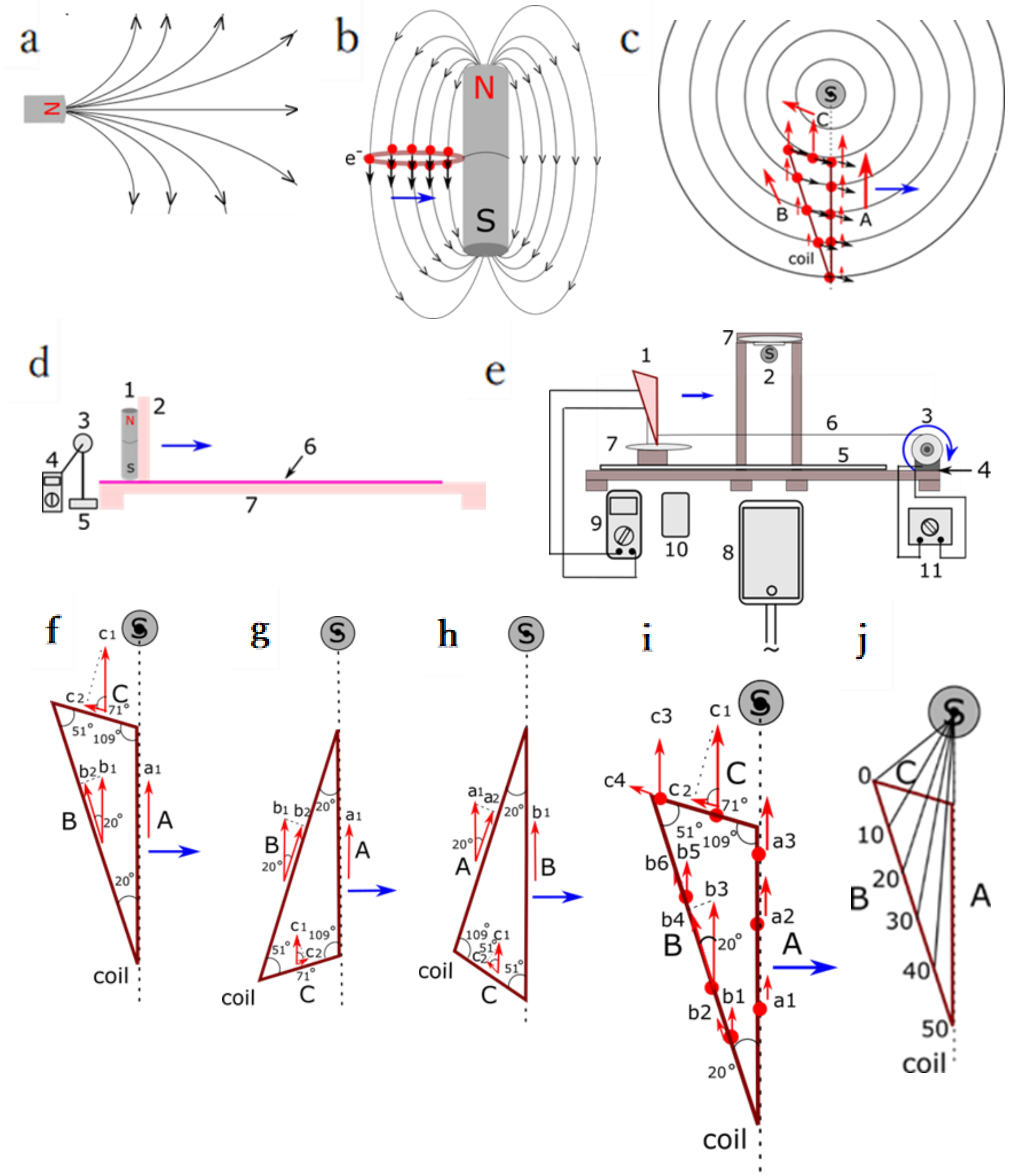

Fig. 1 | Experimental apparatus and electromotive force (EMP) generated in an obtuse triangular coil a: Magnetic lines of force emanating from the magnetic poles. **b:** Red balls, black arrows, and blue arrows indicate electrons in the coil, the direction of electrons, and the direction of coil motion, respectively. **c:** The red arrow represents the magnitude and direction of the EMF, while the blue arrow represents the direction of the coil's travel. **d:** Magnetic flux density measuring device. 1: bar magnet, 2: support plate, 3: probe, 4: gaussmeter, 5: support stand, 6: Scale, 7: wooden stand. **e:** EMF measuring device. 1: obtuse triangle coil, 2: bar magnet, 3: take-up pulley, 4: gearbox with built-in motor, 5: roller stand, 6: kite string, 7: protractor, 8: iPad, 9: digital DC voltmeter, 10: smartphone timer, and 11: DC power supply. The body is made of wood. **f,g,h:** Directions of the obtuse triangular coil are changed. Red arrows indicate the direction and magnitude of the EMF. The blue arrow indicates the direction of the coil's travel. **i:** Represents the direction and magnitude of the EMF generated in the electrons on each side of the obtuse triangular coil. The red spheres represent electrons, the red arrows indicate the direction and magnitude of the EMF, and the blue arrows show the traveling direction of the coil. **j:** Measures the distance from the center of the bar magnet to each point on the B side of the triangle.

2 Experimental Methods

2.1 Experiment 1

To measure the magnetic flux density spreading concentrically from the center of the bar magnet, the magnet was positioned vertically in the magnetic flux density measuring apparatus depicted in Figure 1d. A gaussmeter probe (with the Hall sensor facing upwards) was placed at the bar magnet's center. Measurements were obtained by moving the bar magnet away from its center in 2 mm increments, ranging from 11.5 mm to 21.5 mm. Continuing the measurement process, the bar magnet was moved away from its center in 5 mm increments, ranging from 21.5 mm to 296.5 mm. At each distance from the bar magnet's center, the corresponding magnetic flux density (G) was recorded. It is worth mentioning that the bar magnet used in these experiments was constructed by stacking 23 circular neodymium magnets (each with a diameter of 20 mm and a thickness of 4.8 mm). The maximum magnetic flux density of the north pole was measured to be 1225 G, and the total length of the magnet was 110 mm.

2.2 Experiment 2

I constructed an obtuse triangular coil with angles of 20°, 51°, and 109°, as depicted in Figure 1f. Initially, the plastic coil bases were 20°, 50°, and 110°, but after winding the copper wire, the angles became 20°, 51°, and 109°. The angle of each vertex of the coil was measured using a digital angle meter. The coil wire has a diameter of 0.12 mm, with 100 turns. The length of side A is 123 mm, side B is 149 mm, side C is 54 mm, and the coil width is 5 mm. I attached an obtuse-angled triangular coil (1) to the EMF measuring device shown in Figure 1e and moved it along a roller stand (5) by pulling a string (6) through a winding pulley (3), which was rotated by a gearbox (4) with a built-in motor. The EMF was measured using a digital DC voltmeter (9) at the moment when side A of the coil passed directly beneath the center of the bar magnet (2). Because the voltmeter display has a delay of 0.25 s, I positioned a smartphone timer (10) capable of displaying up to 1/1000th of a second beside the voltmeter and recorded both the timer and the voltmeter readings on the iPad (8). I then examined the video using the SloPro app on the iPad, which provides 50x slow-motion playback, and recorded the voltmeter reading after 0.25 s. The coil was oriented in three different directions (Figures 1f, g, and h), and the distance from the bar magnet to the top of the coil was adjusted to two different values 49 and 64 mm. The EMF was measured six times in the same manner. The coil entered the magnetic field at a speed of 23.97 mm/s. The voltmeter has a resolution of 0.1 mV and an accuracy of ±(0.1%+1). I repeated each measurement five times.

3 Results

The measured values of the distance x (mm) from the center of the bar magnet and the magnetic flux density y (G) in Experiment 1 are presented in Table 1a. Subsequently, the data were plotted in Excel, with distance x (mm) on the x-axis and magnetic flux density y (G) on the y-axis, to obtain an approximate equation. After refining the equation using an application called GeoGebra, I obtained the following approximation: $f(x) = 253.8e^{-0.02798x} + 2$. This is depicted in Figure 2a. Because the actual measured values closely align with the approximation formula from 36.5 to 216.5 mm, I concluded that this approximation formula could be used to derive the integral value in that range. The results of Experiment 2 are summarized in Table 2. Table 1b presents the measured values of the lengths (mm) of sides B and C of the obtuse triangular coil and the corresponding magnetic flux density (G). Figures 2b and 2c show the graphs and the approximate equations for these measurements.

Table 1 | Distance from the magnet and magnetic flux density, and coil length and magnetic flux density

a : Distance from the center of bar magnet and magnetic flux density.
Distance from the magnet (mm) and magnetic flux density (G).

mm	G	mm	G	mm	G
11.5	251.8	101.5	17.1	206.5	2.7
13.5	199.5	106.5	15.4	211.5	2.4
15.5	179.5	111.5	13.9	216.5	2.4
17.5	164.7	116.5	12.3	221.5	2.2
19.5	154.0	121.5	11.0	226.5	2.0
21.5	145.0	126.5	10.0	231.5	1.9
26.5	125.7	131.5	9.0	236.5	1.8
31.5	110.0	136.5	8.2	241.5	1.6
36.5	95.0	141.5	7.6	246.5	1.5
41.5	83.1	146.5	7.0	251.5	1.4
46.5	72.5	151.5	6.3	256.5	1.4
51.5	62.8	156.5	5.8	261.5	1.4
56.5	54.8	161.5	5.2	266.5	1.1
61.5	47.9	166.5	4.8	271.5	1.1
66.5	42.0	171.5	4.5	276.5	1.0
71.5	36.4	176.5	4.2	281.5	1.0
76.5	31.9	181.5	3.7	286.5	0.9
81.5	28.0	186.5	3.4	291.5	0.9
86.5	24.7	191.5	3.3	296.5	0.9
91.5	22.0	196.5	3.0		
96.5	19.4	201.5	2.9		

b: Obtuse triangle coil B side, C side length, and magnetic flux density. Coil length, distance from the magnet (mm), and magnetic flux density (G).

B			C		
Length	mm	G	Length	mm	G
0	59.23	50.39	0	49.00	66.43
10	62.06	46.71	10	46.68	70.75
20	66.27	41.74	20	46.43	71.23
30	71.17	36.65	30	48.17	67.94
40	78.21	30.45	40	51.65	61.82
50	85.14	25.44	54	59.23	50.39
60	92.42	21.12			
70	100.54	17.23			
80	108.69	14.13			
90	117.30	11.53			
100	126.74	9.32			
110	135.28	7.76			
120	144.24	6.48			
130	153.64	5.45			
140	162.46	4.69			
149	172.00	4.06			

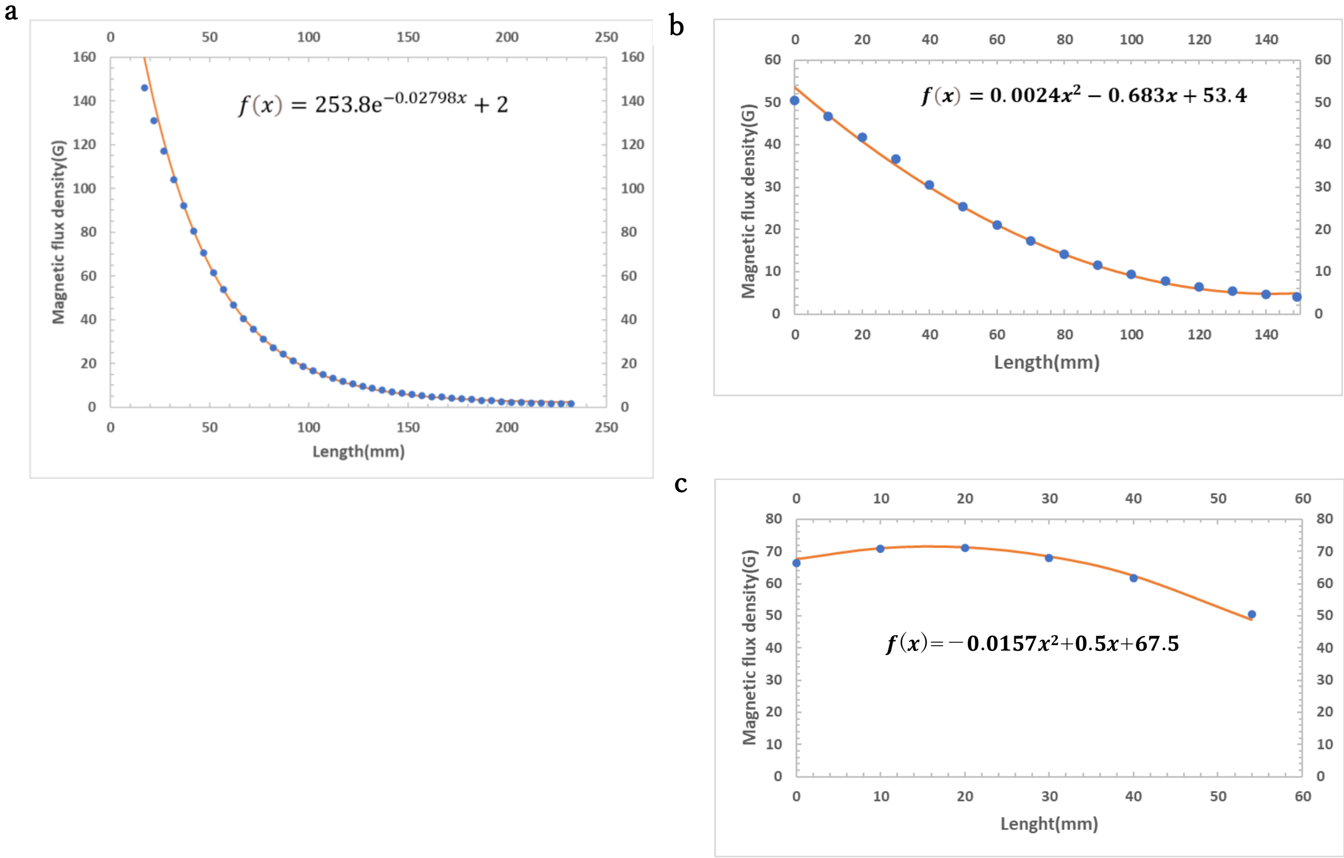

Fig. 2 | a: The graph in Table 1a shows the distance from the center of the bar magnet and the magnetic flux density. The blue points represent the measured values, and the orange curve represents the approximation formula. **b**: The graph in Table 1b shows the length and magnetic flux density of side B of the coil. The blue points represent the measured values, and the orange curve is the approximation formula. **c:** The graph in Table 1b shows the length of side C of the coil and the magnetic flux density. The blue points represent the measured values, and the orange curve is the approximation formula.

Table 2 | Range of mV/G calculated from the integral value of EMF and magnetic flux density of the obtuse triangle coil

	Distance from magnet (mm)	Electromotive force range (mV)	Integral value A	Integral value B	Integral value C	Integral value of whole coil	mV/G range
1  C, B, A	49	[0.23975 , 0.28027]	2474.88	2839.13	1155.86	A+C-B 791.61	[0.000303 , 0.000354]
	64	[0.12986 , 0.16015]	1710.92	2045.07	805.40	A+C-B 471.25	[0.000276 , 0.000340]
2  B, A, C	49	[0.01997 , 0.05004]	2474.88	2454.86	61.99	A+C-B 82.01	[0.000244 , 0.000610]
	64	[0.00998 , 0.03002]	1710.92	1709.26	52.46	A+C-B 54.12	[0.000184 , 0.000555]
3  C, A, B, C	49	[0.00998 , 0.05004]	2375.31	2564.98	117.40	B-A-C 72.27	[0.000138 , 0.000692]
	64	[0 , 0.03002]	1639.61	1787.96	100.49	B-A-C 47.86	[0 , 0.000627]

4 Discussion

In experiment 1, the approximate equation $f(x) = 253.8e^{-0.02798x} + 2$ was obtained. In experiment 2, I questioned why the EMF exhibited variance when changing the orientation of the obtuse triangular coil in three different ways despite maintaining a constant distance from the center of the bar magnet. I hypothesized that modifying the coil's orientation would change the direction in which the EMF was produced based on the inclination of its sides. This led to the formulation of the model depicted in Figure 1i. As illustrated in the figure, all electrons on side A of the obtuse triangular coil experience an upward EMF, as represented by a1.

However, I reasoned that the electrons on side B would experience an upward EMF, as depicted by b3, which would subsequently decrease to an EMF resembling b4 owing to the coil's inclination. In this scenario, b4 = b3 · cos20°. Similarly, side C is also affected by the coil's inclination, resulting in c2 = c1 · cos71°. Subsequently, I inferred that the EMFs of the entire coil, a1 and c2, are generated counterclockwise, as viewed in the diagram, while b4 is generated clockwise. Consequently, they cancel each other out, leaving the remaining EMF of the entire coil, which is measured by the voltmeter. As illustrated in Figure 1i, upward EMFs ranging from a1 to a3 act on side A of the obtuse triangular coil. The collective sum of these forces can be interpreted as the EMF of the entire side A.

I speculated that if we consider the EMF of the entire side A to be generated by the sum of the magnetic flux densities acting on each electron in side A, I may be able to derive the EMF per unit magnetic flux density. The magnetic flux density acting on the entire side A was determined in experiment 1. The magnetic flux density acting on the entire side A can be calculated by integrating the approximation function $f(x) = 253.8e^{-0.02798x} + 2$ over the length of side A. However, how do I determine the total magnetic flux density on sides B and C? On side B, there is an attenuation of 20°, resulting in b4 = b3 · cos20°.

Furthermore, b2 +b4 +b6 is the EMF of the entire side B. Because side C also experiences an angular attenuation of 71°, I have c2 = c1 · cos71°, and c2 + c4 is the EMF of the entire side C. Therefore, it is assumed that the total magnetic flux density is also susceptible to angular attenuation. While side A is located directly below the center of the bar magnet and remains unaffected by angular attenuation, allowing us to use the approximation formula, this approach is not applicable to sides B and C. To derive an approximate formula for the magnetic flux density acting on sides B and C, I traced the coil onto a sheet of paper and marked the position of the magnet. Subsequently, as depicted in Figure 1j, I made marks every 10 mm on sides B and C and measured the distance from the center of the magnet using a digital caliper capable of measuring to 1/100 mm. These measurements were then substituted into the function $f(x) = 253.8e^{-0.02798x} + 2$ to calculate the magnetic flux density (G) of each point. Using the values of the length x (mm) every 10 mm of sides B and C and the corresponding magnetic flux density y (G) obtained through this process, I derived approximate equations for sides B and C, as depicted in Figures 2b and 2c, respectively. Subsequently, using these approximation formulae, I calculated the integral value of the magnetic flux density on each side to determine the angular attenuation. Notably, I derived approximate equations for sides A, B, and C of the obtuse triangular coil, as shown in Figure 1f, and used them to calculate the integral of the magnetic flux density (G) acting on each side as well as the integral of the entire coil.

Obtuse triangle coil in Fig. 1f (with a 49 mm distance from the center of the bar magnet)

Side A $\int_{49}^{172}(253.8e^{-0.02798x} + 2)\,dx \approx 2474.88$

Side B $\cos20 \cdot \int_{0}^{149}(0.0024x^2 - 0.683x + 53.4)\,dx \approx 2839.13$

Side C $\cos71 \cdot \int_{0}^{54}(-0.0157x^2 + 0.5x + 67.5)\,dx \approx 1155.86$

The integral of the magnetic flux density across the coil is A + C − B, which equals 2474.88 + 1155.86 − 2839.13 = 791.61.
Table 2 lists the coil's EMF, the integral value of the magnetic flux density, and mV/G.
From Table 2, the EMF (mV) of this coil is in the range [0.23975, 0.28027].
Therefore, mV/G is [$\frac{0.23975}{791.61}$, $\frac{0.28027}{791.61}$], which is in the range [0.000303, 0.000354].
Here, the calculations were initially done in mV/G but were later converted to V/T.
The common mV/G range in Table 2 is [0.000303, 0.000340].
Using this value, I will determine the EMF on each side of the obtuse triangular coil located 49 mm away from the center of the bar magnet, as illustrated in Figure 1f.

The EMF on side A is the integral of the magnetic flux density on side A× mV/G.
Thus, 2474.88 · 0.000303 ≈ 0.75, 2474.88 · 0.000340 ≈ 0.84. Therefore, [0.75, 0.84].
Similarly, for side B : 2839.13 ・0.000303≈0.86, 2839.13 ・0.000340≈0.97. Therefore, [0.86, 0.97].
For side C: 1155.86 · 0.000303 ≈ 0.35, 1155.86 · 0.000340 ≈ 0.39, so [0.35, 0.39].
The EMF of the entire coil is A + C – B. Thus, 0.75 + 0.35 – 0.86 = 0.24, 0.84 + 0.39 – 0.97 = 0.26. Therefore, [0.24, 0.26].
The measured EMF is [0.23975, 0.28027]. Thus, it can be said to fall within this range.

5 Conclusion

In this research, I have pursued an investigative approach based on the notion that if the EMF per unit magnetic flux density (V/T) were to assume a constant value, this would serve as indirect evidence corroborating the attenuation of the EMF attributable to the inclination (angle) of the coils on each side of the obtuse triangular coil. By measuring the EMF while systematically varying the coil orientation and its distance relative to the center of the magnet, I discovered that the V/T range remained remarkably consistent across all experimental configurations. In other words, the EMF undergoes attenuation by a factor of cosθ contingent upon the angle of inclination of the coil. Therefore, it is assumed that the EMF per unit magnetic flux density (V/T) will be within a specific range of values. Moreover, I ascertained that the EMF was dictated by the angle of the first side and was not affected by the angle of the next side. In the experiment, the voltage was measured in mV, the coil speed was 23.97 mm/s, the number of coil turns N was 100, and the magnetic flux density was expressed in G (Gauss). The calculated range of mV/G was determined to be [0.000303, 0.000340]. If I recalculate this with voltage in V and magnetic flux density in T (tesla), taking into account the conversions 1 V= 1000 mV and 1T = 10000 G, the corresponding V/T range becomes

$$\left[\frac{0.000303 \div 1000}{1 \div 10000}, \frac{0.000340 \div 1000}{1 \div 10000}\right] \approx [0.00303, 0.00340].$$

Also, if we calculate that the number of turns N of the coil is 1 turn (1/100) and the coil speed is 1 m/s (1/0.02397), I get the value $\left[\frac{0.00303}{100 \times 0.02397}, \frac{0.00340}{100 \times 0.02397}\right] \approx [0.00126, 0.00142]$.

This value represents the EMF (V) per unit magnetic flux density (T) when the number of turns in the coil (N) is 1, and the coil speed (v) is 1 m/s.
Using this, the formula to calculate the EMF V on side A in Figure 1f is

$$V = [-\ N\,v\ \ 126E\text{-}05 \int_a^b f(x)\,dx \sin\theta,\quad -\ N\,v\ \ 142E\text{-}05 \int_a^b f(x)\,dx \sin\theta].$$

N is the number of turns in the coil, and the EMF V is proportional to the number of turns N.
The negative sign (−) originates comes from Faraday's formula for electromagnetic induction. v is the speed at which the coil enters the magnetic field, measured in m/s, and is directly proportional to the EMF V. $f(x)$ is a function of the distance x from the center of the bar magnet and the magnetic flux density T. This function was used to calculate the magnetic flux density T by integrating over the length of side A of the coil (from a to b). Sinθ is the attenuation of the EMF in accordance with the angle θ between the direction of the coil and the magnetic field. An angle of 90° corresponds to the maximum EMF, while an angle of 0° corresponds to the minimum EMF [1]. Under the assumption that the integral value (A +C −B) of the magnetic flux density of the entire coil in Figure 1f is equivalent to T, the formula for calculating the EMF of the entire coil becomes:

V = [-Nv 126E-05 T sinθ, -Nv 142E-05 T sinθ].

Is this formula not a more detailed expression of Faraday's law of electromagnetic induction, $V = -N\frac{\Delta\Phi}{\Delta t}$?

I hope that future research will provide more detailed verification.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declaration of Competing Interest

The author has no competing interests or other interests that might be perceived to influence the results and/or discussion reported in this paper.

CRediT author statement

Shingo Ito: research ideas, experiments, data collection, data analysis, theoretical framework, and writing the paper.

Data availability

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Code availability

NA

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