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Two Derivations of Mass-energy Equivalence Law as E = ½mc²

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Abstract: A mass-energy equivalence is derived in two ways, as *E = ½ mc^2*, different from the relativistic equation, where m is mass and c speed of light in a vacuum. The first derivation considers a particle of charge Q moving at speed v, as having kinetic energy in the increase of its electric field, relative to an observer. Secondly, by the induction electric field being a reactive field $\mathbf{E}_a$, due to backward curling of radial field lines of force from an accelerated charged particle, acting on the same charge to create inertial force $Q\mathbf{E}_a$ equal and opposite to accelerating force $m(d\mathbf{v}/dt)$. E is intrinsic energy of mass m.

Keywords: Aberration of Electric Field, Acceleration, Electric Charge, Electric Field, Energy, Force, Mass, Radiation, Relativity, Speed, Velocity.

1. Introduction

Velocity of light **c**, a vector of magnitude (speed) c, will feature prominently in this paper. Vector quantities will be represented by **bold-face type** and scalar quantities by ordinary type or *italicised type*. The renowned physicist, James C. Maxwell, derived an equation for the speed of light in a vacuum, as in equation (1), where μ_o is the permeability and ε_o the permittivity of an electric field in a vacuum.

The mass-energy equivalence law, of special relativity theory, gives energy content E of a particle of mass m as in equation (2). This has become the most famous equation in the world. The mass m in equation (2) is supposed to vary with speed v of the particle, relative to an observer, in accordance with the relativistic mass-velocity formula in equation (3), where m_o is the rest mass (at $v = 0$) and γ is the Lorentz factor. Relativistic kinetic energy is supposed to be accounted for in an increase of mass m of the particle, which becomes infinitely large at the speed of light $v = c$. Kinetic energy K of mass m, is expressed in equation (4), as increase in mass of a moving particle [1, 2].

$c = \sqrt{1/\mu_o\varepsilon_o}$ (1)	$E = mc^2$ (2)	$m = m_o/\sqrt{1-(v/c)^2} = \gamma m_o$ (3)	$K = mc^2 - m_o c^2 = m_o c(\gamma - 1)$ (4)

This paper, invoking aberration of electric field, a missing link in physics, shows that mass of a moving particle remains constant at the rest mass m_o, and the limiting speed c for a particle of charge Q accelerated to velocity **v** by an electric field $\mathbf{E}_o$, is the result of radiation. Radiation reaction force is $-QE_o\mathbf{v}/c$ and radiation power is $QE_o v^2/c$, in rectilinear motion. At the speed of light c, radiation reaction force becomes equal and opposite to the impressed force $Q\mathbf{E}_o$. This makes the accelerating force zero and the particle moves with constant speed c, radiating energy, equal to potential energy lost. So, speed of light is a terminal speed imposed by radiation reaction force [3 – 5].

The first derivation invokes aberration of electric field, a missing link in physics. The second derivation uses Faraday's law of electromagnetic induction. The induction electric field $\mathbf{E}_a$ due to a particle of charge Q and mass m moving at time t with velocity **v** and acceleration $d\mathbf{v}/dt$, is supposed to act on the same charge to produce the inertial force $Q\mathbf{E}_a$ equal and opposite the accelerating force $m(d\mathbf{v}/dt)$.

2. Coulomb's Law and Aberration of Electric Field

Figure 1 depicts Coulomb's law for showing force **F**, dynamic electric field $\mathbf{E}_v$, and electrostatic field $\mathbf{E}_o$ for a particle of charge Q moving at time t with velocity **v**, relative to an observer. **F**, $\mathbf{E}_v$ and $\mathbf{E}_o$ are respectively given by equations (5), (6) and (7), in accordance with aberration of electric field.

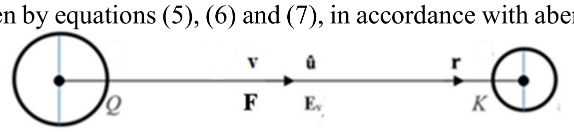

Figure 1: Electric field $\mathbf{E}_v$ of charge Q at velocity **v** relative to charge K, distance r apart.

$\mathbf{F} = \dfrac{QK}{4\pi\varepsilon_o cr^2}(\mathbf{c}+\mathbf{v}) = \dfrac{E_o}{c}(\mathbf{c}+\mathbf{v})$ (5)	$\mathbf{E}_v = \dfrac{Q}{4\pi\varepsilon_o cr^2}(\mathbf{c}+\mathbf{v}) = \dfrac{E_o}{c}(\mathbf{c}+\mathbf{v})$ (6)	$\mathbf{E}_o = \dfrac{Q}{4\pi\varepsilon_o r^2}\hat{\mathbf{u}} = \dfrac{Q}{4\pi\varepsilon_o r^2}\dfrac{\mathbf{c}}{c} = E_o\dfrac{\mathbf{c}}{c}$ (7)

Figure 2 depicts aberration of electric field for a particle of charge Q moving at O with velocity **v** relative to an observer. In Figure 2, electrostatic field $\mathbf{E}_o$ with light velocity **c**, at angle θ to velocity **v**, appears displaced to ON by aberration angle α as given by equation (8), such that the vector sum (**c** + **v**) is along ON, in the direction of unit vector **û**. Dynamic field $\mathbf{E}_v$ is expressed in equation (9).

$\sin\alpha = (v/c)\sin(\theta - \alpha)$ (8)	$\mathbf{E}_v = (E_o/c)(\mathbf{c}+\mathbf{v}) = \hat{\mathbf{u}}(E_o/c)\sqrt{c^2+v^2+2cv\cos\theta}$ (9)

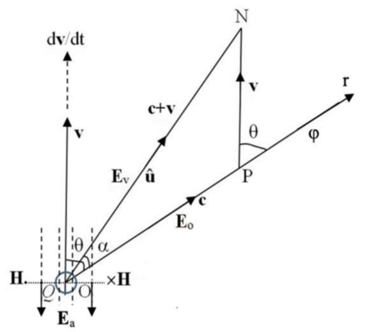

Fig 2: Aberration of electric field due to motion of charge Q with potential φ, moving at time t with velocity **v** and acceleration $d\mathbf{v}/dt$, creating magnetic field **H**, dynamic field $\mathbf{E}_v$ and induction field $\mathbf{E}_a$, where $\sin\alpha = (v/c)\sin(\theta - \alpha)$

3. Intrinsic Energy of a Particle

If a particle of charge Q assumes a configuration, it is likely to be an impregnable spherical shell of radius a and mass m as the smallest in nature. The straight radial electrostatic field lines of force pull the surface charge outwards, to maintain a stable structure. This pulling force explains the force of repulsion or attraction between charges and inertial force due to acceleration of a charged particle.

Intrinsic energy E_n, or electrostatic energy of electrostatic field $\mathbf{E}_o$, expressed in terms of the charge Q, is given by volume integral:

$$E_n = (\varepsilon_o/2)\int_V E_o^2(dV) = (\varepsilon_o/2)\int_a^\infty \left(Q/4\pi\varepsilon_o r^2\right)^2\left(4\pi\varepsilon_o r^2\right)(dr) = Q^2/8\pi\varepsilon_o a \qquad (10)$$

The integral is taken form infinity to the surface of a hollow sphere of radius a. This is the work done in creating the charge Q, of mass m, by infinitesimal amounts from infinity to surface of a hollow sphere of radius a.

Energy contained in the dynamic electric field $\mathbf{E}_v$ (equation 9), is supposed to contain the intrinsic energy E_n in equation (10) and the kinetic energy ½ mv^2 of the charge Q of mass m moving with speed v, relative to an observer. Increase in the electric field, from $\mathbf{E}_o$ to $\mathbf{E}_v$, is $E_o\mathbf{v}/c$, as expressed in equation (9). Energy content of field $E_o\mathbf{v}/c$, equal to the kinetic energy, is as in equation (7), giving intrinsic energy E_n in equation (8). Equation (1) and equation (8) give mass m of charge Q as hollow sphere of radius a, in equation (9).

$(\varepsilon_o/2)\int_V E_o^2(v^2/c^2)(dV) = E_n(v^2/c^2) = (1/2)mv^2$ (11)	$E_n = (1/2)mc^2 = Q^2/8\pi\varepsilon_o$ (12)	$m = \mu_o Q^2/4\pi\varepsilon_o a$ (13)

The intrinsic energy E_n in equation (10), for a particle of charge Q, may be written as:

$$E_n = Q^2/8\pi\varepsilon_o a = (1/2)QU \qquad (14)$$

where Q is the charge, created from 0 to Q in its own potential U built from 0 to U.

4. Induction Electric field and Force of Inertia

Magnetic field **H** generated by a moving electric charge Q, is defined as vector product of velocity **v** and the dynamic electric field, $\mathbf{E}_v = (E_o/c)(\mathbf{c} + \mathbf{v})$, (equation 5), with reference to Figure 2. Magnetic field H and magnetic flux intensity $\mathbf{B} = \mu_o\mathbf{H}$, are given by:

$$\mathbf{H} = \varepsilon_o \mathbf{v}\times\mathbf{E}_v = (\varepsilon_o E_o / c)\mathbf{v}\times(\mathbf{c}+\mathbf{v}) = (\varepsilon_o E_o / c)\mathbf{v}\times\mathbf{c} = \varepsilon_o(\mathbf{v}/c)\times\mathbf{E}_o \tag{15}$$

$$\mathbf{B} = \mu_o\mathbf{H} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_v = (\mu_o\varepsilon_o E_o / c)\mathbf{v}\times(\mathbf{c}+\mathbf{v}) = (\mu_o\varepsilon_o E_o / c)\mathbf{v}\times\mathbf{c} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o \tag{16}$$

where vector product $\mathbf{v}\times\mathbf{v} = 0$ and $(E_o/c)\mathbf{c} = \mathbf{E}_o$. Equation (12), with transformation according to vector calculus, is expressed as:

$$\mathbf{B} = \mu_o\mathbf{H} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o = -\mu_o\varepsilon_o\mathbf{v}\times\nabla\varphi = \mu_o\varepsilon_o\nabla\times(\varphi\mathbf{v}) = \nabla\times\mathbf{A} \tag{17}$$

where the electrostatic field $\mathbf{E}_o = -\nabla\varphi$ denotes the gradient of scalar, φ is scalar potential at a point due to the charge, $\nabla\times\mathbf{A}$ denotes the curl of a vector, **A** is the magnetic vector potential and vector product $\mathbf{v}\times\nabla\varphi = -\nabla\times(\mathbf{v}\varphi)$. Equation (17) gives:

$$\mathbf{A} = \mu_o\varepsilon_o\varphi\mathbf{v} \tag{18}$$

where μ_o is permeability and ε_o permittivity of electric field occupying a vacuum, ∇ denotes the gradient of scalar, φ is the scalar electrostatic potential at a point due to the charge, $\nabla\times$ denotes curl of a vector and **A** is the magnetic vector potential.

A changing magnetic flux intensity **B** with time t, causes inductive electric field $\mathbf{E}_a$, given by Faraday's law and equation 13, as:

$$\nabla\times\mathbf{E}_a = -\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}\nabla\times\mathbf{A} = -\nabla\times\frac{\partial A}{\partial t} \tag{19}$$

$$\mathbf{E}_a = -\frac{\partial\mathbf{A}}{\partial t} = -\mu_o\varepsilon_o\varphi\frac{\partial\mathbf{v}}{\partial t} \tag{20}$$

The electric field $\mathbf{E}_a$ in equation (20) is a reactive field due to backward curling of electric field lines of force from a charged particle. Figure 3 shows the straight field lines of force of an isolated stationary charged particle, pulling the surface charge equally outwards. In Figer 4, for a particle moving with constant velocity **v**, the lines of force are straight, but increased in the forward direction and decreased backwards.

Figure 5 is for a charged particle under acceleration. As a result of finite speed of light, a sudden increase of velocity is not instantaneously impacted to near and far fields. Consequently, the field lines of force are curled backwards to produce a resultant field $\mathbf{E}_a$, thereby creating a reverse effective force **f** equal to the inertial force $-m(\partial\mathbf{v}/\partial t)$, so that:

$$\mathbf{f} = Q\mathbf{E}_a = -m\frac{\partial\mathbf{v}}{\partial t} \tag{21}$$

Equation (21) explains the origin of inertia as property of a body composed of electric charges, not of the space surrounding a body.

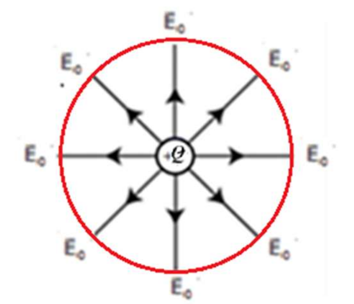

Fig. 3: Radial electrostatic field $\mathbf{E}_o$ of isolated stationary charge +Q.

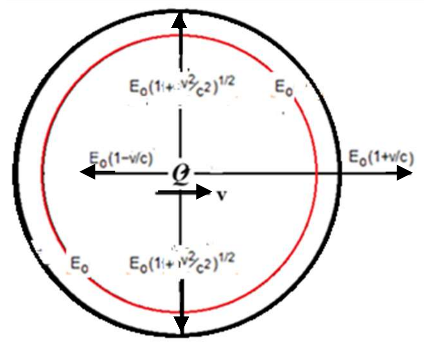

Fig. 4: Electric field lines of force of charge Q at velocity **v** relative to an observer.

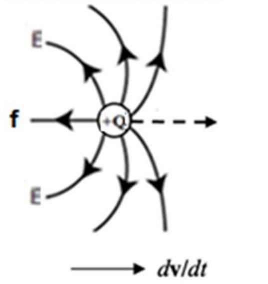

Fig. 5: Electric field lines of force of a particle of charge Q moving with acceleration ($d\mathbf{v}/dt$) and inertial force **f**

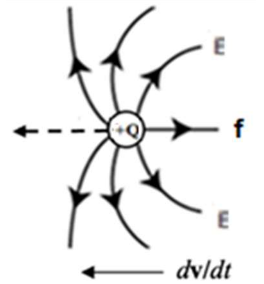

Fig. 6: Electric field lines of force of a particle of charge Q moving with deceleration ($d\mathbf{v}/dt$) and inertial force **f**

Substituting equation (20) into equation (21), gives:

$$\mathbf{f} = Q\mathbf{E}_a = -\mu_o\varepsilon_o\varphi Q\frac{\partial\mathbf{v}}{\partial t} = -m\frac{\partial\mathbf{v}}{\partial t} \tag{22}$$

$$m = \mu_o\varepsilon_o\varphi Q \tag{23}$$

In equation (23), the product φQ exists only at the location of charge Q in space, where $\varphi = U$. Equations (1), (10) and (23) give:

$$E_n = (1/2\mu_o\varepsilon_o)m = (1/2)mc^2 \tag{24}$$

5. Results and Discussion

- The proposition that a charged particle, like an electron, is an indestructible spherical shell with straight radial electric field lines of force (Figure 3) pulling the surface charge outwards, explains forces of repulsion and attraction and inertia, where curled.
- Curling of electric field lines of force is much more realistic than curving of space-time continuum in general relativity..
- In Figure 4 the field is increased, doubling at the speed of light in the forward direction, but reducing to zero backwards.
- Equation (21) explains the origin of inertia being a result of finite speed of light, causing a delay in effects and backward curling of radial electric field lines of force from an accelerated charged particle, producing a reverse effective force (Figure 5).
- Equation (8) and (24) are derivations of mass-energy equivalence as $E_n = (1/2)mc^2$, where E_n is the intrinsic energy of mass m.
- An electrostatic field $\mathbf{E}_o$ is a medium of density $\mu_o(\varepsilon_o\mathbf{E}_o)^2$ and pressure $\varepsilon_o(\mathbf{E}_o)^2$, on propagation of light at speed $c = \sqrt{1/\mu_o\varepsilon_o}$.
- An electrical force, transmitted at the speed of light, accelerates a charged particle, at constant mass, to that speed and no faster.

6. Conclusion

The paper has given two derivations of mass-energy equivalence law as $E_n = (1/2)mc^2$, where E_n is intrinsic energy of mass m.

References: [1]. https://dx.doi.org/10.33140/ATCP.05.04.01 [2]. https://dx.doi.org/10.33140/ATCP.01.03.00002
[3]. doi.org/10.33140/ATCP.03.02.01 [4]. doi.org/10.47363/JPSOS/2023(5)186 [5]. doi.org/10.33140/ATCP.02.04.10