CRITICAL ANALYSIS OF THE FOUNDATIONS OF PURE MATHEMATICS

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ABSTRACT:
The critical analysis of the foundations of pure mathematics is proposed. The unity of formal logic and of rational dialectics is methodological basis of the analysis. The main result is as follows: the concept of "mathematical quantity" – starting and central concept in mathematics – is meaningless, erroneous, and inadmissible one because it represents the following formal-logical and dialectical-materialistic errors: negation of existence of the essential sign (feature) of concept (i.e., negation of existence of the essence (content, intension) of the concept) and negation of existence of measure of material object. The correct metrical-geometrical and physical interpretations of mathematical relationships show that the set of standard mathematical functions represents error, mistake, and blunder. Therefore, pure mathematics does not satisfy the general-scientific criterion of truth: practice is criterion of truth. The obtained results lead to the conclusion that the generally accepted foundations of pure mathematics are a false one, and they should be reconsidered.
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INTRODUCTION

Recently, the progress of sciences, engineering, and technology has led to rise of a new problem: the problem of rationalization of the fundamental sciences (for example, physics, mathematics). Rationalization of sciences is impossible without rationalization of thinking and critical analysis of the foundations of sciences within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics.

As is well known, mathematics is widely and successfully used in the natural sciences. However, it does not mean that the problem of validity of pure mathematics is now completely solved, or that the foundations of mathematics are not in need of formal-logical and dialectical-materialistic analysis. Critical analysis within the framework of the correct methodological basis shows [1-12] that the foundations of theoretical physics and of mathematics (for example, classical geometry, the Pythagorean theorem, differential and integral calculus, vector calculus, trigonometry, theory of negative numbers) contain formal-logical errors.

In my opinion, the formal-logical errors in pure mathematics arise because mathematics abstracts the quantitative aspect from the qualitative aspect of real objects. Mathematics ignores the dialectical-materialistic principle of unity of the quantitative and qualitative aspects [13-17]. Therefore, mathematics does not satisfy the general-scientific criterion of truth: practice is criterion of truth. This gives reason to assert that pure "mathematics is the doctrine where it is not known that we talk about and whether it is true that what we speak" (Bertrand Russell).

In this connection, the problem of complete understanding of the essence of pure mathematics and, consequently, the problem of critical analysis of the foundations of pure mathematics within the framework of the correct methodological basis arise. In my opinion, standard mathematics can not be considered as a science if there is no formal-logical and dialectical substantiation of it. The purpose of the present work is to propose the critical analysis of the foundations of pure mathematics within the framework of the correct methodological basis and to afford an opportunity to make sure of the essence of pure mathematics.

1. THE METHODOLOGICAL BASIS

As is known, correct methodological basis of science is the unity of formal logic and of rational dialectics. Use the correct methodological basis is a necessary condition for correct analysis to make distinction between truth and falsehood. However, this fact is ignored by majority of scientists until now. Therefore, the main propositions of formal logic and of rational dialectics which are used in the present work must be stated.

1.1. The basic principles of formal logic

1. Formal logic is science of the laws of correct thinking as well as means of cognition of reality. Correct thinking represents uncontradictory, coherent, consistent, and sequential thinking. The conclusions resulting from correct thinking are true statements which reflect correctly the objective reality in the process of scientific cognition of the world. The basic formal-logical laws are the following four laws: the law of identity, the law of lack (absence) of contradiction, the law of excluded middle, the law of sufficient reason.

2. Thinking is the highest form of human cognitive activity which represents the process of reflection of objective reality in human consciousness. Human thinking is performed with the help of concepts and has different forms.

3. The form of thought reflecting and fixing the essential sings (features) of things, objects, and phenomena of reality is called concept. In other words, the concept is the thought that reflects things, objects from viewpoint of the general and essential signs (features). (Thing is an object that can be in relation to anything or have some property).

4. The essential sings (features) of the concept are chosen (are singled out) in objects and phenomena by thought. The essential sings (features) characterize the objects of given kind. Non-essential sings (features) do not characterize the objects of given kind. The characteristic which is used to determine similarity or difference of objects of thought is called essential sing (feature). In the most general view, sings (features) of objects can be reduced to properties (for example, large, small, white, black, good, bad, soft, hard, etc.), states (for example, state of rest, state of motion, energetic state, equilibrium state, etc.), actions (for example, it works, he reads, she performs her duties, etc.), and results of actions (for example, have scored success, have benefited, etc.), etc.

5. The first basic form of thought is a concept. Concepts are formed (created) with the help of logical methods such as analysis and synthesis, abstraction and generalization. Analysis is the mental decomposition (dissection) of the object of thought in terms of the elements, the choice (separation) of either sing, and the consideration of it separately. Analysis does not give knowledge of object or of phenomenon as a whole. Synthesis is the mental integration (association, combination, junction) of the elements of the object or of the phenomenon. Synthesis provides knowledge of object or of phenomenon as a whole (as a unity of parts, as a system). But this knowledge is not the reliable and complete one. Abstraction is the mental separation, the mental extraction of the certain, the essential sings (features) of object or of phenomenon and passing over all other sings (i.e., abandonment of all other sings (features) without consideration). Generalization is the mental transition from sings (features) of individual, separate, single objects to sings (features) belonging to whole groups (classes) of these objects. Abstraction is the mental separation, the mental extraction of the certain, the essential sings (features) of object or of phenomenon and passing over all other sings (i.e., abandonment of all other sings (features) without consideration). Generalization is the mental transition from sings (features) of individual, separate, single objects to sings (features) belonging to whole groups (classes) of these objects.

6. All the concepts can be divided into the following separate types: single concepts and general concepts. The concept that relates to the only one certain object, separate phenomenon, separate event is called single (individual) concept. The concept that embraces (covers) a group (class) of similar things, objects is called general concept.

7. Each concept has two aspects: the scope (volume) of the concept and the content of the concept. The scope (volume) of the concept is all the objects and phenomena which can be embraced (covers) by given concept. The scope (volume) of the general concepts is expressed in the form of a logical class. The concept content is a set of all the essential sings (features) of objects, phenomena embraced (covered) by the concept.

8. All the concepts can be divided into the following separate types: concrete concepts and abstract concepts. Concrete concept is the concept that relates to groups, classes of objects, phenomena or to the separate objects, phenomena. Abstract concept is the concept of properties of objects or phenomena if these properties are taken as the separate (independent) object of thought and are abstracted from objects.

9. There is a special kind of concepts that is called categories. Categories are the scientific concepts reflecting the most common properties of objects and phenomena, the most common and essential relations and connections in reality. For example, the concepts of "matter", "movement", "content", "form", "causality", "freedom", "necessity", "randomnicity", "essence", "phenomenon" are the categories.

10. There are the following relations between the concepts: identity relation; relation of subordination; relation of collateral subordination; relation of partial coincidence; relation of disagreement. (For example, the relation of disagreement exists between contradictory concepts and opposite concepts).

11. The second, more complicated, form of thought is a proposition. The proposition is the logical form of expression of thought. The proposition is the logical content of grammatical sentence. The proposition is a statement about the objects and phenomena of objective reality. The statement states the existence or absence of certain features (sings) of objects and of phenomena. The proposition has the following two properties: (a) the proposition either asserts or denies (negates); (b) the proposition is either true or false. The proposition is always assertion or negation. The proposition is true if it reflects correctly the reality; and the proposition is false if it reflects incorrectly the reality. Every proposition represents a system of concepts. There are three elements in every proposition: subject, predicate, connective. The subject of the proposition is that what one states about. The predicate of the proposition is that what one states on the subject. The connective is an indication of the relation between subject and predicate. In any proposition, subject and predicate are concepts connected by connective. The connective in any proposition expressed by the word "is" or "is not".

12. The third form of thought is an inference. The inference represents connection of propositions, which makes it possible to derive a new proposition from given one or more propositions. Those propositions from which one derives the new proposition are called premises, and the new proposition derived from the premises is called conclusion. Relation between the premises and the conclusion is relation between reason (basis) and consequence (logical corollary): the premises are the reason (basis) from which the conclusion follows as a consequence (logical corollary). Consequently, the inference is based on the law of sufficient reason.

Depending on number of premises, all the inferences are divided into two groups: immediate inferences and mediated inferences. The immediate inference is the inference in which the conclusion is consequence of one premise. The mediated inference is the conclusion in which a new proposition is derived from two or more propositions.

13. The mediated inferences can be of two types: deductive and inductive. The mediated deductive inference is called syllogism if a conclusion is derived from two premises. The inference is called inductive inference if the premises indicate features of separate objects or groups of separate objects, and the conclusion is extended to other objects of the same kind. Deduction and induction are in inseparable connection with each other and supplement each other. Mathematics uses mainly method of deduction.

14. Scientific induction is based on the determination of the causes. Therefore, the problem of causal connection of phenomena is important for scientific induction. The causal connection of phenomena is that one phenomenon is a cause another phenomenon, and a change in the first phenomenon entails a change in the second phenomenon too. The phenomenon which necessarily entails another phenomenon is called cause, and the second phenomenon which is entailed by this cause is called effect of this cause. Thus, the connection of cause and of effect is a connection of two phenomena, two facts. In order to determine the cause of the phenomenon studied, one should use two basic logical methods of the inductive research: intercomparison of the circumstances in which given phenomenon occurs; comparison of these circumstances (in which given phenomenon occurs) with other circumstances (similar in other relations) in which given phenomenon do not occur.

15. The validity (trueness) of some proposition is determined with the help of proof. The proof is determination of the validity (trueness) of some proposition by the use of other true propositions from which the validity (trueness) of the given proposition follows. The proofs are based on the logical law of sufficient reason. The proof represents an indication of sufficient reason for whatever proposition. Whatever proof consists of three parts: thesis, arguments, demonstration (manifestation). The proposition is called thesis if one proves validity of this proposition. The propositions which are used for the proof of the thesis are called arguments (i.e., sufficient reason). Derivation of thesis from arguments is called demonstration (manifestation). In other words, demonstration (manifestation) is the propositions that show why the given thesis is substantiated (grounded) by the given arguments.

1.2. The basic principles of rational dialectics

1. Rational dialectics (i.e., corrected dialectical materialism) is a science of programmed (predetermined) development: the science of the most common types of connections and laws of the development of the nature, of human society, and of thought. The universal connection exists not only in the material world – in the nature and society – but also in thinking. Connection and interdependence of the forms of thought (for example, concepts) is (in the final analysis) reflection of the universal connection and of interdependence of the phenomena of the objective world in human consciousness. Since concepts are reflection of objects in human consciousness, the concepts are interconnected, and they can not be taken in isolation from each other. Concepts must correspond to the natural and social processes, must reflect their contents.

2. The basic laws of dialectics are as follows: the law of unity and struggle of opposites; law of transition of quantitative changes into qualitative changes; law of negation of negation. The law of transition of quantitative changes into qualitative changes is essential to analyze the foundations of mathematics. There are also the most common laws of dialectics, which do not belong to the basic ones. The paired (relative) categories of dialectics – necessity and chance, possibility and reality, form and content, essence and phenomenon, etc. – are the theoretical reflection of non-basic laws of dialectics. All the laws and categories of dialectics represent forms of thought, forms of cognition of the objective world, forms of reflection of the objective world in the human consciousness.

3. As is known, the cognitive psychical activity of man is performed in the following way (by the scheme):

(sensation, perception, representation) $\rightarrow$
(concept) $\rightarrow$ ***(theory)*** $\rightarrow$ ***(practice).***

Sensation is a result of influence (effect) of the outside world to the sense-organs of man; perception is an immediate (direct) sensuous reflection of the reality in the consciousness of man; representation is an image of an object or phenomenon (which is not perceived at given instant of time) in the consciousness of man. Thinking is carried out with the help of concepts. Concept is the form of thought reflecting and fixing the essential signs (features) of objects and phenomena of objective reality. Theory is a system of concepts.

4. The unity of sensuous and rational moments in the cognition is that sensuous cognition is the starting point, the first stage of the cognitive activity. A man, even at the level of logical thinking, continues to rely on (rest upon) sensuously perceivable material in the form of visual images, of various schemes, of symbols, on sensuous form of language.

5. Material activity of people represents practice. Practice is (first of all) a sensuous-objective activity aimed at satisfying human needs. Theoretical activity is derived from practice. Social practice is a starting and ending points of theory. The unity of theory and of practice is a starting point of epistemology. Practice is a driving force in development of cognition.

6. Social practice is criterion of truth. The criterion of truth can be found neither in the object of cognition nor in the consciousness of the subject. Practice is the experience of all humanity in its historical development. The absoluteness of practice as criterion of truth is that all knowledge proven by practice is an objective truth. But, at every given stage (step) of theoretical study, practice can not corroborate completely or refute all theoretical propositions – in this sense, practice is relative. Only the unity of formal logic and of practice can corroborate completely or refute all theoretical propositions at every given stage (step) of theoretical study.

7. The law of transition of quantitative changes into qualitative changes is essential to analyze the foundations of mathematics. The essence of this law is as follows: quantitative and qualitative changes represent the dialectical unity (interconnection) of the opposite and interdependent aspects.

Quality is inherent determinacy in the objects and phenomena. Quality is the organic unity of the properties, signs (features), and characteristics that makes it possible for to distinguish given object or phenomenon from the other ones. In other words, quality is the unity of structure and of elements. "There are not qualities, but only objects with qualities" (Friedrich Engels). Quality expresses specific character of an object or phenomenon in whole. Quality is not only holistic characteristic but also a relatively stable set of signs (features) which determines the specificity of given object. Quality is holistic characteristic of an object or phenomenon; and the property is one of the aspects (partial characteristics) of the object or phenomenon. Some properties express the qualitative determinacy of the object; other properties express the quantitative determinacy.

Quantity is inherent determinacy in the objects and phenomena, which expresses the number of inherent properties of objects and of phenomena, the sum of component parts of objects and of phenomena, the amount, the degree of intensity, the scale of development, etc. In other words, quantity is determinacy in objects and phenomena, expressed by a number. For example, noting in the object properties such as volume, weight, length, speed, etc., man ascertains simultaneously quantitative expression of these properties as well. The quantities of volume, weight, length, speed, etc. are the quantitative characteristic of these properties.

8. Quality and quantity are dialectically connected. They represent the unity of opposites. The qualitative determinacy does not exist without the quantitative determinacy, and vice versa. The unity of qualitative and quantitative determinacy is manifested in measure. The measure denotes existence of the interdependence of qualitative and quantitative aspects of the object or phenomenon. The measure expresses the limits (boundaries) within which objects and phenomena are themselves. Each state has its own measure. The violation of the measure leads to a change in the state. The transition from one state to another is a movement. Leading place belongs to quality in the unity of qualitative and quantitative determinacy. Quality determines the framework of quantitative changes. The qualitative changes can only result from the quantitative changes (i.e. quantitative movement).

9. The law of transition of quantitative changes into qualitative changes is essential to analyze systems. The important theoretical proposition of system analysis is as follows. The properties of the system determine the properties of the elements; and the properties of the elements characterize the properties of the system. The main problem is that the dependences of properties (qualitative and quantitative determinacy) of the system on number of the elements and on the qualitative and quantitative determinacy of the elements are not reliably known. (From this point of view, the Universe (System) cannot be cognized by mankind (the element of the System).

10. The law of transition of quantitative changes into qualitative changes is essential to analyze the foundations of mathematics. The question of the fundamental applicability of mathematical methods in all the areas of scientific cognition must be decided on the basis of the law of interdependence of qualitative and quantitative determinacy. The following fundamental statement results from this law: the operation of abstraction of quantitative determinacy from qualitative determinacy is inadmissible mental operation.

2. ANALYSIS OF THE CONCEPT OF QUANTITY

The starting (prime, initial, basic) and the central concept of pure and applied mathematics (i.e., the mathematical formalism of the natural sciences) is the concept of quantity. The quantity is a characteristic, property, sign (feature). In pure mathematics, the quantity is the quantitative determinacy expressed by unnamed (dimensionless) numbers. In physics and engineering, the quantity is the quantitative determinacy expressed by named (dimensional) numbers and is called physical quantity.

2.1. The physical quantity

Essential sign (feature, property) of material object determines a concept; and the concept characterizes the essential sign (feature, property) of the material object ***M***. The basic concept in theoretical physics is the concept of physical quantity characterizing the property of the material object ***M***. Physical quantity is a characteristic (essential sign, feature, property) of material object ***M***, which represents the measure: the unity of qualitative determinacy (i.e., dimension) and of quantitative determinacy (i.e., named numbers). Length, area, volume, weight, temperature, speed, force, etc. are the examples of physical quantities (i.e., quantities which have physical meaning). The concepts of length, area, volume, weight, temperature, speed, force, etc. are not identical concepts. There exists the identity relation between concepts of "physical quantity" and "physical measure": concepts of "physical quantity" and "physical measure" are identical ones.

1. The concept of “physical quantity (length, area, volume, weight, temperature, speed, force, energy, etc.) characterizing given (determinate, separate, single) material object ***M***” is single and concrete concept. This concrete physical quantity can be measured and has relevant dimension, for example: metre (m), area (m^2), volume (m^3), kilogram (kg), kelvin (K), speed ($m\,s^{-1}$), newton ($kg\,m\,s^{-2}$), joule ($kg\,m^2\,s^{-2}$). The dimension characterizes the unity of the qualitative and quantitative determinacy of physical quantity. The expression “concrete physical quantity can be measured” implies that concrete physical quantity can take on numerical values.

2. The concept of “physical quantity (length, area, volume, weight, temperature, speed, force, etc.) characterizing uncertainty large number (set) of material objects” is a general and concrete concept. This concrete physical quantity can not be measured because it characterizes not determinate (not individual, not single) material object, but material object in general (i.e., uncertain, indeterminate object). The expression “concrete physical quantity can not be measured” implies that general and concrete physical quantity does not take on numerical values.

In accordance with the law of interdependence of qualitative determinacy and of quantitative determinacy of material object, the qualitative determinacy of the physical quantity does not exist (in practice) without the quantitative determinacy of this quantity. This implies that general concept of “concrete physical quantity” characterizes measure even if this quantity can not be measured.

3. The concept of “physical quantity (length, area, volume, weight, temperature, speed, force, etc.)” is single (individual) and abstract concept if it expresses a property (essential sign) abstracted from given (determinate, separate, individual, single) material object ***M*** (i.e., if the property is considered as an independent object of thought). This abstract physical quantity determining the mentioned concept can not be measured because it exists only in thought.

In accordance with the law of interdependence of qualitative determinacy and quantitative determinacy of material object, the qualitative determinacy of the physical quantity does not exist (in practice) without the quantitative determinacy of this quantity. This implies that the single (individual) concept of “abstract physical quantity” characterizes abstract measure even if this quantity exists only in thoughts.

4. The concept of “physical quantity (length, area, volume, weight, temperature, speed, force, etc.)” is general and abstract concept if it expresses a property (essential sign) abstracted from the indeterminate set of material objects (i.e., if property is considered as independent object of thought). This abstract physical quantity determining the mentioned concept can not be measured because it exists only in thoughts.

In accordance with the law of interdependence of qualitative determinacy and quantitative determinacy of material object, the qualitative determinacy of the physical quantity does not exist (in practice) without the quantitative determinacy of this quantity. This implies that the general concept of "abstract physical quantity" characterizes abstract measure even if this quantity exists only in thoughts.

2.2. The mathematical quantity

As is known, pure mathematics does not operate with the physical quantities. Mathematical concepts and formulations of mathematical propositions and of theories are abstracted from material objects and concrete nature (i.e., physical meaning) of quantities. Mathematical concepts take into account only the numerical values of quantities, i.e., the concepts of “length”, “square”, “volume”, “weight”, “temperature”, “speed”, “force”, etc. are mathematically considered as identical concepts. This implies that length, area, volume, mass, temperature, speed, force, etc. are not the essential signs (features) of the mathematical concepts. In other words, the concept of physical quantity does not exist in pure mathematics; and dimensionless (unnamed, abstract) numbers represent the only sign (feature) of mathematical concepts. This implies that pure mathematics operates with abstract quantity in general, i.e., abstract dimensionless quantity which exists only in thoughts. This abstract dimensionless quantity is called mathematical quantity. In my view, the concept of “mathematical quantity” – the starting (prime, initial, basic) and central concept of pure and applied mathematics – is not free from objections.

2.2.1. Formal-logical objection

Formal-logical objection is as follows.

1. Numbers are a result of measurement of the concrete physical quantity which has dimension and characterizes the determinate single material object. One makes measurement with the help of device destined for measurement

of given concrete physical quantity. But measurement of given concrete physical quantity with the help of device which is not destined for measurement of this concrete physical quantity can not be made (i.e., numbers do not exist in this case). Moreover, the devices which could measure abstract and dimensionless quantities do not exist. This implies that the dimensionless (unnamed, abstract) numbers do not exist in practice: they exist only in thoughts. Therefore, the mathematical quantity represents fictitious, dummy quantity.

2. Within the framework of formal logic, one can give the following genetic definition of the concept of "mathematical quantity" (As is known, the genetic definition of concept shows how given concept arises). The concept of "mathematical quantity" arises as the result of the following mental operations: (a) abstraction of "quantitative determinacy of the physical quantity" from the "physical quantity" under the assumption that "the quantitative determinacy of the physical quantity" is an independent (separate) object of thought; (b) abstraction of "amount (i.e., abstract number)" from the "quantitative determinacy of the physical quantity" under the assumption that "number (i.e., abstract number)" is a independent (separate) object of thought. In this case, unnamed, abstract numbers represent the only sign (feature) of the concept of "mathematical quantity". However, this sign (feature) is not the essential sign (feature) of the material objects. Furthermore, the content of the concept of "mathematical quantity" is equal to zero, and the scope (denotation, volume) of this concept is infinitely large. This implies that the essence of the concept of "mathematical quantity" is that the concept of "mathematical quantity" contradicts to the standard logical definition of concept. Consequently, the mathematical quantity is a fictitious, dummy quantity.

Thus, the concept of mathematical quantity is vacuous, meaningless, fallacious, and inadmissible concept in science because it represents the following formal-logical and dialectical-materialistic errors: negation of the standard proposition of existence of the essential sign (feature) of concept (i.e., negation of existence of the attribute, essence of concept); and negation of existence of measure of a material object.

3. Relation between the concepts of "abstract physical quantity" and of "mathematical quantity" is the relation of disagreement. Really, the relation of disagreement exists between such concepts which have different scopes (volumes) and contents under the stipulation that the signs (features) entering into the content of one concept ("mathematical quantity") deny (negate) the signs (features) entering into the content of another concept ("abstract physical quantity"). Therefore, the concept of mathematical quantity contradicts to the concept of measure and represents a formal- logical error.

Thus, the formal-logical objection is that mathematical quantity represents fictitious, dummy quantity.

4. Mathematical quantities are literal notation of dimensionless (unnamed, abstract) numbers: for example, x and y denote unnamed numbers. Relation between the quantities x and y is determined as follows: (a) the concept of mathematical operation (addition, subtraction, multiplication, division) on quantities is introduced (defined); (the concept of mathematical operation is the dialectical concept that denotes rational action of human); (b) the quantity x is treated as an independent quantity (i.e., argument) on which mathematical operations are made; and the quantity y is treated as the result of whole set of mathematical operations on the quantity x; the quantity y is called dependent quantity (i.e., the function of the argument x). Relationship between the argument and the function is written in the following symbolic form: $y = f(x)$ where the symbol f is literal notation of whole set of mathematical operations on the quantity x; symbol f is called function characteristic. From this point of view, the unnamed (absolute) numbers are the values which quantities x and y take. Therefore, the expression $y = f(x)$ satisfies the formal-logical law of identity:

(absolute number) = (absolute number).

5. The values which argument x and function y take can be ordered with the help of tabular form of notation (representation). But the main form of representation of functions in mathematics is analytical form. For example, the power function has the following analytical form: $y = x^n$, $n = 2, 3, \ldots$. As is known, fundamental

postulate in mathematics is the following assertion: the concepts of mathematical function and of function graph (i.e., geometrical line) are closely connected: the mathematical function $y = f(x)$ generates the graph of the function (i.e., geometrical line), and the graph (i.e., . geometrical line) generates the mathematical function. This assertion signifies that the concepts of function and of function graph are identical ones in standard mathematics. However, in my opinion, this assertion is wrong because: (a) the term "generate" has no formal-logical and dialectical-materialistic meaning; (b) the concepts of function and of function graph are not identical ones.

2.2.2. Dialectical-materialistic objection

Dialectical-materialistic objection is as follows.

1. In accordance with the law of the interdependence (interconnection) of quantitative determinacy and of qualitative determinacy of material object, quantitative determinacy does not exist (in practice) without qualitative determinacy, and vice versa. (As is known, the unity of qualitative and quantitative determinacy of material object is called measure. The measure signifies interdependence (interconnection) of qualitative and of quantitative aspects of material object). Abstraction of the quantitative determinacy from the qualitative determinacy (i.e., rupture of interconnection) signifies negation of the existence of the measure and is therefore inadmissible mental operation.

2. As is known, the use of pure mathematics in the natural sciences is carried out by means of the geometrical, physical, etc. interpretations of mathematical concepts, relationships, and theories. The generally accepted interpretation is that the mathematical quantities in the expression $y = f(x)$ are identified with geometrical (non-metric) or physical (metric) quantities. However, in my opinion, mathematical quantities in the expression $y = f(x)$ are not identical with geometrical (non-metric) or physical (metric) quantities. In other words, the relation between the concepts of "function $y = f(x)$" and "graph of function $y = f(x)$" is not the identity relation. In this connection, the question of criterion of validity (trueness, truth) of mathematical relationships arises. In my opinion, the answer is as follows: practice is criterion of validity (trueness, truth) of physical relationships; physical relationships are criterion of validity (trueness, truth) of metric geometry; metric geometry is criterion of validity (trueness, truth) of non-metric geometry; non-metric geometry is criterion of validity (trueness, truth) of mathematical relationships.

3. Interpretation is the establishment of relation between the concepts of "function" and "graph of function". Geometrical (non-metric) interpretation of mathematical function $y = f(x)$ is based on use of graphical representation of function $y = f(x)$ in the non-metric system of coordinates. Since any non-metric system of coordinates represents a material system, the graph of the function $y = f(x)$ in the coordinate system is a material object. Then the relation between the concepts of "function $y = f(x)$" and "graph of function $y = f(x)$" is the special case of the relation between the concepts of "abstract quantity" and "material object". It is known, the following relations exist between the concepts: identity relation; relation of subordination; relation of collateral subordination; relation of partial coincidence; relation of disagreement (for example, relation of disagreement exists between the contradictory concepts and the opposite concepts). In my opinion, the relation between the concepts of "abstract quantity" and "material object" is the relation of disagreement. Therefore, the relation between these concepts should be analyzed on the basis of the paired categories of dialectics: content and form, essence and phenomenon.

4. The sense of the standard geometrical (non-metric) interpretation of function $y = f(x)$ is most clearly manifested in the special case of the Cartesian coordinate system XOY. The Cartesian non-metric coordinate system XOY represents the material system consisting of two identical rulers (segments of straight lines) X, Y

on the plane XOY. Rulers X and Y are connected as follows: rulers are intersected forming the angle 90°. The point O of intersection of the rulers represents zero of rulers. The set of marks and of unnamed numbers on the rectilinear rulers X and Y forms the non-metric coordinate scales. The numbers on the scales is arranged in ascending order. The grid on the plane XOY represents the grid which is displayed on the computer screen in the drawing mode (Figure 1).

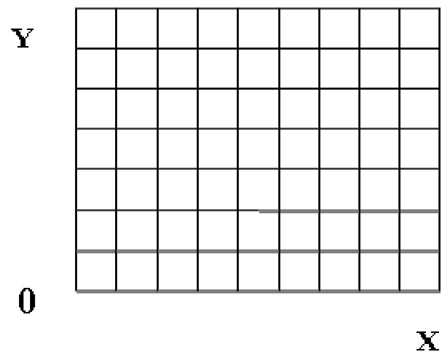

Figure 1. Cartesian non-metric coordinate system.

The grid on the plane is the grid which is displayed on the computer screen in the drawing mode.
Any material object ***M*** on the plane XOY is characterized by coordinates x_M and y_M (i.e., straight line segments).

5. Graphic (non-metric) interpretation of function $y = f(x)$ is carried out in the following way: (a) one puts the mathematical quantities x. y (i.e., abstract, unnamed numbers) in correspondence with the segments of material lines (i.e., unnamed numbers on the material lines) – abscissas and ordinates x_M, y_M of points of the material object ***M*** in non-metric system XOY; (b) one puts the segments of material lines (i.e., unnamed numbers on the material lines) – abscissas and ordinates x_M, y_M of points of the material object ***M*** in non-metric system XOY – in correspondence with the mathematical quantities x. y (i.e., abstract, unnamed numbers). (Note: the ordered numbers on the scales X and Y represent the unnamed numbers. Ordering of numerical values of mathematical quantities x, y is carried out by means of numbers on the scales X, Y).

6. The concepts of "correspondence" and "establishment of correspondence" can not be defined within the framework of formal logic and of materialistic dialectics. From the formal-logical and dialectical-materialistic points of view, the concept of "correspondence" does not signify "identity": the mathematical (abstract) quantities x, y do not characterize the material point ***M*** in the system XOY; the quantities x_M, y_M which characterize material point ***M*** in the system XOY are not mathematical (abstract) quantities.

However, standard mathematics postulates that the relationships $x \equiv x_M$, $y \equiv y_M$, $y_M = f(x_M)$ represent the graphical interpretation of function $y = f(x)$. This postulate contains formal-logical contradiction which is that the mathematical (i.e. abstract) quantities x, y are not abstracted from material objects and characterize the material object. The formal-logical contradiction is eliminated if the law of lack (absence) of contradiction is satisfied: $x \neq x_M$, $y \neq y_M$. This law expresses the fact that the quantities x, y and x_M, y_M have different meanings.

Therefore, the true meaning of the graphic (material) interpretation of function $y = f(x)$ is the following non-identical replacement: $x \to x_M$, $y \to y_M$, $y = f(x) \to y_M = f(x_M)$ under the stipulation that $x \neq x_M$, $y \neq y_M$.

7. The process of geometrical construction an object (point, line, and figure) in the non-metric system XOY is a material activity of man. Therefore, any object (point, line, and figure) constructed in the non-metric system XOY is a material object not having measure (i.e., not having qualitative determinacy). If the material object has no qualitative determinacy, then the difference between big and small objects in the coordinate system XOY is expressed by the number of cells in (into) the object. An explanation is that the big cell is identical to the small cell in the non-metric system XOY. In this case, the graph of the function represents only material form (i.e., the material manifestation of the essence) of the function $y = f(x)$, and the function $y = f(x)$ represents the abstract content (i.e., the essence) of the material object.

8. The concepts of "essence" and of "phenomenon" are not identical ones and are paired categories of dialectics. In this point of view, the concepts of "function $y = f(x)$" and "graph of function $y = f(x)$" are not identical ones and satisfy the formal-logical law of lack (absence) contradictions:

"function $y = f(x)$" ≠ "graph of function $y = f(x)$"

because

"concept of essence" ≠ "concept of phenomenon".

Thus, the correct interpretation of the relation between the concepts of "function $y = f(x)$" and "graph of function $y = f(x)$" represents the assertion that function $y = f(x)$ is the abstract essence of the material object, and graph of function $y = f(x)$ is the material manifestation of the abstract essence. This statement is written in the following symbolic form:

$$x \to x_M, \quad y \to y_M, \quad y = f(x) \to y_M = f(x_M), \quad x \neq x_M, \quad y \neq y_M.$$

For example:

(a) The linear function (i.e., abstract essence) $y = ax + b$ is manifested in the form of straight line (i.e., material line) in the non-metric system XOY.

(b) The quadratic function (i.e., abstract essence) $y = ax^2 + bx + c$ is manifested in the form of parabola (i.e. material line) in the non-metric system XOY.

(c) The parabolas, lemniscates, conchoids, cycloids, etc. exist as the material lines in the non-metric system XOY.

9. The following assertions are logical corollary of this result: if graph is material manifestation of the essence of mathematical function $y = f(x)$, then the graph represents the geometrical (non-metric) criterion of validity (trueness, truth) of mathematical function $y = f(x)$; if set of non-metric subsystems XOY, XOZ, and YOZ forms full material system $OXYZ$, then there is only three-dimensional geometry in reality, and multi-dimensional geometry is a mathematical fiction, fabrication.

Metric geometry, geometrical (metric) and physical (metric) interpretations of mathematical function $y = f(x)$ represent the criterion of validity (trueness, truth) of non-metric geometry and of geometrical (non-metric) interpretation of mathematical function $y = f(x)$.

10. The correct geometrical (metric) and physical (metric) interpretations of mathematical function $y = f(x)$ are carried out in the following way: one puts mathematical quantities x, y in correspondence with physical (metric) quantities x_M, y_M which characterize the material object ***M***. The concept of "correspondence between mathematical (abstract) quantities x, y and physical (metric) quantities x_M, y_M" signifies the relation between essence and phenomenon, respectively. Therefore, the concept of "correct interpretation" signifies the following replacement of quantities: $x \to x_M$, $y \to y_M$ under the condition that $x \neq x_M$, $y \neq y_M$. In this case, the relationship between physical quantities has the following form:

$$y_M = f(x_M).$$

The criterion of validity (trueness, truth) of the relationships between physical (metric) quantities is practice.

11. If physical quantities x_M, y_M have the dimension of length ("metre"), then the geometrical (metric, practical) interpretation of the physical formula $y_M = f(x_M)$ represents criterion of validity (trueness, truth) of physical formula. The geometrical (metric) interpretation of physical formula $y_M = f(x_M)$ in the Cartesian metric coordinate system XOY is possible under the following conditions: the coordinate scales X and Y have identical dimensions of length ("metre"); numbers on the scales X and Y represent ordered concrete numbers; one puts physical quantities x_M, y_M in correspondence with the abscissa and ordinate of the material point ***M***; the establishment of correspondence leads to ordering of values taken by the physical quantities x_M, y_M. The process of construction an object (point, line segment, and figure) in the system XOY represents a material activity of man. Therefore, any object (point, line segment, and figure) constructed in the system XOY is a material and concrete object. The length of any line segment constructed in the system XOY has dimension of "metre". If length of any line segment had a different dimension (or had no dimension), then the line segment would not exist in the system XOY. In other words, points of this line segment would not exist and would not have the projections in the system XOY.

12. Scales X and Y cannot have different dimensions. If the scale X had dimension of "metre" and the scale Y had the dimension of "square metre" (or "cubic metre", "kilogram", "second", "newton", "degree", etc.), then the point O of intersection of the scales would have both dimensions at the same time: "metre" and "square metre". Also, the scales X and Y under superimposition would have both dimensions at the same time. However, it would be contradict to formal logic and practice because the scales X and Y cannot be superimpose. Consequently, the scales X and Y of the system XOY must be identical to each other. And scales having different dimensions are not the elements of the system of coordinates. Therefore, the set of values of quantities "square metre", "cubic metre", "kilogram", "second", "newton", "degree", etc. cannot form a line, figure, and plane in the system XOY. The line, figure, and the plane are not the geometrical locus for values of such quantities. The values of such quantities cannot be ordered in the system XOY.

13. In accordance with the formal-logical law of identity, the left-hand and right-hand sides of physical relationship $y_M = f(x_M)$ must have identical qualitative determinacy (i.e., identical dimension). Some examples explaining this requirement are as follows.

<u>Example 1</u>.

Linear and exponential functions

$$y_M = k\,x_M,$$

$$y_M = a_M \exp(x_M / b_M), \quad y_M = a_M \exp(-x_M / b_M)$$

(where quantities x_M and y_M have the dimension of "metre"; k is a dimensionless coefficient; a_M and b_M are coefficients having dimension of "metre") satisfy the formal-logical law of identity:

(dimension of length) = (dimension of length).

(The dimensionless coefficient k (i.e., unnamed number) does not exist in the system XOY, but the quantity $k\,x_M$ (having dimension) exists in the system XOY). These physical relationships are materially manifested (i.e., graphs exist) in the system XOY.

<u>Example 2</u>.

Power function $y_M = x_M^2$ does not satisfy the formal-logical law of identity,

(dimension of length) = (dimension of length),

if x_M and y_M have dimension of length ("metre"). If x_M and y_M have the dimension of length ("metre"), then this function is not manifested (i.e., graph does not exist) in the system XOY. If y_M has dimension of area, then the function $y_M = x_M^2$ satisfies the formal-logical law of identity:

(dimension of area) = (dimension of area).

But in this case, the power function is not manifested (i.e., graph does not exist) in the system XOY.

Example 3.

Quadratic function $y_M = a x_M^2 + b x_M + c$ does not satisfy the formal-logical law of identity,

(dimension of length) = (dimension of length),

if x_M and y_M have dimension of length ("metre"), and coefficients have no dimension. In this case, the quadratic function are not manifested (i.e., graph does not exist) in the system XOY.

If x_M and y_M have dimension of length ("metre") and coefficients have the dimensions

$$[a] = 1/m, \quad [b] = "No", \quad [c] = m,$$

then the quadratic function satisfy the formal-logical law of identity. In this case, the quadratic function is manifested (i.e., graph exists) in the system XOY.

If y_M has dimension of area, and coefficients have dimensions

$$[a] = "No", \quad [b] = m, \quad [c] = m^2,$$

then the quadratic function satisfy the formal-logical law of identity:

(dimension of area) = (dimension of area).

But, in this case, the quadratic function is not manifested (i.e., graph does not exist) in the system XOY.

Example 4.

Equation of circle $x_M^2 + y_M^2 = r_M^2$ does not satisfy the formal-logical law of identity,

(dimension of length) = (dimension of length),

and, therefore, the equation is not manifested (i.e., graph does not exist) in the system XOY. This equation satisfies the formal-logical law of identity:

(dimension of area) = (dimension of area).

But, in this case, the equation is not manifested (i.e., graph does not exist) in the system XOY.

The equation of circle is manifested (i.e., graph exists) in the system XOY if and only if this equation has linear form:

$$y_M = \sqrt{r_M^2 - x_M^2}.$$

In this case, the equation satisfies the formal-logical law of identity:

(dimension of length) = (dimension of length).

14. In accordance with the formal-logical law of lack (absence) contradiction, left-hand and right-hand sides of physical relationship $y_M = f(x_M)$ must not have different qualitative determinacy (i.e., dimension). But if the left-hand and right-hand sides of physical relationship $y_M = f(x_M)$ have different qualitative determinacy (i.e., dimension), then it signifies impossibility of physical interpretation of mathematical function $y = f(x)$. Example explaining this statement is as follows: semicubical parabola, lemniscates, conchoids, cycloids, etc. can not be physically interpreted. In this case, mathematical function is a fiction.

It follows from this that the operations on physical quantity x_M such as

$$x_M^n, \quad x_M^{(1/n)}, \quad n = 2, 3, ...$$

lead to a change in the qualitative determinacy of the physical quantity because the quantities x_M, x_M^n, and $x_M^{(1/n)}$ have a different dimension. This implies that such operations are not mathematical (i.e. quantitative) ones, but physical (qualitative) operations.

Thus, the dialectical-materialistic objection is that mathematical quantity and relationships between mathematical quantities do not satisfy the physical criterion of truth. In order that mathematical quantity and relationships between mathematical quantities will satisfy the physical criteria of truth, one must make physical interpretation of mathematical quantity and relationships between mathematical quantities. The physical interpretation leads to the fact that mathematical quantity and relationships between mathematical quantities are transmuted (converted, transformed) into physical quantity and relationships between physical quantities. In other words, the interpretation is the restoration of measure, i.e., restoration of the unity of qualitative and of quantitative determinacy.

3. DISCUSSION

1. As is known, formal logic is the general science of the laws of correct thought. The laws of formal logic represent the theoretical generalization and reflection of practice in human consciousness. Consequently, formal logic exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of formal logic.
2. Dialectical materialism is the general science of the most common (general) kinds of connections and laws of development of nature, of human society, and of thought. The laws of dialectics represent the theoretical generalization and reflection of practice in human consciousness. Consequently, dialectics exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of dialectics.
3. The only correct methodological basis of science is the unity of formal logic and of rational dialectics. Mathematics is a science if and only if its foundations are formulated within the framework of correct methodological basis.
4. Pure mathematics is partial, special, non-general, non-common, and abstract science. Today, there is no complete understanding of the essence of pure mathematics. In my opinion, the essence of mathematics can be understood only within the framework of correct methodological basis. The metric-geometrical and physical interpretations of mathematical concepts, theorems, and theories within the framework of correct methodological basis disclose the essence of mathematics.
5. The concept of mathematical quantity is starting and central concept in mathematics. As the critical analysis shows, mathematical quantity and relationships between mathematical quantities do not satisfy the criterion of truth. In order that mathematical quantity and relationships between mathematical quantities satisfy the physical criterion of truth, one must interpret physically mathematical quantity and relationships between mathematical quantities. Interpretation is operation of restoration of measure, i.e., restoration of the unity of qualitative and quantitative determinacy of material object. Mathematics without operation of restoration of measure represents a fiction, a useless intellectual game.
6. In my opinion, the correct definition of the essence of mathematics is as follows. Correct mathematics is a concretization of the basic principles of materialistic dialectics: the correct mathematics is the science of quantitative determinacy of the measure of a material object.

CONCLUSION

Thus, the correct analysis of the foundations of mathematics is possible only within the framework of correct methodological basis: the unity of formal logic and of rational dialectics. The analysis leads to the following results:

1) the concept of mathematical quantity – the starting and the central concept in pure mathematics – is meaningless, mistaken, and inadmissible concept in science because it contradicts to the basic propositions (statements) of formal logic and of materialistic dialectics. The concept of mathematical quantity represents the following the formal-logical and dialectical-materialistic errors: negation of the existence of the essential sign (feature) of concept (i.e., negation of existence of the essence (content, intension) of concept) and negation of existence of measure of material object;

2) criterion of validity (trueness, truth) of pure mathematics is metric-geometrical and physical interpretations of mathematical concepts, theorems, and theories; and criterion of validity (trueness, truth) of metric geometry, physical concepts, and physical theories is practice;

3) the correct geometrical and physical interpretations of mathematical relationships are the operation of establishing the relation between mathematical and physical quantities. Interpretation is operation of restoration of measure (i.e., interpretation is operation of restoration of the unity of qualitative and of quantitative determinacy) of material object. Mathematics without operation of restoration of measure can not be used in the natural sciences and is useless intellectual game.

4) the correct metric-geometrical and physical interpretations of mathematical relationships show that a set of standard mathematical functions represents error, mistake, and blunder.

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