Abstract. Ampère’s Circuital Law in its differential form can be obtained by taking the curl of the Biot-Savart Law. The question is whether or not this will expose Maxwell’s displacement current explicitly. This matter will be investigated in both the Lorenz gauge and the Coulomb gauge.

The Biot-Savart Law

I. The argument begins with the Biot-Savart Law in the form,

\[ \mathbf{B} = \mu/4\pi \int_V (\mathbf{J}dV)\times\hat{r}/r^2 \]  \hspace{1cm} (1)

Equation (1) is a measure of the magnetic flux density, \( \mathbf{B} \), at a point in space due to a region of electric current density, \( \mathbf{J} \), at a distance \( r \) from the point in question. Dr. Zhong-Cheng Liang of the Nanjing University of Posts and Telecommunications recently wrote a paper entitled “Dark matter and real-particle field theory”, [1], in which he takes the curl of an equation which has the same mathematical form as equation (1), and he obtains a result which has a mathematical form equivalent to Ampère’s Circuital Law,

\[ \nabla\times\mathbf{B} = \mu (\mathbf{J} + \varepsilon \partial\mathbf{E}_S/\partial t) \] \hspace{1cm} (2)

where,
\[ \nabla \cdot E_S = \frac{\rho}{\varepsilon} \]  

(3)

Dr. Liang intends his equations to apply to gravity and not to electromagnetic theory, nevertheless, his ingenious mathematical manipulations have indicated the existence of an extra \( \varepsilon \frac{\partial E_S}{\partial t} \) term in Ampère’s circuital law that is not picked up in the textbooks when the curl of the Biot-Savart law is taken. This extra term becomes significant in the vicinity of electric currents where the charge density is changing in time, such as on the plates of a capacitor. One will no doubt immediately counter that this extra term is none other than Maxwell’s displacement current, but they will miss the point. The textbooks add Maxwell’s displacement current to Ampère’s circuital law \textit{only after} the curl of the Biot-Savart law has already been taken, and besides, \( \varepsilon \frac{\partial E_S}{\partial t} \) cannot be Maxwell’s displacement current anyway, since the electrostatic \( E_S \) is not compatible with Faraday’s law.

Maxwell’s Displacement Current

II. Dr. Liang was working in the \textit{Lorenz gauge}, which means that his starting point was a momentum equation in the mathematical form,

\[ A_L = \frac{\mu}{4\pi} \int_V (JdV)/r \]  

(4)

We might therefore refer to the term \( \varepsilon \frac{\partial E_S}{\partial t} \) as \textit{Displacement Current in the Lorenz gauge}, but it’s not Maxwell’s displacement current. Maxwell, when deriving the electromagnetic wave equation in connection with displacement current, used the Coulomb gauge. Displacement current needs to be compatible with \textit{Faraday’s Time Varying Law of Induction} if it is to be used to derive the electromagnetic wave equation, and this means that it should take the form \( \varepsilon \frac{\partial E_K}{\partial t} \), such that,

\[ E_K = -\frac{\partial A_C}{\partial t} \]  

(5)

where \( A_C \) is the electromagnetic momentum/magnetic vector potential in the Coulomb gauge. Maxwell’s displacement current is in fact a special case of the \( J \) term in equation (2). It is not the additional \( \varepsilon \frac{\partial E_S}{\partial t} \) term. In order to justify displacement current as \( J = \varepsilon \frac{\partial E_K}{\partial t} \), we need to work in the Coulomb gauge where \( A_C = J \) and where \( \nabla \cdot A_C = 0 \). We also need the agency of Maxwell’s all-pervading sea of tiny molecular vortices, [2], [3], [4], where we will identify \( A_C \) with the circumferential
momentum circulating around the edge of the vortices. It’s then just a simple case of defining $B$ as the vorticity of this circulating current, as in,

$$\nabla \times A_C = B$$

Then further taking the curl of $B$, this expands to,

$$\nabla \times \nabla \times A_C = \nabla (\nabla \cdot A_C) - \nabla^2 A_C$$

Dr. Liang demonstrated, *in the Lorenz gauge*, the equivalent of,

$$\nabla (\nabla \cdot A_L) = \varepsilon \partial E_S / \partial t$$

Hence, in the Coulomb gauge where $\nabla \cdot A_C = 0$, we can drop the $\varepsilon \partial E_S / \partial t$ term and equation (2) then reduces to the simple form,

$$\nabla \times B = \mu J$$

We then simply equate $J$ with the displacement current $\varepsilon \partial E_K / \partial t$, since $J = A_C = -\varepsilon \partial^2 A_C / \partial t^2$. Displacement current is an oscillatory phenomenon in the dynamic state, [5].

**Conclusion**

III. In the Lorenz gauge, in regions of space where there is zero electric current density, the differential form of Ampère’s Circuital Law cannot be used for the purposes of deriving the electromagnetic wave equation in the manner that James Clerk Maxwell did. The current density term, $J$, only applies to the source current and not to the space beyond where we are investigating the physical nature of the wireless electromagnetic radiation. Danish physicist Ludwig Lorentz himself realized that light is an electric current, but he assumed space to be merely a bad conductor rather that a physical medium for the propagation of light waves, [6]. Maxwell’s approach on the other hand was quite different. While Maxwell acknowledged that at a point in space where wireless electromagnetic waves are passing through, we need a real electric current to be physically present at that location, he solved this matter by connecting this displacement current with the elasticity in an all-pervading background dielectric medium.
A variation of the Biot-Savart Law will now be used in order to explain Maxwell’s displacement current. See Appendix I for the details. In this case, the magnetic flux density, $B$, is based on the vorticity of the tiny fine-grained aethereal vortices, that according to Maxwell, pervade all of space. The circumferential current density circulating around the edge of Maxwell’s tiny vortices is denoted here by $A_C$, and being transverse to the polar origin at the centre of these vortices, the divergence of $A_C$ must be zero, and so we will be working in the Coulomb gauge. In the dynamic state when radiation is passing through, the vortices are undergoing an oscillatory angular acceleration, and electric fluid (aether) of which the dipolar vortices are comprised, is being transferred from vortex to vortex, [7], [8]. Electromagnetic radiation is therefore an aether momentum, and due to its oscillatory nature, it can be equated with $-\varepsilon \partial^2 A_C / \partial t^2$ or $\varepsilon \partial E_K / \partial t$, where $E_K$ is the electromotive force, $-\partial A_C / \partial t$, generated in line with Faraday’s time-varying law of induction, and where $\varepsilon$ is the elasticity factor. The term $\varepsilon \partial E_K / \partial t$ is Maxwell’s Displacement Current.

This results in Ampère’s circuital law adopting the simple form,

$$\nabla \times B = \mu \varepsilon \partial E_K / \partial t$$

with the Coulomb gauge guaranteeing that both sides of the equation will have zero divergence.

While electromagnetic radiation operates in the Coulomb gauge, the Lorentz transformations on the other hand operate in the Lorenz gauge. The Lorentz transformations are mathematically equivalent to a classical longitudinal Doppler shift in the frequency of a light beam over a return path, but they actually relate to the frequencies and dimensions of the constituent molecular vortices that comprise the light carrying medium and how these are distorted by the interaction of bodies in motion along with their associated electrostatic and electromagnetic fields. The rigid solid characteristic of the sea of vortices is manifested in the context of EM waves to the extent that there is no deformation, and so the radial inter-particle bonding forces are not invoked in the wave analysis, whereas when it comes to matter in motion through the sea of vortices, the interaction is of a different nature, and the vortex sea exhibits a more fluid-like characteristic in which these forces do become significant.
References


[3] O’Neill, John J., “PRODIGAL GENIUS, Biography of Nikola Tesla”, Long Island, New York, 15th July 1944, Fourth Part, paragraph 23, quoting Tesla from his 1907 paper “Man’s Greatest Achievement” which was published in 1930 in the Milwaukee Sentinel, “Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance”.

[4] Whittaker, E.T., “A History of the Theories of Aether and Electricity”, Chapter 4, pages 100-102, (1910) “All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools. ”


[7] The 1937 Encyclopaedia Brittanica article on ‘Ether’ discusses its structure in relation to the cause of the speed of light. It says, “POSSIBLE STRUCTURE. ___ The question arises as to what that velocity can be due to. The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would
transmit waves of the same general nature as light waves _i.e., periodic disturbances across the line of propagation_ and would transmit them at a rate of the order of magnitude as the vortex or circulation speed - - - -”


https://www.researchgate.net/publication/335169091_Wireless_Radiation_Beyond_the_Near_Magnetic_Field

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See also “The Double Helix and the Electron-Positron Aether” (2017)


Appendix I
(The Biot-Savart Law in the Coulomb Gauge)

“The Double Helix Theory of the Magnetic Field” [9], is essentially Maxwell’s sea of aethereal vortices but with the vortices replaced by rotating electron-positron dipoles. Within the context of a single rotating electron-positron dipole, the angular momentum can be written as \( \mathbf{H} = \mathbf{D} \times \mathbf{v} \), where \( \mathbf{D} \) is the displacement from the centre of the dipole and \( \mathbf{v} \) is the circumferential velocity. When elastically bonded to other dipoles within the wider electron-positron sea, the displacement \( \mathbf{D} \) will be related to the transverse elasticity through Maxwell’s fifth equation, \( \mathbf{D} = \varepsilon \mathbf{E} \). A full analysis can be seen in the articles “Radiation Pressure and E = mc²” [10], and “The 1855 Weber-Kohlrausch Experiment” [11]. If we substitute \( \mathbf{D} = \varepsilon \mathbf{E} \) into the equation \( \mathbf{H} = \mathbf{D} \times \mathbf{v} \), this leads to,

\[
\mathbf{H} = -\varepsilon \mathbf{v} \times \mathbf{E}_C
\]  

(11)

See Appendix II regarding why the magnitude of \( \mathbf{v} \) should necessarily be equal to the speed of light. Equation (11) would appear to be equivalent to the Biot-Savart Law if \( \mathbf{E}_C \) were to correspond to the
Coulomb electrostatic force. However, in the context, \( \mathbf{E}_C \) will be the centrifugal force, \( \mathbf{E}_C = \mu \mathbf{v} \times \mathbf{H} \), and not the Coulomb force. If we take the curl of equation (11) we get,

\[
\nabla \times \mathbf{H} = -\varepsilon [\mathbf{v}(\nabla \cdot \mathbf{E}_C) - \mathbf{E}_C(\nabla \cdot \mathbf{v}) + (\mathbf{E}_C \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{E}_C]
\]

(12)

Since \( \mathbf{v} \) is an arbitrary particle velocity and not a vector field, this reduces to,

\[
\nabla \times \mathbf{H} = -\varepsilon [\mathbf{v}(\nabla \cdot \mathbf{E}_C) - (\mathbf{v} \cdot \nabla)\mathbf{E}_C]
\]

(13)

Since \( \mathbf{v} \) and \( \mathbf{E}_C \) are perpendicular, the second term on the right-hand side of equation (13) vanishes. In a rotating dipole, the aethereal flow from positron to electron will be cut due to the vorticity, the separate flows surrounding the electron and the positron will be passing each other in opposite directions, and so the Coulomb force of attraction will be disengaged. Hence, the two particles will press against each other with centrifugal force while striving to dilate, since the aether can’t pass laterally through itself, and meanwhile the two vortex flows will be diverted up and down into the axial direction of the double helix, [9]. Despite the absence of the Coulomb force in the equatorial plane, \( \mathbf{E}_C \) is still nevertheless radial, and like the Coulomb force, as explained in Appendix III, it still satisfies Gauss’s Law, this time with a negative sign in the form,

\[
\nabla \cdot \mathbf{E}_C = -\rho / \varepsilon
\]

(14)

Substituting into equation (13) leaves us with,

\[
\nabla \times \mathbf{H} = \rho \mathbf{v} = \mathbf{J} = \mathbf{A}_C
\]

(15)

and hence since \( \mathbf{B} = \mu \mathbf{H} \) then,

\[
\nabla \times \mathbf{B} = \mu \mathbf{J} = \mu \mathbf{A}_C
\]

(16)

which is Ampère’s Circuital Law in the Coulomb gauge as per equation (9).
Appendix II
(The Speed of Light)

Starting with the Biot-Savart law in the Coulomb gauge, \( \mathbf{H} = -\epsilon \mathbf{v} \times \mathbf{E}_C \), where \( \mathbf{E}_C = \mu \mathbf{v} \times \mathbf{H} \), means that we can then write \( \mathbf{H} = -\epsilon \mu \mathbf{v} \times (\mathbf{v} \times \mathbf{H}) \). It follows therefore that the modulus \( |\mathbf{H}| \) is equal to \( \epsilon \mu \mathbf{v}^2 \mathbf{H} \) since \( \mathbf{v} \), \( \mathbf{E}_C \), and \( \mathbf{H} \) are mutually perpendicular within a rotating electron-positron dipole. Hence, from the ratio \( \epsilon \mu = 1/c^2 \), it follows that the circumferential speed \( \mathbf{v} \) must be equal to \( c \) within such a rotating dipole. In other words, the ratio \( \epsilon \mu = 1/c^2 \) hinges on the fact that the circumferential speed in Maxwell’s molecular vortices is equal to the speed of light.

Appendix III
(Gauss’s Law for Centrifugal Force)

Taking the divergence of the centrifugal force, \( \mathbf{E}_C = \mu \mathbf{v} \times \mathbf{H} \), we expand as follows,

\[
\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = \mu [\mathbf{H} \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{H})]
\]

(17)

Since \( \mathbf{v} \) refers to a point particle in arbitrary motion, and not to a vector field, then \( \nabla \times \mathbf{v} = 0 \), and since \( \nabla \times \mathbf{H} = \mathbf{J} = \rho \mathbf{v} \), it follows that,

\[
\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\mu \rho \mathbf{v} \cdot \mathbf{v}
\]

(18)

then substituting \( \mathbf{v} = c \) as per Appendix II,

\[
\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\mu \rho c^2
\]

(19)

and substituting \( c^2 = 1/\mu \epsilon \), this leaves us with,

\[
\nabla \cdot (\mu \mathbf{v} \times \mathbf{H}) = -\rho/\epsilon
\]

(20)

which is a negative version of Gauss’s law for centrifugal force.